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2.4 Potensial Listrik

Sifat khusus dari muatan listrik E yaitu

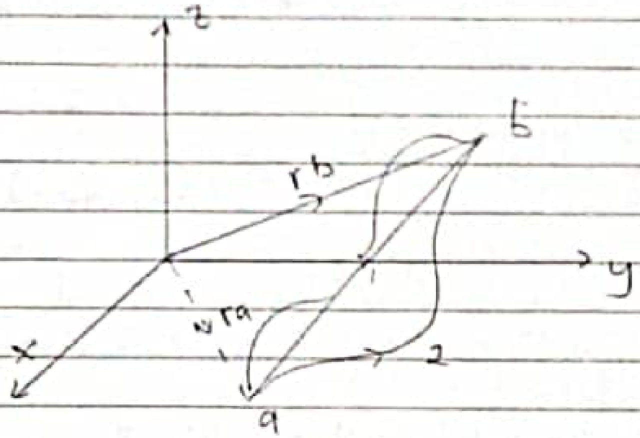
$\nabla \times \vec{E} = 0$. Ini artinya medan $\vec{E}$ merupakan gradien suatu skalar jadi.

$\vec{E} \sim \nabla \phi$. maka $\nabla \times \vec{E} = 0$

Menurut teorema stokes

$\oint \vec{E}\, d\vec{p} = \int \nabla \times \vec{E}\, d\vec{s} = 0$

artinya $\oint_a^b \vec{E}\, dp$ tidak bergantung pada pilihan jalan a ke b



$E = \nabla V$

$E \cdot dp = -\nabla V \cdot d\vec{p}$

$= -dV$

$\int \vec{E}\, d\vec{p} = \int_b^a -dV$

$\int \vec{E}\, dp = V_b - V_a$

V_a lintasannya kelutup

$V_b - V_a = 0$

$\int \vec{E} = d\vec{p} = 0$

$|r_a \cdot r_b|^2$

baru ke persamaan stokes

$\int E\, d\vec{p} = \int \nabla \times E\, d\vec{s}$

$d\vec{s} \neq 0$, $\nabla = \vec{E} = 0$

Hal ini berlaku untuk medan $\vec{E}$ dari satu muatan titik di titik asal atau dari setiap konfigurasi muatan yang statis

Bila lintasan a ke b tertutup, melalui jalan 1 dan jalan 2 harga $\oint \vec{E}\, d\vec{p} \neq 0$, karena $(r_1 \neq r_2)$

Selanjutnya : $\vec{E} = -\nabla V$. (E merupakan gradien potensial)

$E = -\nabla V$

$\nabla \vec{E} = -\nabla \cdot \nabla V$

$\nabla \vec{E} = -\nabla^2 V$

$\frac{\rho}{\varepsilon_0} = -\nabla^2 V$

$\nabla^2 V = \frac{\rho}{\varepsilon_0}$

$E = -\nabla V$

$E \cdot dp = -\nabla V\, dp$

$\int_a^b E\, dp = \int_a^b dV$

$= V \int_a^b dV$

$= V_b - V_a$

$V_b = V_a$

$\nabla V \cdot dp = dV$

$E = -\nabla V$

$\int \vec{E}\, d\vec{p} = -\int \nabla V\, d\vec{p} = -\int dV$, jadi persoalan diatas $V_b = k\dfrac{Q}{r^2}$

$$\left.\begin{aligned} V(a) &= -\int_{\infty}^{a} \vec{E} \cdot d\vec{p} \\ V(b) &= -\int_{a}^{b} \vec{E}\, d\vec{p} \end{aligned}\right] \quad V_{ab} = V_b - V_a = -\int_{\infty}^{b} \vec{E}\, d\vec{p} + \int_{\infty}^{a} \vec{E}\, d\vec{p}$$

$$= -\int_{r}^{b} E \cdot \vec{\varphi}$$

$$= -\int_{\infty}^{b} E\, dP - \int_{a}^{\infty} E \cdot d\vec{p}$$

$$= -\int_{a}^{b} \vec{E}\, d\vec{p}$$

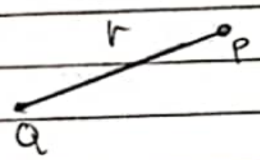
$$V_b - V_a = -\int_{a}^{b} \vec{E}\, dp$$

$$\boxed{V(P) = -\int \vec{E}\, d\vec{p}} \quad \text{rumus umum}$$

Jadi potensial di suatu titik P

$$V(p) = -\int_{\infty}^{P} \vec{E}\, d\vec{p}$$

misal :



$$V(p) = -\int_{\infty}^{r} \vec{E}\, d\vec{p}$$

$$= -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}\, dr.$$

$$V(p) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$

Untuk sejumlah muatan $q_1, q_2, q_3 \cdots q_4$ masing-masing pada jarak $r_1, r_2, r_3 \cdots r_n$, maka.

