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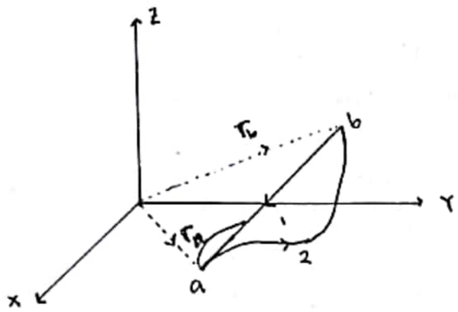
Kelistrikan dan Kemagnetan

▷ Potensial listrik

Sifat khusus dari muaran listrik $\vec{E}$ yaitu

$\nabla \times \vec{E} = 0$, Medan $\vec{E}$ merupakan gradien suatu skalar, jadi
$\vec{E} \sim \nabla \phi$, Maka: $\nabla \times \vec{E} = 0$.



Menurut teorema stokes

$\oint \vec{E}\, d\vec{p} = \int \nabla \times \vec{E}\, d\vec{s} = 0$

artinya $\oint_a^b \vec{E} \cdot d\vec{p}$ tidak bergantung pada pilihan jalan a ke b.

$\vec{E} = -\nabla V$

$\vec{E} \cdot 4 = -\nabla V \cdot d\vec{p}$

$= -dV$

$\int E\, d\vec{p} = -\int_a^b -dV$

$\int \vec{E}\, d\vec{p} = V_b - V_a$

tertutup

$V_b = V_a$

$V_b - V_a = 0$

$\oint \vec{E} \cdot d\vec{p} = 0$

$\oint \vec{E}\, d\vec{p} = \int \nabla \times E\, d\vec{s}$

$d\vec{s} \neq 0$,

$\nabla \times E = 0$

Jika $V_a \neq V_b$

$$\left.\begin{aligned} V(a) &= -\int_{b}^{a} \vec{E} \cdot d\vec{p} \\ V(b) &= -\int_a^b \vec{E}\, d\vec{p} \end{aligned}\right] V_{ab} = V_b - V_a = -\int_a^b \vec{E}\, dp + \int_a^a \vec{E}\, d\vec{p} = -\int_a^b \vec{E} \cdot d\vec{p}$$

$$= -\int_\theta^b E\, dp - \int_\theta^b E\, d\vec{p} = -\int_a^b \vec{E} \cdot d\vec{p}$$

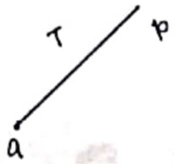
$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{p}$$

$\boxed{V_{(b)} = -\int E \cdot d\vec{p}}$ ⇒ Rumus umum

Jadi, potensial di suatu titik P.

$$V_{(P)} = -\int_{\infty}^{P} \vec{E}\, d\vec{p} \, .$$

Misal :



$$V_{(p)} = -\int_{\infty}^{T} \vec{E} \cdot dp$$

$$= -\int_{\infty}^{T} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}\, dr$$

$$V(p) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$

Untuk sejumlah muatan $q_1, q_2, q_3 \ldots q_n$ masing-masing pada jarak $r_1, r_2, r_3, \ldots r_n$.

maka :

$$V_{(p)} = \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots \frac{q_n}{r_n} \right]$$

potensial oleh distribusi muatan

- untuk kawat

$$V(p) = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda\, dr}{r}$$

- distribusi oleh permukaan

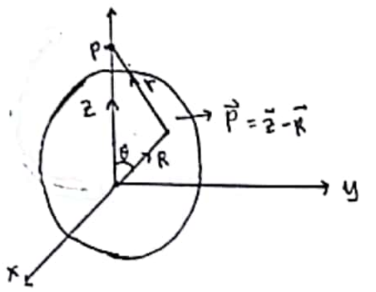
$$V(p) = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma\, d\vec{s}}{r}$$

- distribusi oleh volume

$$V(p) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho\, d\tau}{r}$$

contoh :

- potensial oleh muatan permukaan



Bola konduktor bermuatan homogen dengan rapat muatan σ. Berapa potensial dititik P

$$V_{(p)} = \int \frac{1}{4\pi\varepsilon_0} \frac{\sigma\, d\vec{s}}{r}$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \int \frac{R^2 \sin\theta\, d\phi\, d\theta}{(z^2+R^2-2Rz\cos\theta)^{\frac{1}{2}}}$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0} \int_0^{\pi}\int_0^{2\pi} \frac{d\phi \sin\theta\, d\theta}{(z^2+R^2-2Rz\cos\theta)^{\frac{1}{2}}}$$

$$= \frac{\pi\sigma R^2}{4\pi\varepsilon_0} \frac{1}{2} \int_0^{\pi} \frac{1}{Rz} \frac{d(-2Rz\cos\theta)}{(z^2+R^2-2Rz\cos\theta)^{\frac{1}{2}}}$$

$$= \frac{\sigma R}{4\varepsilon_0 z} \left(2(z^2+R^2-2Rz\cos\theta)^{\frac{1}{2}} \right)_0^{\pi}$$

$$= \frac{\sigma R}{2\varepsilon_0 z} \left[(z^2+R^2+2Rz)^{\frac{1}{2}} - (z^2+R^2-2Rz)^{\frac{1}{2}} \right]$$

$$V_{(p)} = \frac{\sigma R}{2\varepsilon_0 z} \left\{ \left[(R+z)^2\right]^{\frac{1}{2}} - \left[(R-z)^2\right]^{\frac{1}{2}} \right\}$$

