Date

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Prodi : Pendidikan Fisika B

Potensial listrik

Sifat khusus dari medaan listrik $\vec{E}$ yaitu :

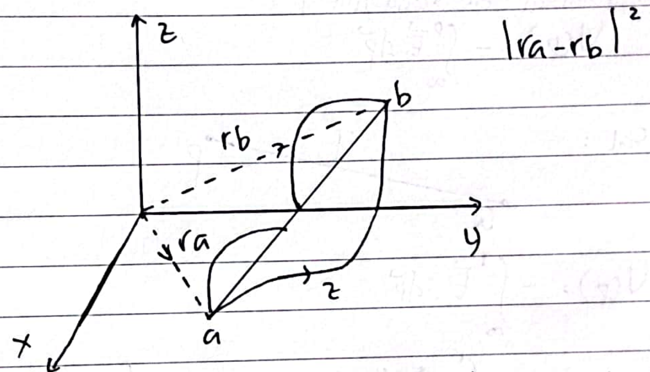
$\nabla \times \vec{E} = 0$, artinya medan $\vec{E}$ merupakan gradien suatu saklar, Jadi,

$\vec{E} \sim \nabla \phi$, maka : $\nabla \times \vec{E} = 0$

menurut teorema stokes :

$\oint \vec{E} d\vec{p} = \int \nabla \times \vec{E} d\vec{s} = 0$ → (caranya setelah lihat gambar)

artinya $\oint_a^b \vec{E} \cdot d\vec{r}$ tidak bergantung pada pilihan jalan a ke b



$\vec{E} = -\nabla V$

$E \cdot d\vec{p} = -\nabla V \cdot d\vec{p}$

$= -dV$

$\int \vec{E} d\vec{p} = \int_a^b -dV$

$\int \vec{E} \cdot d\vec{p} = V_B - V_a$

V_b lintasan tertutup

Hal ini berlaku untuk medan $\vec{E}$ dari suatu muatan titik di titik asal atau dari setiap konfigurasi muatan yg statis

$V_b = V_a$

$V_b - V_a = 0$

$\int \vec{E} \cdot d\vec{p} = 0$ → baru ke pers. stokes

Selanjutnya : $\vec{E} = -\nabla V$ ($\vec{E}$ mrp. gradien potensial)

$E = -\nabla V$

$\nabla \cdot \vec{E} = -\nabla \cdot \nabla V$

$\nabla \cdot \vec{E} = -\nabla^2 V$

$\rho / \varepsilon_0 = -\nabla^2 V$

$\rightarrow \nabla^2 V = \frac{\rho}{\varepsilon_0}$

$E = -\nabla V$

$E \cdot dp = -\nabla V \cdot dp$

$\int_a^b E dp = \int_a^b dV$

$= \int_a^b dV = V_b - V_a$

$V_b = V_a$

$V_b = k \frac{Q}{r^2}$

$\int E \cdot d\vec{p} = \int F \times E ds$

$d\vec{s}$ Fo

$V \times \vec{E} = 0$

Jika $V_a \neq V_b$

VOS

$\nabla V \cdot dp = dV$

$E = -\nabla V$

$\int \vec{E} \cdot d\vec{p} = -\int \nabla V \cdot d\vec{p} = -\int dV$, Jadi Persoalan diatas

$V(a) = -\int^{a} \vec{E} \cdot d\vec{p}$

$V(b) = -\int_{\infty}^{b} \vec{E}\, d\vec{p}$

$V_{ab} = V_b - V_a = -\int_{\infty}^{b} \vec{E} \cdot dp + \int_{\infty}^{a} \vec{E} \cdot d\vec{p} = -\int_{a}^{b} \vec{E} \cdot d\vec{p}$

$= -\int^{b} E\, dp - \int^{\infty} E \cdot d\vec{p} = -\int^{b} \vec{E}\, d\vec{p}$

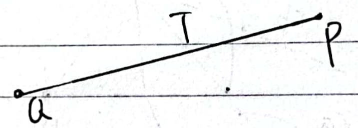
$V_b - V_a = -\int_{a}^{b} \vec{E} \cdot d\vec{p}$

$\boxed{V(b) = -\int \vec{E} \cdot d\vec{p}}$ Rumus umum

Jadi Potensial di suatu titik P

$V(p) = -\int_{\infty}^{P} \vec{E}\, d\vec{p}$

misal :



