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Resume lima pert 6

Potensial Listrik

sifat khusus dari medan listrik $\vec{E}$ yaitu :

$\nabla \times \vec{E} = 0$, artinya medan $\vec{E}$ merupakan gradien suatu skalar,

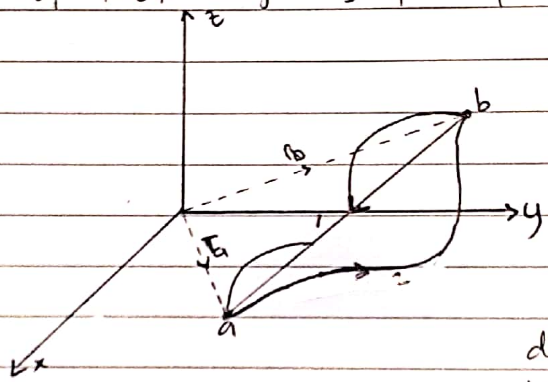
sehingga :

$\vec{E} \sim \nabla \phi$, maka : $\nabla \times \vec{E} = 0$

menurut teorema stokes

$\oint \vec{E}\, d\vec{p} = \int \nabla \times \vec{E}\, d\vec{s} = 0$

artinya, $\oint \vec{E}\, d\vec{p}$ tidak bergantung pada pilihan jalan a ke b



$|r_a - r_b|^2$

Berlaku untuk medan $\vec{E}$ dari suatu muatan titik di titik asal atau dari setiap konfigurasi muatan yg statis.

$\vec{E} = -\nabla V$

$\vec{E}\, d\vec{p} = -\nabla V\, d\vec{p}$

$= -dV$

$\int \vec{E}\, dp = \int_a^b -dV$

$\int \vec{E}\, d\vec{p} = V_b - V_a$

Y tinjauan tertutup,

sehingga, $V_b = V_a$

$V_b - V_a = 0$

sehingga, $\int \vec{E}\, d\vec{p} = 0$

↓

baru lepas stokes

$\int \vec{E}\, d\vec{p} = \int \nabla \times E\, ds$

$d\vec{s} \neq 0$, maka

$\nabla \times \vec{E} = 0$

Jika $V_a \neq V_b$

Bila lintaran a ke b tertutup melalui jalan 1 dan jalan 2 harga $\oint \vec{E}\, dp \neq 0$, karena ($r_1 \neq r_2$) maka, $\vec{E} = -\nabla V$, ($\vec{E}$ mirip gradien potensial)

$E = -\nabla V$

$\nabla \vec{E} = -\nabla \nabla V$

$\nabla \vec{E} = -\nabla^2 V$

$\rho / \varepsilon_0 = -\nabla^2 V$

$\hookrightarrow \nabla^2 V = -\frac{\rho}{\varepsilon_0}$

$E = -\nabla V$

$E\, dp = -\nabla V\, ds$

$\int_a^b E\, dp = \int_a^b dV$

$\int_a^b E\, dp = V_b - V_a$

$\boxed{V_b = V_a}$

$\nabla V\, dp = dv$

$E = -\nabla V$

$\int \vec{E}\, d\vec{p} = -\int \nabla V\, d\vec{p} = -\int dV$, jadi permulaan diatas

$\boxed{V_b = k\frac{Q}{r^2}}$

$\rightarrow V_{(a)} = \int_\infty \vec{E}\, dp$

$V_{(b)} = -\int_\infty^b E\, dp$

$V_{ab} = V_b - V_a = \int_a^b E\, dp + \int_\infty E\, dp = -\int_a^b \vec{E}\, d\vec{p}$

$= -\int_\infty E\, dp - \int_\infty^a \vec{E}\, dp = -\int_a^b \vec{E}\, d\vec{p}$

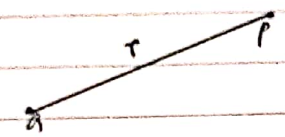
$$V_b - V_a = -\int_a^b \vec{E}\, d\vec{p}$$

$\boxed{V_{(b)} = -\int \vec{E}\, d\vec{p}}$ Rumus umum

Jadi potensial di suatu titik P.

$$V_{(p)} = -\int_{\infty}^{p} \vec{E}\, d\vec{p}$$

Misal :



$$V_{(p)} = -\int_{\infty}^{r} \vec{E}\, d\vec{p}$$

$$= -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}\, dr$$

$$V_{(p)} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$

untuk sejumlah muatan $q_1, q_2, q_3 \dots q_n$ masing² pada jarak $r_1, r_2, r_3 \dots r_n$, maka:

$$V_{(p)} = \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \dots \frac{q_n}{r_n} \right] \Rightarrow \text{penjumlahan biasa}$$

Potensial oleh distribusi muatan.

