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NPM : 2013022002

Kelas : B

28 / 8 / 2022

2.4 Potensial listrik

Syal khusus dari $\vec{E}$ yaitu : $\nabla \times \vec{E} = 0$, ini artinya medan E merupakan gradien suatu skalar. Jadi, $\vec{E} \sim \nabla\phi$, maka : $\nabla \times \vec{E} = 0$.

Menurut teorema stokes $\Rightarrow \oint \vec{E}\, d\vec{p} = \int \nabla \times \vec{E}\, d\vec{s} = 0$.

artinya $\oint \vec{E}\, d\vec{p}$ tidak bergantung pada pilihan jalan a ke b.

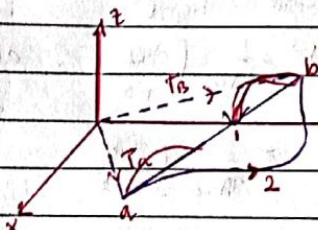
$\rightsquigarrow$ $\vec{E} = -\nabla V$

$E \cdot dp = -\nabla V \cdot d\vec{p}$

$= -dV$

$\int E\, dp = \int -dV$

$\int \vec{E} \cdot d\vec{p} = V_b - V_a$



y (lintasan tertutup)

$\rightsquigarrow V_{(a)} = \int^{a} \vec{E} \cdot d\vec{p}$

$V_{(b)} = -\int_{\infty}^{b} \vec{E} \cdot d\vec{p}$

$\cdot)\ V_{ab} = V_b - V_a = -\int_{\infty}^{b} \vec{E} \cdot dp + \int_{\infty}^{a} \vec{E}\, dp$

$= -\int_a^b \vec{E} \cdot d\vec{p}$

$= -\int_a^b \vec{E}\, d\vec{p}$

Misal :

Q ——T—— P

$V_{(p)} = -\int_{\infty}^{T} \vec{E} \cdot d\vec{p} = -\int_{\infty}^{T} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}\, dr$

$\cdot)\ V(p) = -\int \vec{E} \cdot d\vec{p}$ Rumus Umum :

Jadi, potensial di titik P :

$V_{(p)} = -\int_{\infty}^{P} \vec{E}\, d\vec{p}$

$V_{(p)} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$

Potensial oleh distribusi muatan

1) Untuk kawat $\Rightarrow V_{(p)} = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda\, dr}{r}$

2) Untuk distribusi oleh permukaan $\Rightarrow V_{(p)} = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma\, d\vec{s}}{r}$

3) Distribusi oleh volume $\Rightarrow V(p) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho\, d\tau}{r}$

Contoh :

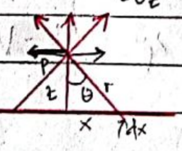
Potensial oleh muatan pada kawat

a) Kawat panjang sekali

$$E_p = \frac{2\lambda}{4\pi\varepsilon_0 z} \Rightarrow V_p = -\int_{\infty}^{a} E \cdot dp = \frac{-2\lambda}{4\pi\varepsilon_0} \int_{\infty}^{z} \frac{dz}{z} = \frac{-2\lambda}{4\pi\varepsilon_0} \ln z$$

b) Kawat dengan panjang tertentu, misalnya $2L$. Dari perhitungan di depan :

$$E_p = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda L}{z\sqrt{z^2 + L^2}}$$



$\cdot)\ V = \frac{2}{4\pi\varepsilon_0} \int_0^L \frac{\lambda\, dx}{r}$

$= \frac{2\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(z^2 + x^2)^{1/2}}$

$x = z \operatorname{tg} \theta$

$dx = z \sec^2 \theta\, d\theta$

$= \frac{2\lambda}{4\pi\varepsilon_0} \int_0^L \frac{z \sec^2\theta\, d\theta}{z(\operatorname{tg}^2\theta + 1)\, L}$

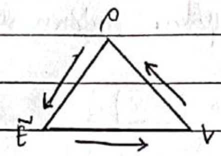
$= \frac{2\lambda}{4\pi\varepsilon_0} \int_0^L \sec\theta\, d\theta$

