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Kelas : B

UTS Kelistrikan dan kemagnetan.

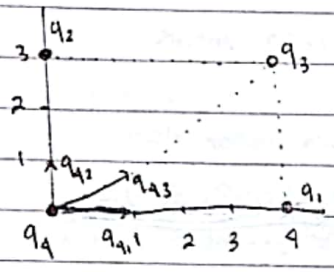
1) Diketahui: $q_1(4,0) = 2$ stat C

$q_2(0,3) = 5$ stat C

$q_3(4,3) = 10$ stat C

$q_4(0,0) = -1$ statC

Tentukan medan listrik pada posisi (0,0), dimana jarak dalam centimeter. dengan $k = 1$ dyne.cm^2/stat.C^2.



a). Gaya Coulomb yang dialami oleh muatan q_4

$$\vec{F} = q_4 k \left\{ \frac{q_1}{r_{01}^2}\hat{r}_{01} + \frac{q_2}{r_{02}^2}\hat{r}_{02} + \frac{q_3}{r_{03}^2}\hat{r}_{03} \right\}$$

$$= -1(1)\left\{ \frac{2}{16}\hat{i} + \frac{5}{9}\hat{j} + \frac{10}{25}\left(\frac{4\hat{i}+3\hat{j}}{5}\right)\right\}$$

$$= -1\left\{ \frac{2}{16}\hat{i} + \frac{40}{125}\hat{i} + \frac{5}{9}\hat{j} + \frac{30}{125}\hat{j}\right\}$$

$$= -\hat{i}\,(0,044\hat{i} + 0,79\hat{j})$$

Jadi, besarnya gaya $F = -\{(0,44)^2 + 0,79)^2\}^{\frac{1}{2}}$ pada arah $-\arctan\left(\frac{0,79}{0,44}\right)$ atau 60,88°.

b). Medan listrik pada titik (0,0) atau muatan q_4

* Medan listrik oleh q_1

$$\vec{E}_1 = k.\frac{q_1}{r_1^2}.\hat{r} = 1.\frac{2}{16}\left(\frac{4\hat{i}}{4}\right) = \frac{8}{64}\hat{i}$$

* Medan listrik oleh q_2

$$\vec{E}_2 = k.\frac{q_2}{r_2^2}.\hat{r}$$

$$\vec{E}_2 = 1.\frac{5}{9}\left(\frac{3\hat{j}}{5}\right)$$

$$\vec{E}_2 = \frac{15}{25}\hat{j}$$

* Medan listrik oleh q_3

$$\vec{E}_3 = k.\frac{q_3}{r_3^2}.\hat{r} = 1.\frac{10}{25}\left(\frac{4\hat{i}+3\hat{j}}{5}\right)$$

$$= \frac{40\hat{i}}{125} + \frac{30\hat{j}}{125}$$

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Medan listrik oleh q_1, q_2 dan q_3

$$\vec{E} = \frac{8}{64}\hat{i} + \frac{15}{27}\hat{j} + \left(\frac{40}{125}\hat{i} + \frac{30}{125}\hat{j}\right)$$

$$= 0{,}44\hat{i} + 0{,}79\hat{j}$$

$\vec{E} = \sqrt{0{,}44^2 + 0{,}79^2}$ pada arah $\text{arc.tg}\left(\frac{0{,}79}{0{,}44}\right)$ atau $60{,}88°$.

c) potensial listrik pada titik (0,0)

$$V = K.\frac{q_1}{r_1} + K.\frac{q_2}{r_2} + K.\frac{q_3}{r_3}$$

$$= 1\left(\frac{2}{4} + \frac{5}{3} + \frac{10}{5}\right)$$

$$= 4{,}166 \text{ dyne.cm/stat c}$$

d). Besar usaha yang dilakukan untuk memindahkan keempat muatan tersebut.

$W_1 = 0$

$$W_2 = K.\frac{q_1}{r_1^2}.q_2 = 1.\frac{2}{5}.5 = 2 \text{ dyne.cm}$$

$$W_3 = 1\left(\frac{2}{3} + \frac{5}{4}\right).10 = 19{,}16 \text{ dyne.cm}$$

$$W_4 = K\left(\frac{q_1}{r_{14}} + \frac{q_2}{r_{24}} + \frac{q_3}{r_{34}}\right).q_4 = 1\left(\frac{2}{4} + \frac{5}{3} + \frac{10}{5}\right).-1 = -4{,}16 \text{ dyne.cm}$$

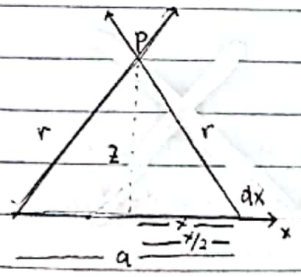
maka,

$$W = W_1 + W_2 + W_3 + W_4$$

$$= 0 + 2 + 19{,}16 + (-4{,}16)$$

$$= 17 \text{ dyne.cm}$$

2).