$$V(p) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \dots \frac{q_n}{r_n}\right] \rightarrow \text{penjumlahan biasa}$$

Potensial oleh distribusi muatan

1) Untuk kawat : $V(p) = \dfrac{1}{4\pi\varepsilon_0} \displaystyle\int \frac{\lambda\, dr}{r}$

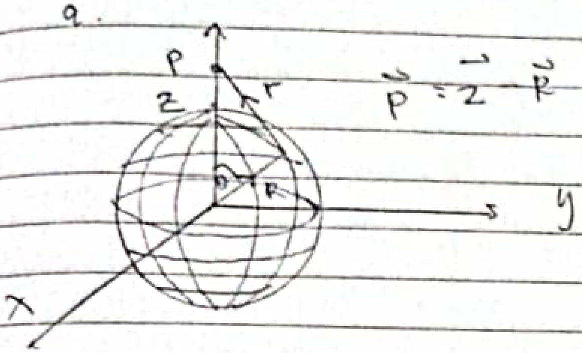
2) Untuk distribusi oleh permukaan : $V(p) = \dfrac{1}{4\pi\varepsilon_0} \displaystyle\int \frac{\sigma\, ds}{r}$

3) Distribusi oleh volume $V(p) = \dfrac{1}{4\pi\varepsilon_0} \displaystyle\int \frac{\rho\, d\sigma}{r}$

Contoh

1. Potensial oleh muatan permukaan bersambung

a.



$\vec{p} = \vec{z} - \vec{R}$

bola konduktor bermuatan homogen dengan rapat muatan σ

Berapa potensial di titik P.

Solusi : distribusi muatan pada permukaan bola, maka

$$V(P) = \int \frac{1}{4\pi\varepsilon_0} \frac{\sigma\, d\vec{s}}{r}$$

$|\vec{P}| = \pm(z^2 + R^2 - 2Rz\cos\theta)^{\frac{1}{2}}$

$$= \frac{\sigma}{4\pi\varepsilon_0} \int \frac{R^2 \sin\theta\, d\theta\, d\phi}{(z^2+R^2-2Rz\cos\theta)^{\frac{1}{2}}}$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0} \int_0^{\pi}\int_0^{2\pi} \frac{d\phi \sin\theta\, d\theta}{(z^2+R^2-2Rz\cos\theta)^{\frac{1}{2}}}$$

$$= \frac{2\pi\sigma R^2}{4\pi\varepsilon_0} \cdot \frac{1}{2} \int_0^{\pi} \frac{1}{Rz} \frac{d(-2Rz\cos\theta)}{(z^2+R^2-2Rz\cos\theta)^{\frac{1}{2}}}$$

$$= \frac{\sigma R}{2\varepsilon_0 z} \left(2(z^2+R^2-2Rz\cos\theta)^{\frac{1}{2}} \right)_0^{\pi}$$

$$= \frac{\sigma R}{2\varepsilon_0 z} \left[(z^2+R^2-2Rz\cos\theta)^{\frac{1}{2}} - (z^2+R^2-2Rz)^{\frac{1}{2}} \right]$$

$$\therefore V(P) = \frac{\sigma R}{2\varepsilon_0 z} \left\{ \left[(R+z)^2\right]^{\frac{1}{2}} - \left[(R-z)^2\right]^{\frac{1}{2}} \right\}$$

Bila $z > R$ maka

$[(R-z)^2]^{\frac{1}{2}} = z - R$, sehingga

$$V(P) = \frac{\sigma R}{2\varepsilon_0 z}\left[(R+z) - (z-R)\right]$$

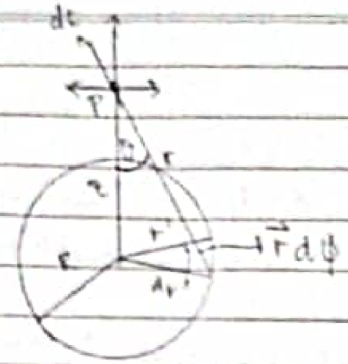
$$V(P) = \frac{\sigma R^2}{\varepsilon_0 z}$$

