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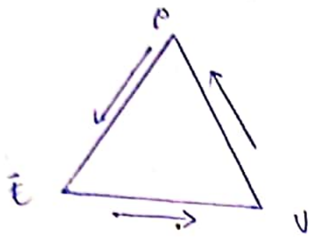
Prodi : Pendidikan Fisika (B)

2. Hubungan antara $\vec{E}$, ρ dan V



$$\nabla \times \vec{E} = 0$$
$$\nabla \cdot E = \frac{1}{\varepsilon_0}\rho$$
> Syarat elektrostatika

$$\Delta V = -\frac{\rho}{\varepsilon_0}$$

$$\rho = \varepsilon_0 \Delta \cdot E$$

Bentuk Integralnya

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ}{r^2} \quad ; \quad dQ = \rho\, d\tau$$
$$= \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{r^2}$$

$$V = \int \vec{E} \cdot d\vec{P}$$

Hubungan V dengan ρ

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{r}$$

Solusi hubungan diatas

1. Misal E diketahui V dan ρ dapat dihitung

$$V = -\int E\, dp$$
$$\rho = \varepsilon_0 \nabla \cdot E$$

2. Misal V diketahui, E da ρ dapat dicari

$$E = -\nabla V$$
$$\rho = -\frac{1}{\varepsilon_0}\nabla^2 V$$

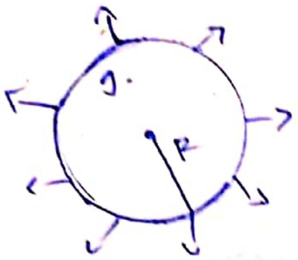
3. Misal ρ diketahui, E dan V dapat dihitung

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{r}$$

Contoh

1. Bola konduktor dengan jari² R dengan muatan q seperti gambar, Berapa W?



$r < R$, $E = 0$

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$; $r \le R$

$W = \frac{1}{2}\int \rho V d\tau$

$= \frac{1}{2}\frac{3q}{4\pi R^3}\frac{Q}{4\pi\varepsilon_0 R}\int_0^R 4\pi r^2 dr$

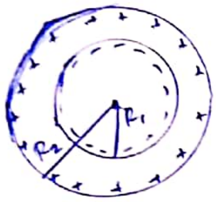
$W = \frac{q^2}{8\pi\varepsilon_0 R}$

$V = \frac{3}{4}\pi R^3$

$Q = \rho V$

$\rho = \frac{3Q}{4\pi R^3}$

2) Bola konduktor berongga mempunyai jari-jari R_1 dan R_2 seperti gambar. Berapa energi yang tersimpan dalam konduktor tersebut.



$r < R_1$

$W_d = +\frac{1}{2}\int \rho V d\tau$

$= +\frac{1}{2}\int_{R_1}^{0}\frac{3Q}{4\pi R_1^3}\frac{Q}{4\pi\varepsilon_0 R_1}\cdot 4\pi r^2 dr$

$= -\frac{Q^2}{8\pi\varepsilon_0 R_1}$

$Q = \rho V$

$Q = \rho\left(\frac{4}{3}\pi R^3\right)$

$\rho = \frac{3Q}{4\pi R^3}$

$r > R_2$

$W_l = \frac{1}{2}\int_0^{R_2}\frac{3Q}{4\pi R_2^3}\frac{Q}{4\pi R_2}\cdot 4\pi r^2 dr$

$= \frac{Q^2}{8\pi\varepsilon_0 R_2}$

$W_d = \frac{1}{2}\int_0^{R_1}\frac{3Q}{4\pi R_1^3}\frac{Q}{4\pi\varepsilon_0 R_1}\,4\pi\rho^2 d\rho$

$= \frac{3Q^2}{4\pi\varepsilon_0 R_1^4}\int_0^{R_1} r^2 g$

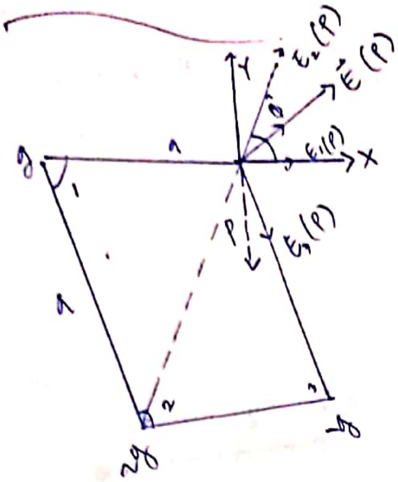
$= \frac{\cancel{3}}{\cancel{3}}\frac{Q^3}{4\pi\varepsilon R_1^4}R_1^3 = \frac{Q^3}{4\pi\varepsilon R_1}$