$$\vec{E}(P) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\, dx}{r^2}\hat{r}$$

$$r = (z^2 + x^2)^{\frac{1}{2}}$$

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2+x^2)^{\frac{1}{2}}}$$

$$* \vec{E}(P) = \frac{1}{4\pi\varepsilon_0}\int_0^{a/2} \frac{2\lambda\, dx}{(z^2+x^2)}.\frac{z}{(z^2+x^2)^{\frac{1}{2}}}.K$$

$$= \frac{1}{4\pi\varepsilon_0}\int_0^{a/2} \frac{2\lambda z}{(z^2+x^2)^{3/2}}\, dx.K$$

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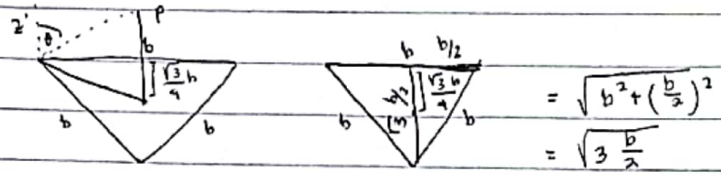
$$= \frac{2\lambda z}{4\pi\varepsilon_0}\left[\frac{x}{z^2\sqrt{z^2+x^2}}\right]\Bigg|_0^{\frac{a}{2}} k$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda a}{z\sqrt{z^2+\frac{a^2}{4}}}\cdot k \Rightarrow \text{medan listrik di titik P}$$

3). P'



$$= \sqrt{b^2+\left(\frac{b}{2}\right)^2}$$

$$= \sqrt{3\,\frac{b}{2}}$$

Menggunakan kawat yang panjangnya 2L

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\lambda L}{z^2}$$

$$P = P'\cos\theta$$

$$P = 3p'\cos\theta$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{3\lambda L}{z^2}$$

$$\vec{E} = 3\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{2\lambda L}{z'^2}\cos\theta$$

$$\vec{E} = 3\,\frac{1}{4\pi\varepsilon_0}\cdot\frac{2\lambda\left(\frac{\sqrt{3}}{4}b\right)\cdot z}{\left(z^2+\left(\frac{\sqrt{3}}{4}b^2\right)\sqrt{z^2+\left(\frac{\sqrt{3}}{4}b\right)^2}\right.}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{3\lambda\,\frac{\sqrt{3}}{2}b\cdot\lambda}{\left(z^2+\left(\frac{3}{16}b^2\right)\sqrt{z^2+\frac{3}{16}b^2}\right.}$$

Karena:

$$L = \frac{\sqrt{3}}{4}b$$

$$z' = r = \sqrt{z^2+\left(\frac{\sqrt{3}}{4}b\right)^2}$$

$$\cos\theta = \frac{z}{\sqrt{z^2+\frac{\sqrt{3}}{4}b}}$$

4). Apa yang anda ketahui tentang Hukum Gauss. (jelaskan secara Verbal, lalu gambar permukaan Gauss, dan jelaskan secara simbolik (rumus) sesuai definisi dan gambar yang anda kemukakan. Dimana hukum Gauss diterapkan, dan beri contoh!

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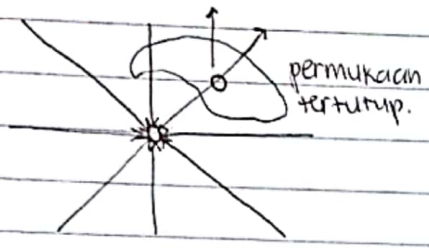
Hukum Gauss.

Hukum Gauss adalah hukum yang menentukan besarnya sebuah fluks listrik melalui sebuah bidang.



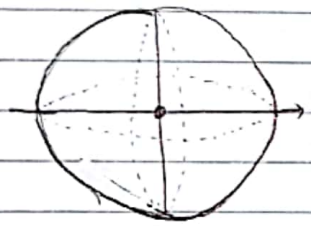
Jumlah garis gaya keluar (fluks listrik total) dari suatu permukaan tertutup sebanding dengan jumlah muatan listrik yang dilingkupi oleh permukaan tertutup itu.

$$\oint \vec{E} \cdot d\vec{A} = \frac{\Sigma q}{\varepsilon_0}$$

$$\oint \vec{E} \cos\theta \, dA = \frac{\Sigma q}{\varepsilon_0}$$

Hukum Gauss berguna untuk menentukan medan listrik disekitar objek-objek yang bermuatan.

