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Resume Potensial listrik

Sifat khusus dari ~~medan~~ ^{medan} listrik $\vec{E}$ yaitu:

$\nabla \times \vec{E} = 0$, ini artinya medan $\vec{E}$ merupakan gradien suatu skalar, jadi $\vec{E} \sim \nabla Q$, maka $\nabla \times \vec{E} = 0$

Menurut teorema Stokes

$$\oint \vec{E} \cdot d\vec{p} = \int \nabla \times \vec{E} \cdot d\vec{S} = 0$$

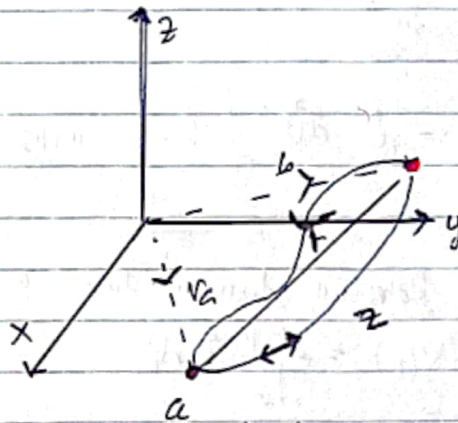
Artinya $\oint_a^b \vec{E} \cdot d\vec{p}$ tidak bergantung pada pilihan jalan a ke b

$$\vec{E} = -\nabla V$$

$$E \cdot d\vec{p} = -\nabla V \cdot d\vec{p} = -dV$$

$$\int \vec{E} \cdot d\vec{p} = \int_a^b -dV$$

$$\int \vec{E} \cdot d\vec{p} = V_b - V_a$$



Utl lintasan tertutup

$$V_b = V_a$$

$$V_b - V_a = 0$$

Hal ini berlaku untuk medan $\vec{E}$ dari suatu muatan titik di titik asal atau dari setiap konfigurasi muatan yang statis.

$\int \vec{E} \cdot d\vec{p} = 0$ bila lintasan a ke b tertutup, melalui jalan 1 ada 2
↓
harga $\oint \vec{E} \cdot d\vec{p} \neq 0$, karena (r, ϕ, r_z)

harus lepas selanjutnya : $\vec{E} = -\nabla V$. ($\vec{E}$ merupakan gradien potensial)

Stokes

$$\oint \vec{E} \cdot d\vec{p} = \int \nabla \times \vec{E} \cdot d\vec{S}$$

$$d\vec{S} \neq 0, \text{ maka } \nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\nabla \times \vec{E} = -\nabla \times \nabla V$$

$$\nabla \times \vec{E} = -\nabla^2 V$$

$$\rho/\epsilon_0 = -\nabla^2 V$$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

$$\vec{E} = -\nabla V$$

$$E \cdot d\vec{p} = -\nabla V \cdot d\vec{p}$$

$$\int_a^b \vec{E} \cdot d\vec{p} = \int_a^b -dV$$

$$\int_a^b dV = V_b - V_a$$

$$= V_b - V_a$$

Jika $V_a + V_b$

$$W \cdot d\vec{p} = dV$$

$$E = -\nabla V$$

$$\int \vec{E} \cdot d\vec{p} = - \int \nabla V \cdot d\vec{p} = - \int dV, \text{ jadi potensial di atas } V_b = \frac{Q}{r_b}$$

$$V(a) = - \int_{\infty}^a \vec{E} \cdot d\vec{p}$$

$$V(b) = - \int_{\infty}^b \vec{E} \cdot d\vec{p}$$

$$V_{ab} = V_b - V_a = - \int_{\infty}^b \vec{E} \cdot d\vec{p} + \int_{\infty}^a \vec{E} \cdot d\vec{p} = - \int_a^b \vec{E} \cdot d\vec{p}$$

$$= - \int_a^b \vec{E} \cdot d\vec{p} - \int_a^b \vec{E} \cdot d\vec{p} = - \int_a^b \vec{E} \cdot d\vec{p}$$

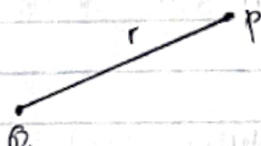
$$V_b - V_a = - \int_a^b \vec{E} \cdot d\vec{p}$$

$$\boxed{V(p) = - \int \vec{E} \cdot d\vec{p}} \rightarrow \text{Rumus umum}$$

Jadi, potensial di suatu titik P

$$V(p) = - \int_{\infty}^P \vec{E} \cdot d\vec{p}$$

Misal :


$$\begin{aligned} V(p) &= - \int_{\infty}^r \vec{E} \cdot d\vec{p} \\ &= - \int_{\infty}^r \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2} dr \\ V(p) &= \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \end{aligned}$$

Untuk sejumlah muatan $q_1, q_2, q_3, \dots, q_n$ masing-masing pada jarak $r_1, r_2, r_3, \dots$

Maka .

$$V(p) = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \dots + \frac{q_n}{r_n} \right]$$

Potensial oleh distribusi muatan

1) Untuk kerapatan

$$V(p) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda dr}{r}$$

2) Untuk distribusi permukaan

$$V(p) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma dS}{r}$$

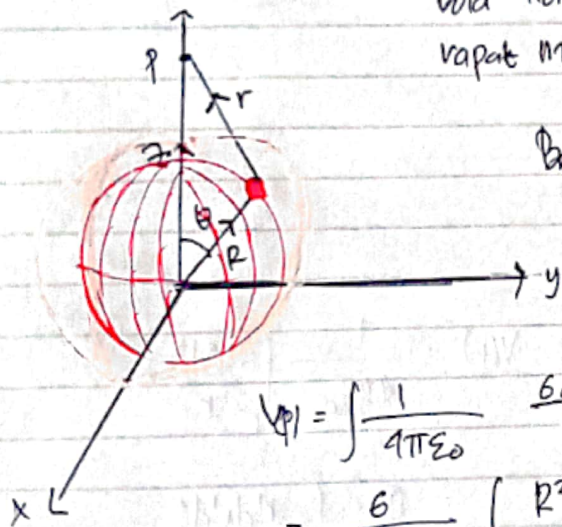
3) distribusi oleh volume

$$V(p) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho d\tau}{r}$$

Contoh :

1. Potensial oleh muatan permukaan.

u.



bola konduktor bermuatan homogen dengan rapat muatan σ .