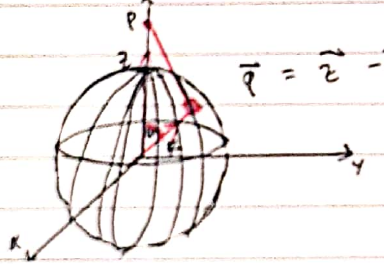
1) untuk kawat $\quad V(p) = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda\, dr}{r}$

2) untuk distribusi oleh permukaan $\quad V(p) = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma\, d\vec{s}}{r}$

3) untuk distribusi oleh volume $\quad V(p) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho\, d\tau}{r}$

Contoh.

① Potensial oleh muatan permukaan bersambung . . .



$\vec{r} = \vec{z} - \vec{R}$

bola konduktor bermuatan homogen dengan rapat σ. Berapa potensial ~~tertentu~~ dititik P.

Solusi : distribusikan muatan pada permukaan bola, maka,

$$V_{(p)} = \int \frac{1}{4\pi\varepsilon_0} \frac{\sigma\, a\vec{\tau}}{r}$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \int \frac{R^2 \sin\theta\, d\theta\, d\phi}{(z^2 + R^2 - 2Rz\cos\theta)^{1/2}}$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0} \int_0^{\pi} \int_0^{2\pi} \frac{d\phi \sin\theta\, d\theta}{(z^2 + R^2 - 2Rz\cos\theta)^{1/2}}$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0} \frac{1}{2} \int_0 \frac{1}{Rz} \frac{d(-2Rz\cos\theta)}{(z^2 + R^2 - 2Rz\cos\theta)^{1/2}}$$

$$= \frac{\sigma R}{2\varepsilon_0 z} \left[(z^2 + R^2 + 2Rz)^{1/2} - (z^2 + R^2 - 2Rz)^{1/2} \right]$$

$$V_{(p)} = \frac{\sigma R}{2\varepsilon_0 z} \left\{ \left[(R+z)^2\right]^{1/2} - \left[(R-z)^2\right]^{1/2} \right\}$$ Bila $z > R$, maka $\left[(R-z)^2\right]^{1/2} = z - R$

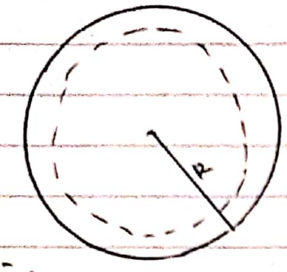
sehingga : $V_{(p)} = \frac{\sigma R}{2\varepsilon_0 z} \left\{ (R+z) - (z-R) \right\} = \frac{\sigma R}{\varepsilon_0 z}$

Bila $R > z$, maka $\left[(R-z)^2\right]^{1/2} = R - z$

$V_{(p)} = \sigma R / 2\varepsilon_0 z \quad \left[(R+z) - (R-z) \right]$

$V_{(p)} = \sigma R / \varepsilon_0$

(2) Potensial oleh muatan volume.



Bola dengan rapat muatan ρ = konstan, dgn jari² R.

Hitung potensial pada :

a) $r < R$

b) $r > R$

Solusi :

$r<R$, $E_1 \Rightarrow \oint E_1\, d\vec{s} = \frac{1}{\varepsilon_0}\rho V$

$$E_1 (4\pi r^2) = \frac{1}{\varepsilon_0}\rho\left(\frac{4}{3}\pi r^3\right)$$

$$\vec{E}_1 = \frac{1}{3\varepsilon_0}\rho r\,\hat{r}_0$$

$r>R$, $E_2 \Rightarrow \oint E_2\, d\vec{s} = \frac{1}{\varepsilon_0}\rho_V$

$$E_2(4\pi r^2) = \frac{1}{\varepsilon_0}\rho\left(\frac{4}{3}\pi R^3\right)$$

$$\vec{E}_2 = \frac{1}{3\varepsilon_0 r^2}\rho R^3\,\hat{r}_0$$

$$V_1 = -\int \vec{E}\, dp = -\int_{\infty}^{R} \vec{E}_2 d\vec{r} - \int_{R}^{r}\vec{E}_1 d\vec{r}$$

$$= \int_{\infty}^{R}\frac{1}{3\varepsilon_0 r^2}\rho R^3 dr - \int_{R}^{r}\frac{1}{3\varepsilon_0}\rho r\, dr$$

