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Kelas : B

2.9 potensial listrik

Sifat khusus dari medan listrik $\vec{E}$ yaitu :

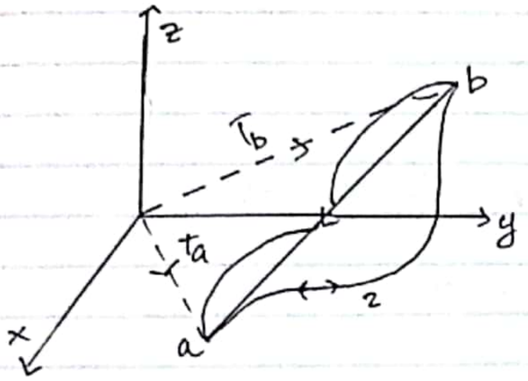
$\nabla \times A = 0$. Ini artinya medan $\vec{E}$ merupakan gradien suatu skalar.

$\vec{E} = \nabla\phi$, maka $\nabla \times \vec{E} = 0$

Menurut teorema stokes

$\oint \vec{E}\, d\vec{p} = \int \nabla \times \vec{E}\, d\vec{s} = 0$

artinya $\oint_a \vec{E} \cdot d\vec{p}$ tidak bergantung pada pilihan jalan a ke b



Hal ini berlaku untuk ~~muatan~~ medan $\vec{E}$ dari suatu hambatan titik di titik asal atau dari setiap konfigurasi muatan yang statis.

Bila lintasan a ke b tertutup melalui jalan 1 dan jalan 2 maka $\oint E \cdot d\vec{p} \neq 0$, karena $(r_1 \neq r_2)$

Selanjutnya : $\vec{E} = -\nabla V$. ($\vec{E}$ mrp. gradien potensial)

$E = -\nabla V$

$\nabla \cdot \vec{E} = -\nabla \cdot \nabla V$

$\nabla \cdot \vec{E} = -\nabla^2 V$

$\rho/\varepsilon_0 = -\nabla^2 V$

$\Rightarrow \nabla^2 V = -\frac{\rho}{\varepsilon_0}$

$= -\int_0^d E\, dL$

$E = -\nabla V$

$E \cdot d\rho = -\nabla V\, dL$

$\int_0^d E\, dL = \int_0^d dV$

$= 0 \int_0^d dV = V_b - V_a$

$= V_c = V_d$

$\nabla V \cdot d\rho = dV$

$E = -\nabla V$

$\int \vec{E} \cdot d\vec{p} = -\int \nabla V \cdot d\vec{p} = -\int dV$. jadi persoalan diatas $V_v = \frac{Q}{r^2}$

$V(a) = -\int_\infty^a \vec{E} \cdot d\vec{p}$

$V(b) = -\int_\infty^b E \cdot d\vec{p}$

$\rightarrow V_{ab} = V_b - V_a = -\int_\infty^b \vec{E} \cdot dp + \int_\infty^a \vec{E} \cdot d\vec{p} = -\int_a^b \vec{E} \cdot d\vec{p}$

$= -\int_\infty^a E\, dp - \int_a^\infty E \cdot d\vec{p} = -\int_a^b \vec{E} \cdot d\vec{p}$

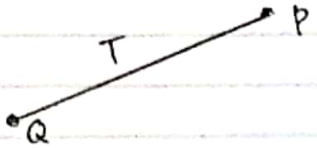
$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{p}$$

$$\boxed{V_{(P)} = -\int \vec{E} \cdot d\vec{p}}$$ Rumus Umum

Jadi potensial di suatu titik P

$$V_{(P)} = -\int_{\infty}^{P} \vec{E} \, d\vec{p}$$

Misal :



$$V_{(P)} = -\int_{\infty}^{r} \vec{E} \cdot d\vec{p}$$

$$= -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \, dr$$

$$V_{(P)} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$

Untuk sejumlah muatan $q_1, q_2, q_3 \cdots q_4$ masing² pada jarak $r_1, r_2, r_3 \cdots r_4$ maka

$$V_{(P)} = \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots \frac{q_4}{r_4} \right]$$ => penjumlahan biasa

potensial oleh distribusi muatan

1). Untuk kawat

$$V_{(P)} = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda \, dr}{r}$$

2). Untuk distribusi permukaan

$$V_{(P)} = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma \, d\vec{s}}{r}$$