Berapa potensial di titik P.

Solusi : distribusi muatan pada permukaan bola ; maka

$$V(p) = \int \frac{1}{4\pi\epsilon_0} \frac{\sigma dS}{r}$$

$$= \frac{\sigma}{4\pi\epsilon_0} \int \frac{R^2 \sin \theta d\theta d\phi}{(z^2 + R^2 - 2Rz \cos \theta)^{1/2}} \rightarrow |\vec{r}|$$

$$= \frac{\sigma R^2}{4\pi\epsilon_0} \int_0^\pi \int_0^{2\pi} \frac{d\phi \sin \theta d\theta}{(z^2 + R^2 - 2Rz \cos \theta)^{1/2}}$$

$$= \frac{2\pi \sigma R^2}{4\pi\epsilon_0} \cdot \frac{1}{2} \int_0^\pi \frac{1}{Rz} \frac{d(-2Rz \cos \theta)}{(z^2 + R^2 - 2Rz \cos \theta)^{1/2}}$$

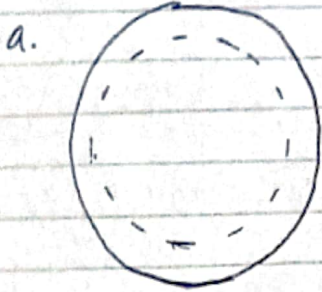
$$= \frac{\sigma R}{\epsilon_0 z} \left[z(z^2 + R^2 - 2Rz \cos \theta)^{1/2} \right]_0^\pi$$

$$= \frac{\sigma R}{2\epsilon_0 z} \left[(z^2 + R^2 + 2Rz)^{1/2} - (z^2 + R^2 - 2Rz)^{1/2} \right]$$

Potensial diatas dapat juga dihitung dengan rumur :

$$V(p) = - \int \vec{E} \cdot d\vec{z}$$

2) Potensial oleh muatan Volume



bola dengan rapat muatan $\rho = \text{konstan}$ dengan jari-jari R . Hitung Potensial listrik pada

i $r < R$

i $r > R$

Solusi :

$$r < R, \quad E_1 \rightarrow \oint \vec{E}_1 \cdot d\vec{s} = \frac{1}{\epsilon_0} \rho V$$

$$E_1 (4\pi r^2) = \frac{1}{\epsilon_0} \rho \left(\frac{4}{3} \pi r^3 \right)$$

$$E_1 = \frac{1}{3\epsilon_0} \rho r \hat{s}$$

$$r > R, \quad E_2 \rightarrow \oint \vec{E}_2 \cdot d\vec{s} = \frac{1}{\epsilon_0} \rho V$$

$$E_2 (4\pi r^2) = \frac{1}{\epsilon_0} \rho \left(\frac{4}{3} \pi R^3 \right)$$

$$E_2 = \frac{1}{3\epsilon_0 r^2} \rho R^3 \hat{s}$$

$$V_1 = - \int \vec{E} \cdot d\vec{r} = - \int_{\infty}^R \vec{E}_2 \cdot d\vec{r} - \int_R^r \vec{E}_1 \cdot d\vec{r}$$

$$= - \int_{\infty}^R \frac{1}{3\epsilon_0 r^2} \rho R^3 dr - \int_R^r \frac{1}{3\epsilon_0} \rho r dr$$

$$= - \frac{\rho R^3}{3\epsilon_0} \int_{\infty}^R \frac{dr}{r^2} - \frac{\rho}{3\epsilon_0} \int_R^r dr$$

$$= \frac{\rho R^2}{3\epsilon_0} - \frac{\rho}{6\epsilon_0} (r^2 - R^2)$$

$$= \frac{\rho R^2}{3\epsilon_0} + \frac{\rho R^2}{6\epsilon_0} - \frac{\rho r^2}{6\epsilon_0} = \frac{2\rho R^2}{6\epsilon_0} + \frac{\rho R^2}{6\epsilon_0} - \frac{\rho r^2}{6\epsilon_0}$$

$$V_1 = \frac{\rho r^2}{\epsilon_0} - \frac{\rho r^2}{6\epsilon_0}$$

$$\therefore V_1 = \frac{\rho}{\epsilon_0} \left(R^2 - \frac{r^2}{6} \right)$$

$$V_2 = - \int_{\infty}^r \frac{1}{3\epsilon_0 r^2} \rho r^3 dr$$

$$= - \frac{\rho R^3}{3\epsilon_0} \int_{\infty}^r \frac{dr}{r^2}$$

$$\therefore V_2 = \frac{\rho R^3}{3\epsilon_0 r}$$

$$\frac{3}{6} \frac{\rho r^2}{\epsilon_0} - \frac{\rho r^2}{6\epsilon_0}$$

$$= \frac{1}{6\epsilon_0} (3\rho R^2 - \rho r^2)$$

$$= \frac{\rho}{6\epsilon_0} (3R^2 - r^2)$$