Bila $R > z$, maka $[(R-z)^2]^{\frac{1}{2}} = R - z$

$$V(P) = \frac{\sigma R}{2\varepsilon_0 z}\left[(R+z) - R + z\right]$$

$$V(P) = \frac{\sigma R}{\varepsilon_0}$$

b)



$$V(p) = \frac{1}{4\pi\varepsilon_0} \int_s \frac{\sigma\, d\vec{S}}{r}$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \int_s \frac{r'\,dr'\,d\phi}{(z^2+r'^2)^{\frac{1}{2}}}$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \int_0^R \int_0^{2\pi} \frac{d\phi\, r'\,dr'}{(z^2+r'^2)^{\frac{1}{2}}}$$

$$= \frac{2\pi\sigma}{4\pi\varepsilon_0} \int_0^R \frac{1}{2} \frac{d(r'^2)}{(z^2+r'^2)^{\frac{1}{2}}}$$

$$= \frac{\sigma}{2\varepsilon_0} \left[\frac{1}{2}\, 2\, (z^2+r'^2)^{\frac{1}{2}} \right]_0^R$$

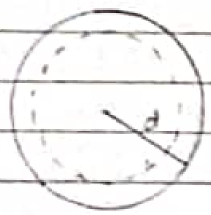
$$V(p) = \frac{\sigma}{2\varepsilon_0} \left[\sqrt{z^2+R^2} - z \right]$$

Persoalan diatas dapat juga dihitung dengan rumus:

$$V(p) = -\int \vec{E}_z \cdot d\vec{z}$$

2 Potensial oleh muatan volume

a)



bola dengan rapat muatan ρ = konstan, dengan jari-jari R

Hitung potensial pada

(i) $r < R$

(ii) $r > R$

Solusi

$r < R$, $E_1 \rightarrow \oint E_1\, d\vec{S} = \frac{1}{\varepsilon_0} \rho V$

$$E_1 (4\pi r^2) = \frac{1}{\varepsilon_0} \rho \left(\frac{4}{3} \pi r^3 \right)$$

$$E_1 = \frac{1}{3\varepsilon_0} \rho r \hat{r}_0$$

$r > R$, $E_2 \rightarrow \oint E_2\, d\vec{S} = \frac{1}{\varepsilon_0} \rho V$

$$E_2 (4\pi r^2) = \frac{1}{\varepsilon_0} \rho \left(\frac{4}{3} \pi R^3 \right)$$

$$\vec{E}_2 = \frac{1}{3\varepsilon_0 r^2} \rho R^3 \hat{r}_0$$

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$$V_1 = -\int \vec{E}\, d\vec{r} = -\int_\infty^R \vec{E}_2\, d\vec{r} - \int_R^r \vec{E}_1 \cdot d\vec{r}$$

$$= -\int_\infty^R \frac{1}{3\varepsilon_0 r^2}\rho R^3\, dr - \int_R^r \frac{1}{3\varepsilon_0}\rho r\, dr$$

$$= -\frac{\rho R^3}{3\varepsilon_0}\int_\infty^R \frac{dr}{r^2} - \frac{\rho}{3\varepsilon_0}\int_R^r r\, dr$$

$$= \frac{\rho R^2}{3\varepsilon_0} - \frac{\rho}{6\varepsilon_0}(r^2 - R^2)$$

$$= \frac{\rho R^2}{3\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$= \frac{2\rho R^2}{6\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$= \frac{3}{6}\frac{\rho R^2}{\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$= \frac{1}{6\varepsilon_0}(3\rho R^2 - \rho r^2)$$

$$= \frac{\rho}{6\varepsilon_0}(3R^2 - r^2)$$

$$V_1 = \frac{\rho R^2}{\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$\therefore V_1 = \frac{\rho}{\varepsilon_0}\left(R^2 - \frac{r^2}{6}\right)$$