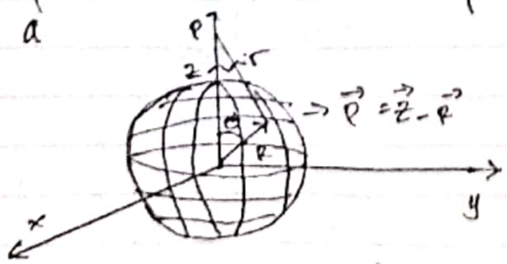
3). distribusi oleh volume

$$V_{(P)} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho \, d6}{r}$$

Contoh :

1. Potensial oleh muatan permukaan

. a



bola konduktor bermuatan homogen dgn rapat muatan σ

Berapa potensial di titik P

Solusi : distribusi muatan pada permukaan bola, maka

$$V_{(P)} = \int \frac{1}{4\pi\varepsilon_0} \frac{\sigma d\vec{s}}{r}$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0} \cdot \int_s \frac{R^2 \sin\theta \, d\theta \, d\phi}{(z^2+R^2-2Rz\cos\theta)^{1/2}}$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0} \cdot \int_0^{\pi}\int_0^{\pi} \frac{d\phi \sin\theta \, d\theta}{(z^2+R^2-2Rz\cos\theta)^{1/2}}$$

$$= \frac{2\pi\sigma R^2}{4\pi\varepsilon_0} \cdot \frac{1}{2} \int_0^{\pi} \frac{1}{Rz} \frac{d(-2Rz\cos\theta)}{(z^2+R^2-2Rz\cos\theta)^{1/2}}$$

$$= \frac{\sigma R}{4\varepsilon_0 z}\left(2(z^2+R^2-2Rz\cos\theta)^{\frac{1}{2}}\right)_0^{\pi}$$

$$= \frac{\sigma R}{2\varepsilon_0 z}\left[(z^2+R^2-2Rz)^{\frac{1}{2}} - (z^2+R^2-2Rz)^{\frac{1}{2}}\right]$$

$$\therefore V_{(P)} = \frac{\sigma R}{2\varepsilon_0 z}\left\{[(R+z)^2]^{\frac{1}{2}} - [(R-z)^2]^{\frac{1}{2}}\right\}$$

Bila $z > R$, maka

$[(R-z)^2]^{\frac{1}{2}} = z - R$, sehingga

$$V_{(P)} = \frac{\sigma R}{2\varepsilon_0 z}\left[(R+z)-(z-R)\right]$$

$$= \frac{\sigma R^2}{\varepsilon_0 z}$$

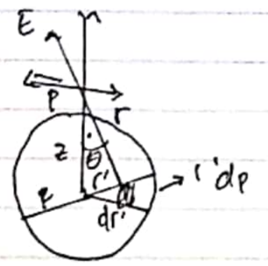
Bila $R > z$, maka

$[(R-z)^2]^{\frac{1}{2}} = R - z$

$$V_{(P)} = \frac{\sigma R}{2\varepsilon_0 z}\left[(R+z) - R + z\right]$$

$$V_{(P)} = \frac{\sigma R}{\varepsilon_0}$$

b.



$$V_{(P)} = \frac{1}{4\pi\varepsilon_0}\int_s \frac{\sigma d\vec{s}}{r}$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\int_s \frac{r' dr' d\phi}{(z^2+r'^2)^{\frac{1}{2}}}$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\int_0^R\int_0^{2\pi} \frac{d\phi \, r' dr'}{(z^2+r'^2)^{\frac{1}{2}}}$$

$$= \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^R \frac{1}{2}\frac{d(r'^2)}{(z^2+r'^2)^{\frac{1}{2}}}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\frac{1}{2}\cdot 2(z^2+r'^2)^{\frac{1}{2}}\right]_0^R$$

$$V_{(P)} = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{z^2+R^2} - z\right]$$

Persoalan diatas dapat juga dihitung dengan rumus $V_{(P)} = -\int \vec{E}_z \cdot d\vec{z}$