$W = W_d + W_l$

$= \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$

$= \frac{Q^3}{8\pi\varepsilon_0}\left(\frac{R_1 - R_2}{R_2 \cdot R_1}\right)$

Soal dan Pembahasan

1. 

a.) Hitung kuat medan pada titik (E_P), jika keempat sisi Japran genjang itu Sama.

b.) Hitung Pula Potensial dititik P ($V_{(P)}$)

Solusi

$$\vec{E}_1(P) = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\hat{r}_{01}$$

$$\vec{E}_2(P) = \frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2}\hat{r}_{02}$$

$$\vec{E}(P) = \vec{E}_1(P) + \vec{E}_2(P) + \vec{E}_3(P)$$
$$= \vec{E}_x + \vec{E}_y$$

$$E_x = E_1(P) + E_2(P)\cos 60 + E_3(P)\cos 60$$

Misal: $\frac{q}{4\pi\varepsilon_0 a^2} = c$

$$\vec{E}_x = c + 2c\left(\tfrac{1}{2}\right) + c\left(\tfrac{1}{2}\right)$$
$$= \tfrac{5}{2}c\,\hat{j}$$

$$\vec{E}_y = 0 + E_2(P)\sin 60 + E_3\sin 120$$
$$= 2c\left(\tfrac{1}{2}\sqrt{3}\right) - c\left(\tfrac{1}{2}\sqrt{3}\right)$$
$$= \tfrac{1}{2}\sqrt{3}\,c.\hat{k}$$

$$\vec{E}(P) = \left[\left(\tfrac{5}{2}c\right)^2 + \left(\tfrac{1}{2}\sqrt{3}c\right)^2\right]^{\frac{1}{2}}$$
$$= c\left(\tfrac{25}{4} + \tfrac{3}{4}\right)^{\frac{1}{2}}$$
$$= \sqrt{7}\,c\,\hat{\varphi}$$

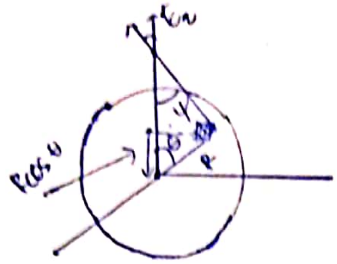
$$\vec{E}(P) = \sqrt{7}\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\hat{\varphi}\ \ N/C$$

$$V_{(P)} = c + 2c - c$$
$$= 2c$$
$$= \frac{2}{4\pi\varepsilon_0}\frac{q}{a^2}$$

2. Hitung kuat medan yang ditimbulkan oleh rapat muatan pada permukaan bola.

Solusi :

Lihat koordinat bola



$$\vec{E}_z = \frac{1}{4\pi\varepsilon_0} \iint \frac{\sigma\, d\vec{a} \cos\psi}{r^2}$$

$$\vec{E}_z = \frac{\sigma}{4\pi\varepsilon_0} \int_0^{\pi}\int_0^{2\pi} \frac{R^2 \sin\theta\, d\theta\, d\phi\, (z - R\cos\theta)}{(R^2+z^2-2Rz\cos\theta)^{3/2}}$$

$$E_z = \frac{1}{4\pi\varepsilon_0}(2\pi R^2\sigma)\int_0^{\pi} \frac{(z-R\cos\theta)\sin\theta\, d\theta}{(R^2+z^2-2Rz\cos\theta)^{3/2}}\, d\theta$$

Misal : $u = \cos\theta$

$du = -\sin\theta\, d\theta$; $\theta = 0 \rightarrow u = 1$

$\theta = \pi \Rightarrow u = -1$

$$\vec{E}_z = \frac{1}{4\pi\varepsilon_0}(2\pi R^2\sigma)\int_{-1}^{+1} \frac{(z-Ru)}{(R^2+z^2-2Rzu)^{3/2}}\, du$$

$$= \frac{1}{4\pi\varepsilon_0}(2\pi R^2\sigma)\left[\int_{-1}^{+1} \frac{z\, du}{(R^2+z^2-2Rzu)^{3/2}} - \int_{-1}^{+1} \frac{Ru\, du}{(R^2+z^2-2Rzu)^{3/2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}(2\pi R^2\sigma)\left[\frac{1}{z^2}\frac{(zu-R)}{\sqrt{R^2+z^2-2Rzu}}\right]_{-1}^{+1}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\pi R^2\sigma}{z^2}\left[\frac{(z-R)}{|z-R|} - \frac{(-z-R)}{|z+R|}\right]$$

Dipindai dengan CamScanner