$$= -\frac{\rho R^3}{3\varepsilon_0}\int_{\infty}^{R}\frac{dr}{r^2} - \frac{\rho}{3\varepsilon_0}\int_{R}^{r} r\,dr$$

$$= \frac{\rho R^2}{3\varepsilon_0} - \frac{\rho}{3\varepsilon_0}(r^2 - R^2)$$

$$= \frac{\rho R^2}{3\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$V_1 = \frac{\rho R^2}{\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$\frac{2\rho R^2}{6\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0} + \frac{\rho r^2}{6\varepsilon_0}$$

$$\frac{3}{6}\frac{\rho R^2}{\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$= \frac{1}{6\varepsilon_0}(3\rho R^2 - \rho r^2)$$

$$= \frac{\rho}{6\varepsilon_0}(2R^2 - r^2)$$

$$\# \; V_1 = \frac{\rho}{\varepsilon_0}\left(R^2 - \frac{r^2}{6}\right)$$

maka, $V_2 = -\int_{\infty}^{r} E_2\, dr$

$$= -\int_{\infty}\frac{1}{3\varepsilon_0 r^2}\rho R^3 dr$$

$$= -\frac{\rho R^3}{3\varepsilon_0}\int_{\infty}\frac{dr}{r^2}$$

$$\# \; V_2 = \rho R^3 / 3\varepsilon_0 r$$

(3) Potensial oleh muatan pada kawat

a. kawat panjang sekali

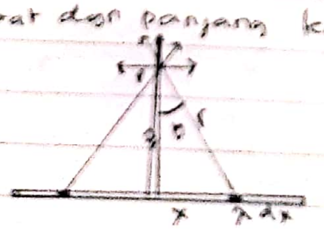
$$E_p = \frac{2\lambda}{4\pi\varepsilon_0}$$

$$V_p = -\int_{\infty}^{a} E\, ap$$

$$= -\frac{2\lambda}{4\pi\varepsilon_0}\int_{\infty}^{z}\frac{dz}{z}$$

$$V_p = -\frac{2\lambda}{4\pi\varepsilon_0}\ln z$$

b. kawat dgn panjang tertentu, misal $2L$ $\quad E_p = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z\sqrt{z^2+L^2}}$



$$V = \frac{2}{4\pi\varepsilon_0}\int_0^L \frac{\lambda\, dx}{r}$$

$$= \frac{2\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(z^2+x^2)^{1/2}}$$

$$\boxed{\begin{aligned} x &= z \operatorname{tg}\theta \\ dx &= z \sec^2\theta\, d\theta \end{aligned}}$$

$$= \frac{2\lambda}{4\pi\varepsilon_0}\int_0^L \frac{z \sec^2\theta\, d\theta}{z(\operatorname{tg}^2\theta+1)^{1/2}}$$

$$= \frac{2\lambda}{4\pi\varepsilon_0}\int_0 \sec\theta\, d\theta$$

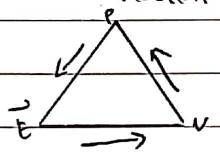
$$= \frac{2\lambda}{4\pi\varepsilon_0}\ln(\sec\theta + \operatorname{tg}\theta) + C$$

$$= \frac{2\lambda}{4\pi\varepsilon_0}\ln\left(\frac{\sqrt{x^2+z^2}}{z} + \frac{x}{z}\right)\Bigg|_0^L$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{\sqrt{L^2+z^2}}{z} + \frac{L}{z}\right)$$

\# Hubungan antara $\vec{E}$, ρ dan V

Dalam elektronika hubungan antara ~~kuat~~ medan dengan rapat muatan volume dan V skalar



$$E = -\nabla V \qquad \nabla \times \vec{E} = 0$$

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \qquad \nabla \cdot \vec{E} = \frac{1}{\varepsilon_0}\rho$$

} syarat elektrostatika

Bentuk integralnya $= \vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ}{r^2}$; $dQ = \rho\, d\tau$

$$= \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{r^2}$$

$V = -\int \vec{E}\, d\hat{\rho}$ hubungan V dengan ρ :

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{r}$$

Solusi segitiga diatas :

1) misal : E diketahui, V dan ρ dapat dihitung

$$V = -\int E\, d\rho$$

$$\rho = \varepsilon_0 \nabla \cdot \vec{E}$$

2) misal : V diketahui, E dan ρ dapat di cari

$$E = -\nabla V$$

$$\rho = -1/\varepsilon_0\, \nabla^2 V$$

3) misal ρ diketahui, E dan V dapat dicari

$$E = 1/4\pi\varepsilon_0 \int \frac{\rho\, d\tau}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{r}$$