$V(p) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{p}$

$= -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}\, dr$

$V(p) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$

Untuk sejumlah muatan $q_1, q_2, q_3 \ldots q_n$ masing² pada jarak $r_1, r_2, r_3 \ldots r_n$

maka, $V(p) = \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots \frac{q_n}{r_n} \right]$ → Penjumlahan biasa

Potensial oleh distribusi muatan

1) untuk kawat

$V(p) = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda\, dr}{r}$

2) untuk distribusi oleh permukaan

$V(p) = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma\, d\vec{s}}{r}$

3) distribusi oleh volume

$V(p) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho\, d\tau}{r}$

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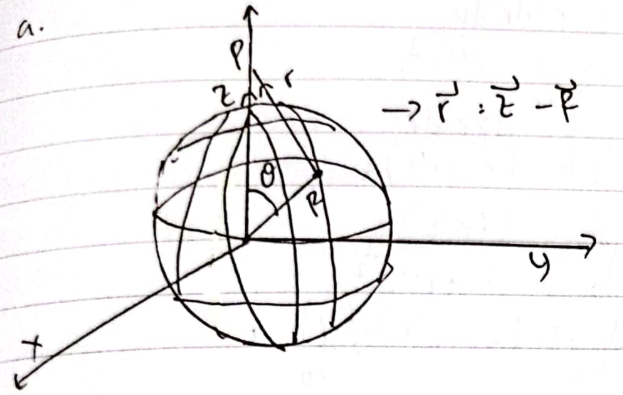
Contoh :

1. Potensial oleh muatan Permukaan

a.



$\rightarrow \vec{r} : \vec{z} - \vec{R}$

bola konduktor bermuatan homogen dgn rapat muatan σ

Berapa partikel di titik P.

Solusi : distribusi muatan pada permukaan bola, maka

$$V(p) : \int \frac{1}{4\pi\varepsilon_0} \frac{\sigma d\vec{s}}{r}$$

$$: \frac{\sigma}{4\pi\varepsilon_0} \int_s \frac{R^2 \sin\theta \, d\theta \, d\phi}{(z^2 + R^2 - 2Rz\cos\theta)^{\frac{1}{2}}}$$

$\rightarrow |\vec{r}| \pm (z^2 + R^2 - Rz\cos\theta)^{\frac{1}{2}}$

$$: \frac{\sigma R^2}{4\pi\varepsilon_0} \int_0^{\pi} \int_0^{2\pi} \frac{d\phi \sin\theta \, d\theta}{(z^2 + R^2 - 2Rz\cos\theta)^{\frac{1}{2}}}$$

$$: \frac{2\pi\sigma R^2}{4\pi\varepsilon_0} \cdot \frac{1}{2} \int_0^{\pi} \frac{1}{Rz} \frac{d(-2Rz\cos\theta)}{(z^2 + R^2 - 2Rz\cos\theta)^{\frac{1}{2}}}$$

$$: \frac{\sigma R}{2\varepsilon_0 z} \left(2(z^2 + R^2 - 2Rz\cos\theta)^{\frac{1}{2}} \right)_0^{\pi}$$

$$: \frac{\sigma R}{2\varepsilon_0 z} \left[(z^2 + R^2 + 2Rz)^{\frac{1}{2}} - (z^2 + R^2 - 2Rz)^{\frac{1}{2}} \right]$$

$$V(p) : \frac{\sigma R}{2\varepsilon_0 z} \left\{ \left[(R+z)^2 \right]^{\frac{1}{2}} - \left[(R-z)^2 \right]^{\frac{1}{2}} \right\}$$

Bila $z > R$, maka

$\left[(R-z)^2 \right]^{\frac{1}{2}} : z - R$, sehingga

$$V(p) : \frac{\sigma R}{2\varepsilon_0 z} \left[(R+z) - (z-R) \right]$$

$$: \frac{\sigma R^2}{\varepsilon_0 z}$$

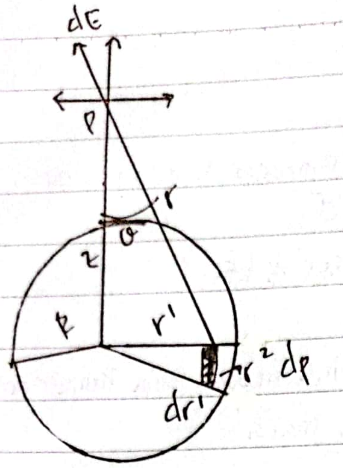
Bila $R > z$, maka

$\left[(R-z)^2 \right]^{\frac{1}{2}} : R - z$

$$V(p) : \frac{\sigma R}{2\varepsilon_0 z} \left[(R+z) - R + z \right]$$

$$V(p) : \frac{\sigma R}{\varepsilon_0}$$

b.