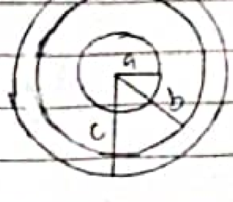
$$V_2 = -\int_\infty^r E_2\, dr$$

$$= -\int_\infty^r \frac{1}{3\varepsilon_0 r^2}\rho R^3\, dr$$

$$= -\frac{\rho R^3}{3\varepsilon_0}\int_\infty^r \frac{dr}{r^2}$$

$$\therefore V_2 = \frac{\rho R^3}{3\varepsilon_0 r}$$

b) bola konduktor kosentris seperti pada gambar



1. $r < a \rightarrow E = 0$
2. $a < r < b$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}_0$$

3. $b < r < c \rightarrow E = 0$
4. $r > c$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}_0$$

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$$V_1 = -\int_{\infty}^{r} \vec{E} \cdot d\vec{p}$$

$$= \frac{-q}{4\pi\varepsilon_0} \left[\int_{\infty}^{c} E_4 \, d\vec{r} + \int_{c}^{b} E_3 \, dr + \int_{b}^{a} E_2 \, dr + \int_{a}^{r} E_1 \, dr \right]$$

$$= \frac{-q}{4\pi\varepsilon_0} \left[\int_{\infty}^{c} \frac{dr}{r^2} + \int_{b}^{a} \frac{dr}{r^2} \right]$$

$$= \frac{-q}{4\pi\varepsilon_0} \left[-\frac{1}{c} + \left(-\frac{1}{a} + \frac{1}{b} \right) \right]$$

$$\therefore V_1 = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b} \right)$$

$$V_2 = \frac{-q}{4\pi\varepsilon_0} \left[\int_{\infty}^{c} \frac{dr}{r^2} + \int_{c}^{b} \frac{dr}{r^2} + \int_{b}^{r} \frac{dr}{r^2} \right]$$

$$= \frac{-q}{4\pi\varepsilon_0} \left[\int_{\infty}^{c} \frac{dr}{r^2} + \int_{b}^{r} \frac{dr}{r^2} \right]$$

$$= \frac{-q}{4\pi\varepsilon_0} \left[-\frac{1}{c} + \left(-\frac{1}{r} + \frac{1}{b} \right) \right]$$

$$\therefore V_2 = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{c} + \frac{1}{r} - \frac{1}{b} \right)$$

$$V_3 = \frac{-q}{4\pi\varepsilon_0} \left[\int_{\infty}^{c} \frac{dr}{r^2} + \int_{c}^{r} \frac{dr}{r^2} \right]$$

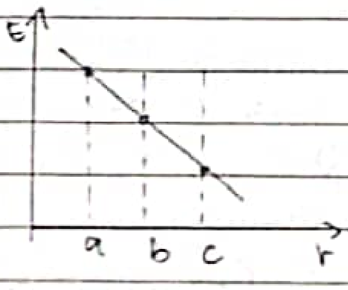
$$= \frac{-q}{4\pi\varepsilon_0} \left(-\frac{1}{c} + \left(-\frac{1}{r} + \frac{1}{c} \right) \right)$$

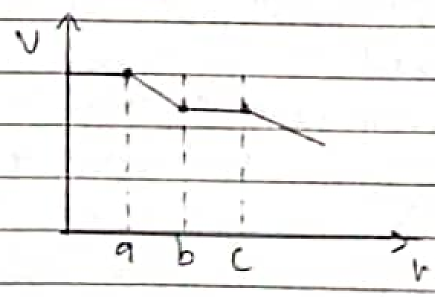
$$= \frac{-q}{4\pi\varepsilon_0 r} \quad ; \quad b < r < c$$

$$V_4 = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \, dr$$

$$V_4 = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

Grafiknya :





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3) Potensial oleh muatan pada kawat

a) Kawat panjang selali

$$E_p = \frac{2\lambda}{4\pi\varepsilon_0 z}$$

$$V_p = -\int_{\infty}^{a} E \cdot dp$$

$$= -\frac{2\lambda}{4\pi\varepsilon_0}\int_{\infty}^{a}\frac{dz}{z}$$

$$V_p = \frac{-2\lambda}{4\pi\varepsilon_0}\ln z$$

b) Kawat dengan panjang tertentu, misal $2L$

Dari perhitungan di depan

$$E_p = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z\sqrt{z^2+L^2}}$$

$$V = \frac{2}{4\pi\varepsilon_0}\int_0^L \frac{\lambda\, dx}{r}$$

$$= \frac{2\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(z^2+x^2)^{\frac{1}{2}}}$$

$$x = z \operatorname{tg}\theta$$

$$dx = z \sec^2\theta\, d\theta$$