$= \frac{2\lambda}{4\pi\varepsilon_0} \ln (\sec\theta + \operatorname{tg}\theta) + C$

$= \frac{2\lambda}{4\pi\varepsilon_0} \ln \left(\frac{\sqrt{x^2+z^2}}{z} + \frac{x}{z} \right) \Big|_0^L$

$\therefore \frac{\lambda}{2\pi\varepsilon_0} \ln \left(\frac{\sqrt{L^2+z^2}}{z} + \frac{L}{z} \right)$

2.5 Hubungan antara $\vec{E}$, ρ dan V



$$E = -\nabla V \qquad \nabla \times \vec{E} = 0$$

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \qquad \nabla \cdot \vec{E} = \frac{1}{\varepsilon_0}\rho$$

syarat elektrostatika

$$\rho = \varepsilon_0 \, \nabla . E$$

•) Bentuk integral :

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ}{r^2} \; ; \; dQ = \rho \, d\sigma$$

$$= \frac{1}{4\pi\varepsilon_0}\int \frac{\rho \, d\sigma}{r^2}$$

$$V = -\int \vec{E} \, dp$$

→ Solusi hubungan segitiga diatas.

1) Misal, E diketahui, V dan ρ dapat dihitung.

$$V = -\int E . dp$$

$$\rho = \varepsilon_0 \, \nabla . E$$

2) Misal : V diketahui E dan ρ dapat dicari.

$$E = -\nabla V$$

$$\rho = -\frac{1}{\varepsilon_0}\nabla^2 V$$

3) Misal, ρ diketahui, E dan V dapat dihitung :

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho \, d\sigma}{r^2}$$

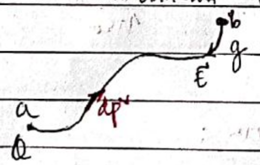
$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho \, d\sigma}{r}$$

2.6 Usaha dan Energi

2.6.1 Pengertian Usaha dan Energi

Usaha adalah kerja yg dilakukan oleh gaya F untuk memindahkan muatan dari suatu tempat ke tempat lain : $W = \int \vec{F} . d\vec{s}$ atau $W = \int (-F) \, d\vec{s}$

Misal : Memindahkan muatan Q dari a ke b



$$\vec{F} = \frac{Qq}{4\pi\varepsilon_0 r^2}\hat{r}_0$$

$$F = -Q.E \; ; \; \vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}_0$$

$$W = \int_a^b (-F) \, dp = -\int_a^b QE . d\vec{p} = -Q\int_a^b \vec{E} \, d\vec{p} \; ; -\int_a^b E \, d\vec{p} = V_b - V_a$$

$\therefore$ $W = Q(V_b - V_a)$

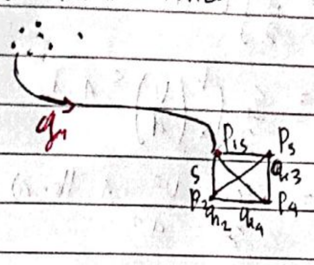
Kalau kita memindahkan muatan dari titik ∞ ke titik P

$$W = -Q\int_\infty^P E . dp$$

$$W = Q . V_p$$

2.6.2 Distribusi Muatan

a) Distribusi muatan titik



$W_1 = q_1 . V_{p_1}$; $V_p = 0$ (mula² tidak ada muatan di P_1)

$W_1 = 0$.

$W_2 = q_2 . V_{p_2}$; $V_{p_2} = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{s}$

$W_3 = q_3 . V_{p_3}$; $V_{p_3} = \frac{q_1}{4\pi\varepsilon_0 s\sqrt{2}} + \frac{q_2}{4\pi\varepsilon_0 s}$

$W_4 = q_4 . V_{p_4}$; $V_{p_4} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{s} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{s\sqrt{2}} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{s}$

Jadi, energi yg diperlukan untuk memindahkan 4 buah muatan seperti diatas adalah.