$$V(P) = \frac{1}{4\pi\varepsilon_0} \int_s \frac{\sigma d\vec{s}}{r}$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \int_s \frac{r' dr' dp}{(z^2 + r'^2)^{\frac{1}{2}}}$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \int_0^R \int_0^{2\pi} \frac{dp\, r' dr'}{(z^2 + r'^2)^{\frac{1}{2}}}$$

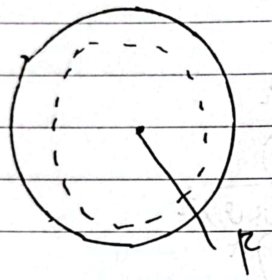
$$= \frac{2\pi\sigma}{4\pi\varepsilon_0} \int_0^R \frac{1}{2} \frac{d(r'^2)}{(z^2 + r'^2)^{\frac{1}{2}}}$$

$$= \frac{\sigma}{2\varepsilon_0} \left[\frac{1}{2} \cdot 2 \, (z^2 + r'^2)^{\frac{1}{2}} \right]_0^R$$

$$V(P) = -\int \vec{E}\, z \cdot d\vec{z}$$

2.) Potensial oleh muatan volume

a.



bola dgn rapat muatan ρ = konstan dgn jari² R

Hitung Potensial Pada

i $r < R$

ii $r > R$

Solusi :

$r < R$, $E_1 \rightarrow \oint E_1 d\vec{s} = \frac{1}{\varepsilon_0} \rho V$

$$E_1 (4\pi r^2) = \frac{1}{\varepsilon_0} \rho \left(\frac{4}{3}\pi r^3\right)$$

$$\vec{E}_1 = \frac{1}{3\varepsilon_0} \rho r \hat{r}_0$$

$r > R$, $E_2 \rightarrow \oint E_2 d\vec{s} = \frac{1}{\varepsilon_0} \rho V$

$$E_2 (4\pi r^2) = \frac{1}{\varepsilon_0} \rho \left(\frac{4\pi R^3}{3}\right)$$

$$\vec{E}_2 = \frac{1}{3\varepsilon_0 r^2} \rho R^3 \hat{r}_0$$

$$V_1 = -\int \vec{E} \cdot d\vec{p} = -\int_\infty^R \vec{E}_2 \cdot d\vec{r} - \int_R^r \vec{E}_1 \cdot d\vec{r}$$

$$= \int_\infty^R \frac{1}{3\varepsilon_0 r^2} \rho R^3 dr - \int_R^r \frac{1}{3\varepsilon_0} \rho r dr$$

$$= -\frac{\rho R^3}{3\varepsilon_0} \int_\infty^R \frac{dr}{r^2} - \frac{\rho}{3\varepsilon_0} \int_R^r r dr$$

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$$= \frac{\rho R^2}{3\varepsilon_0} - \frac{\rho}{6\varepsilon_0}(r^2 - R^2)$$

$$= \frac{\rho R^2}{3\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0} = \underbrace{\frac{2\rho R^2}{6\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0}} - \frac{\rho r^2}{6\varepsilon_0}$$

$$V_1 = \frac{\rho R^2}{\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0} \qquad \frac{3}{6}\frac{\rho R^2}{\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$V_1 = \frac{\rho}{\varepsilon_0}\left(R^2 - \frac{r^2}{6}\right) \qquad = \frac{1}{6\varepsilon_0}(3\rho R^2 - \rho r^2)$$

$$V_2 = -\int_\infty^r E_2 \cdot dr \qquad = \frac{\rho}{6\varepsilon_0}(2R^2 - r^2)$$

$$= -\int_\infty^r \frac{1}{3\varepsilon_0 r^2}\rho R^3 \, dr$$

$$= -\frac{\rho R^3}{3\varepsilon_0}\int_\infty^r \frac{dr}{r^2}$$

$$V_2 = \frac{\rho R^3}{3\varepsilon_0 r}$$