Bila $z > R$, maka

$\left[(R-z)^2\right]^{\frac{1}{2}} = z - R$, sehingga

$$V(p) = \frac{\sigma R}{2\varepsilon_0 R} \left[(R+z) - (z-R) \right]$$

$$= \frac{\sigma R^2}{\varepsilon_0 R}$$

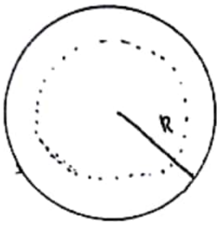
Bila $R > z$, maka

$\left[(R-z)^2\right]^{\frac{1}{2}} = R - z$

$$V_{(p)} = \frac{\sigma R}{2\varepsilon_0 z} \left[(R+z) - R + z \right]$$

$$V(p) = \frac{\sigma R}{\varepsilon_0}.$$

▷ potensial oleh muatan Volume



bola dengan rapat muatan ρ = konstan, dengan Jari-Jari R,
Hitung Potensial pada
i. $r < R$
ii. $r > R$

solusi :

$r < R$, $E_1 \rightarrow \oint E_1 \, d\vec{s} = \frac{1}{\varepsilon_0} \rho V$

$$E_1 (4\pi r^2) = \frac{1}{\varepsilon_0} \rho \left(\frac{4}{3} \pi r^3 \right)$$

$$E_1 = \frac{1}{3\varepsilon_0} \rho r \hat{r}_0$$

$r > R$, $E_2 \rightarrow \oint E_2 \, d\vec{s} = \frac{1}{\varepsilon_0} \rho V$

$$E_2 (4\pi r^2) = \frac{1}{\varepsilon_0} \rho \left(\frac{4}{3} \pi R^3 \right)$$

$$\vec{E}_2 = \frac{1}{3\varepsilon_0 r^2} \rho R^3 \hat{r}_0$$

$$V_1 = -\int \vec{E} \, d\vec{p} = -\int_\infty^R \vec{E}_2 \, d\vec{r} - \int_R^r E_1 \, d\vec{r}$$

$$= -\int_\infty^R \frac{1}{3\varepsilon_0 r^2} \rho R^3 \, dr - \int_R^r \frac{1}{3\varepsilon_0} \rho r \, dr$$

$$= -\frac{\rho R^3}{3\varepsilon_0} \int_\infty^R \frac{dr}{r^2} - \frac{\rho}{3\varepsilon_0} \int_R^r r \, dr$$

$$= \frac{\rho R^2}{3\varepsilon_0} - \frac{\rho}{\varepsilon_0} (r^2 - R^2)$$

$$= \frac{\rho R^2}{3\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$= \frac{2\rho R^2}{6\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$= \frac{1}{6\varepsilon_0} (3\rho R^2 - \rho r^2)$$

$$= \frac{\rho}{6\varepsilon_0} (2R^2 - \rho^2$$

$$V_1 = \frac{\rho R^2}{\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$V_1 = \frac{\rho}{\varepsilon_0} \left(R^2 - \frac{r^2}{6} \right)$$

$$V_2 = -\int_\infty^r E_2 \, dr$$

$$= -\int_\infty^r \frac{1}{3\varepsilon_0 r^2} \rho R^2 \, dr$$

$$= -\frac{\rho R^3}{3\varepsilon_0} \int_\infty^r \frac{dr}{r^2}$$

$$V = \frac{\rho R^3}{3\varepsilon_0 r}$$

▷ potensial oleh muatan pada kawat

a) kawat panjang sekali

$$E_p = \frac{2\lambda}{4\pi\varepsilon_0}$$

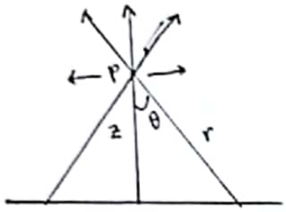
$$V_p = -\int_\infty^a E\, dp$$

$$= \frac{-2\lambda}{4\pi\varepsilon_0} \int_\infty^a \frac{dz}{z}$$

$$V_p = -\frac{2\lambda}{4\pi\varepsilon_0} \ln z$$

b) kawat dengan panjang tertentu, misalnya 2L.

$$E_p = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda L}{z\sqrt{z^2+L^2}}$$



$$V = \frac{2}{4\pi\varepsilon_0} \int_0^L \frac{\lambda\, dx}{r}$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(z^2+x^2)^{\frac{1}{2}}} \quad , \quad x = z\, tg\theta, \quad dx = z \sec^2\theta\, d\theta$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \int_0^L \frac{z \sec^2\theta\, d\theta}{z\,(tg^2\theta+1)}$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \int_0^L \sec\theta\, d\theta$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \ln(\sec\theta + tg\,\theta) + C$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \ln\left(\frac{\sqrt{x^2+z^2}}{z} + \frac{x}{z}\right)\Big|_0^L$$

$$V = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{\sqrt{L^2+z^2}}{z} + \frac{L}{z}\right).$$