- penerapan hukum Gauss
  - Medan pada muatan titik



$$\oint \vec{E} \cdot d\vec{A} = \frac{\Sigma q}{\varepsilon_0}$$

$$\oint \vec{E} \cdot dA = \frac{\Sigma q}{\varepsilon_0}$$

$$E \oint dA = \frac{\Sigma q}{\varepsilon_0}$$

$$E\, 4\pi r^2 = \frac{q}{\varepsilon_0}$$

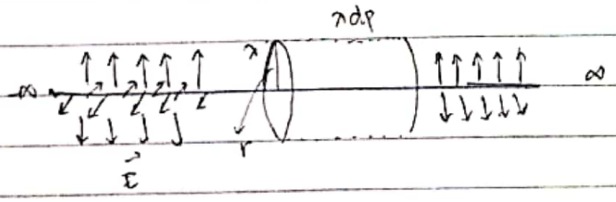
$$E = \frac{q}{4\pi r^2 \varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$ ⇒ medan listrik disekitar muatan.

$$E = \frac{K \cdot q}{r^2}$$

$$E = \frac{1}{4\pi\varepsilon_0} = 8{,}9 \times 10^9 \ N.m^2/C^2$$

- pada kawat lurus panjang dengan rapat-rapat k



a). E(r) pada r > R

b) E(r) pada r < R

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Solusi :

Pada r > R

$$\oint \vec{E}\, d\vec{A} = \frac{1}{\varepsilon_0} \int \lambda\, d\vec{p}$$

$$\vec{E}\,(2\pi r.\ell) = \frac{1}{\varepsilon_0}\lambda\ell$$

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r} \cdot \hat{r}_0 \;;\; r > R$$

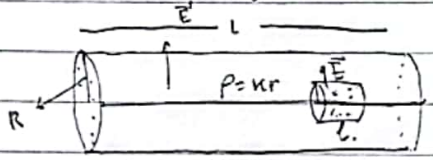
Pada r < R

$$\oint \vec{E}.d\vec{A} = \frac{1}{\varepsilon_0}\int \lambda\, d\vec{p}$$

$$E\,(2\pi r.\ell) = \frac{1}{\varepsilon_0}\lambda\ell$$

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\,\hat{r}_0 \;:\; r < R.$$

- pada silinder dengan jari-jari R



$\vec{E}(r)$ pada r < R

$$\oint \vec{E}.d\vec{A} = \frac{1}{\varepsilon_0}\int_V \rho\, dV$$

$$\vec{E}\,(2\pi r\ell) = \frac{1}{\varepsilon_0}\int_0^{\ell}\int_0^{r}\int_0^{2\pi} (kr)\; r\, d\phi\, dr\, dz$$

$$= \frac{2\pi k}{\varepsilon_0}\int_0^{\ell}\int_0^{r} r^2\, dr\, dz$$

$$= \frac{2\pi k}{3\varepsilon_0}\, r^3 \int_0^{\ell} dz$$

$$E\,(2\pi r\ell) = \frac{2\pi k r^3 \ell}{3\varepsilon_0}$$

$$\vec{E} = \frac{kr^2}{3\varepsilon_0}\cdot\hat{r}_0$$

pada r = R

$$E\,(2\pi RL) = \frac{2\pi k R^3 L}{3\varepsilon_0}\,\hat{r}_0$$

$$E = \frac{1}{3\varepsilon_0} kR^2 \hat{r}_0$$

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pada $r > R$, cara yang sama

$\vec{E}(2\pi rL) = 2\pi k R^3 L \cdot \hat{r}_0$

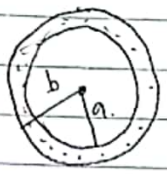
$\vec{E} = \frac{1}{3\varepsilon_0} \frac{kR^3}{r} \cdot \hat{r}_0$

- pada bola konaksial dengan rapat muatan

$\rho = \frac{k}{r^2}$

$E(r)$ pada $r < a$

$E(r) = 0$



$a < r < b$

$\oint \vec{E}\, dA = \frac{1}{\varepsilon_0} \int \rho\, d\tau$

$= \frac{1}{\varepsilon_0} \int \frac{k}{r^2} \cdot r^2 \sin\theta\, d\theta\, dr\, d\phi$

$= \frac{1}{\varepsilon_0} k \int_a^r \int_0^{\pi} \int_0^{2\pi} d\phi \sin\theta\, d\theta\, dr$

$= \frac{4\pi k}{\varepsilon_0} \int_a^r dr$

$E(4\pi r^2) = \frac{4\pi k}{\varepsilon_0} [r - a]$

$\vec{E} = \frac{1}{\varepsilon_0 r^2} k (r-a) \hat{r}_0$

$E(r)$ pada $r > b$

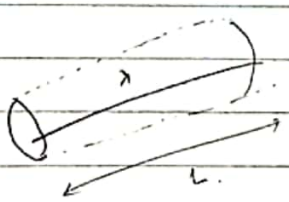
$\vec{E}(4\pi r^2) = \frac{1}{\varepsilon_0} k \int_a^b \int_0^{\pi} \int_0^{2\pi} d\phi \sin\theta\, d\theta\, dr$

$= \frac{4\pi k}{\varepsilon_0} (b-a)$

$\vec{E} = \frac{1}{\varepsilon_0 r^2} \cdot k (b-a) \hat{r}_0$

Contoh Soal.

Kawat lurus sangat panjang memiliki rapat muatan λ. Tentukan medan Listrik yang dihasilkan oleh kawat ini



$q_{enc} = \lambda L$

$E(2\pi r) = \frac{\lambda}{\varepsilon_0}$

$E = \frac{\lambda}{2\pi\varepsilon_0 r} \hat{r}$