$$W = W_1 + W_2 + W_3 + W_4$$
$$= 0 + \frac{q_2 . q_1}{4\pi\varepsilon_0 s} + \left(\frac{q_3 q_1}{4\pi\varepsilon_0 s} + \frac{q_3 q_2}{4\pi\varepsilon_0 s\sqrt{2}}\right) + \left(\frac{q_4 q_1}{4\pi\varepsilon_0 s\sqrt{2}} + \frac{q_4 q_2}{4\pi\varepsilon_0 . s}\right) + \left(\frac{q_4 . q_3}{4\pi\varepsilon_0 s}\right)$$
$$= \frac{q_1}{4\pi\varepsilon_0}\left(\frac{q_2}{s} + \frac{q_3}{s} + \frac{q_4}{s\sqrt{2}}\right) + \frac{q_2}{4\pi\varepsilon_0}\left(\frac{q_3}{s\sqrt{2}} + \frac{q_4}{s}\right) + \left(\frac{q_4}{4\pi\varepsilon_0}\right)\left(\frac{q_3}{s}\right)$$
$$= \sum_{i=1}^{4} q_i \frac{1}{4\pi\varepsilon_0} \sum_{j>i}^{4} \frac{q_j}{r_{ji}} = \frac{1}{2}\sum_{i=1}^{4} q_i \sum_{j=1}^{4} \frac{1}{4\pi\varepsilon_0}\frac{q_j}{r_{ij}} = \frac{1}{2}\sum_{i=1}^{4} q_i V_{p_i}$$

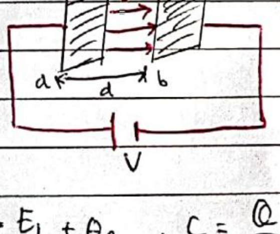
b) Energi distribusi muatan kontinu yg rapat muatan λ → $W = \frac{1}{2}\int \lambda V dl$

c) Energi dg rapat muatan σ → $W = \frac{1}{2}\int \sigma V da$

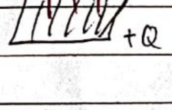
d) Energi dg rapat muatan ρ → $W = \frac{1}{2}\int \rho V d\tau$

2.7. Kapasitor

↳ Konduktor bermuatan $-Q$ dan $+Q$



Perhatikan satu keping yg luasnya A



Garis gaya melalui dua permukaan

$$\oint E\, dA = \oint E_1\, dA + \oint E_2\, dA$$
$$= 2\oint E\, dA$$
$$\frac{Q}{\varepsilon_0} = 2EA$$
$$\frac{\sigma A}{\varepsilon_0} = 2EA$$
$$E_1 = \frac{\sigma}{2\varepsilon_0}$$

$E = E_1 + E_2$; $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$

$= \frac{\sigma}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}$

$= \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

$E = \frac{V}{d}$

$V = E . d$

Energi kapasitor

•) $W_1 = \frac{1}{2}\int \sigma V \, dA$

$= \frac{1}{2}\int \frac{Q}{A} V_1 \, dA$

$= \frac{1}{2} Q V_1$

•) $W_2 = \frac{1}{2}\int -\sigma V \, dA$

$= -\frac{1}{2} Q V_2$

•) $W = W_1 + W_2$

$= \frac{1}{2} Q (V_1 - V_2)$

$W = \frac{\varepsilon_0}{2}\int E^2 \, d\tau$

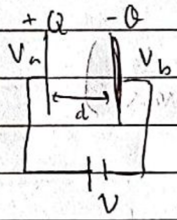
atau $= \frac{\varepsilon_0}{2}\int_a^b \left(\frac{V}{d}\right)^2 A \cdot dx$

$= \frac{\varepsilon_0}{2} \frac{V^2}{d^2} A \, (b-a)$

$= \frac{1}{2} \frac{\varepsilon_0 A}{d} V^2$

$\therefore \; W = \frac{1}{2} Q V^2.$

Contoh



•) $V = V_a - V_b$

$C = \frac{Q}{V}$

$C = \frac{\varepsilon_0 A}{d}$

•) $W = \frac{1}{2} C V^2$

$= \frac{1}{2} Q V$

$W = \frac{1}{2} \frac{Q^2}{C}$