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2.4 Potensial Listrik

Sifat khusus dari muatan listrik E yaitu.

$\vec{\nabla} \times \vec{E} = 0$. Ini artinya medan $\frac{0}{E}$ merupakan gradien suatu skalar, jadi.

$\vec{E} \sim \nabla \phi$ maka $\nabla \times \vec{E} = 0$

Menurut teorema stoker

$\oint \vec{E}\, d\vec{p} \cdot \int \nabla \times \vec{E}\, d\vec{s} = 0$

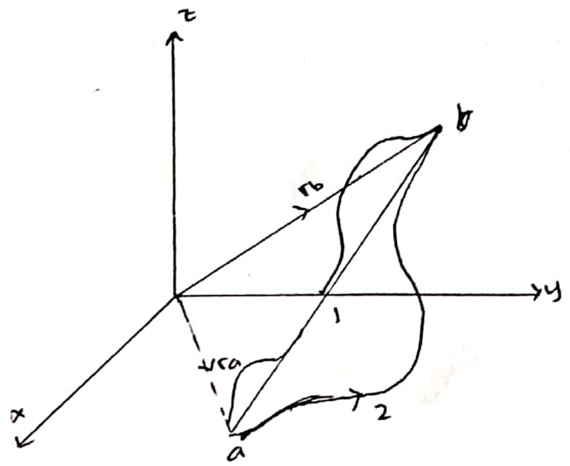
artinya $\oint^b \vec{E}\, dp$ tidak bergantung pada pilihan jalan a ke b

$E = \nabla V$

$E dp = -\nabla V\, d\vec{p} = -dV$

$\int \vec{E}\, d\vec{p} = \int_b^a -dV$

$\int \vec{E}\, d\vec{p} = V_b - V_a$



Va lintarannya tertutup

$V_b = V_a = 0$

$\int \vec{E} \cdot d\vec{p} = 0$

baru ke pers stoker

$\int E\, d\vec{p} = \int \nabla \times E\, d\vec{s}$

$d\vec{s} \neq 0$, $\nabla = \vec{E} = 0$

$|r_a . r_b|^2$

Hal ini berlaku untuk medan $\vec{E}$ dari suatu muatan listrik dititik asal atau dari setiap konfigurasi muatan yg statis.

Bila lintasan a ke b tertutup, melalui jalan 1 dan jalan 2 harga $\oint \vec{E}\, d\vec{p} \neq 0$ karna $(r_1 \neq r_2)$

Selanjutnya : $\vec{E} = -\nabla V$. (E merupakan gradien potensial)

$E = -\nabla V$

$\nabla \vec{E} = -\nabla \cdot \nabla V$

$\nabla \vec{E} = -\nabla^2 V$

$\frac{\rho}{\varepsilon_0}$

$\nabla^2 V = \frac{\rho}{\varepsilon_0}$

$E = -\nabla V$

$E \cdot dp = -\nabla V\, dp$

$\int_a^b E\, dp = -\int_a^b dV$

$= V \int_a^b dV$

$= V_b - V_a$

$V_b = V_a$

$\nabla V \, d\vec{p} = dV$

$E = -\nabla V$

$\int \vec{E} \, d\vec{p} = -\int \nabla V \, d\vec{p} = -\int dV$, jadi persamaan diatas $V_b = k \dfrac{Q}{r^2}$

$$V(a) = -\int_{\infty}^{a} \vec{E} \cdot d\vec{p}$$
$$V(b) = -\int_{a}^{b} \vec{E} \cdot d\vec{p}$$

$$V_{ab} = V_b - V_a = \int_{\infty}^{b} \vec{E} \, d\vec{p} + \int_{\infty}^{a} \vec{E} \, d\vec{p}$$
$$= -\int_{a}^{b} E \cdot \varphi$$
$$= -\int_{\infty}^{b} E \, dp - \int_{a}^{\infty} E \, d\vec{p}$$
$$= -\int_{a}^{b} \vec{E} \, d\vec{p}$$
$$V_b - V_a = -\int_{a}^{b} \vec{E} \, dp$$

$$\boxed{V(P) = -\int \vec{E} \, d\vec{p}}$$ → Rumus umum

Jadi Potensial di suatu titik P

$$V(P) = \int_{\infty}^{P} \vec{E} \, d\vec{p}$$

mural :

$$V(P) = -\int_{\infty}^{r} \vec{E} \, d\vec{p}$$
$$= -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \, dr$$
$$V(P) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$

Untuk sejumlah muatan $q_1, q_2, q_3, \ldots q_n$ masing² pada jarak $r_1, r_2, r_3, \ldots r_n$, maka

$$V(P) = \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots \frac{q_n}{r_n} \right]$$ → Penjumlahan biasa

Potensial oleh distribusi muatan

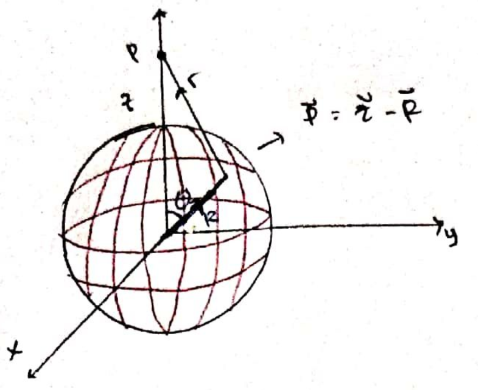
1) untuk kawat : $V(P) = \dfrac{1}{4\pi\varepsilon_0} \displaystyle\int \frac{\lambda \, dr}{r}$

2) untuk distribusi oleh permukaan , $V(P) = \dfrac{1}{4\pi\varepsilon_0} \displaystyle\int \frac{\sigma \, ds}{r}$

3) distribusi oleh volume $V(P) = \dfrac{1}{4\pi\varepsilon_0} \displaystyle\int \frac{\rho \, d\sigma}{r}$

Contoh

1. Potensial oleh muatan permukaan bersambung ...



$\vec{r} = \vec{z} - \vec{R}$

bola konduktor bermuatan homogen dg rapat muatan σ

Berapa potensial di titik P

Jawab: distribusi muatan pada permukaan bola, maka

$$V(P) = \int \frac{1}{4\pi\varepsilon_0} \frac{\sigma \, d\vec{s}}{r}$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \int \frac{R^2 \sin\theta \, d\theta \, d\phi}{(z^2 + R^2 - 2Rz\cos\theta)^{\frac{1}{2}}} \rightarrow |\vec{r}| = \pm (z^2 + R^2 - 2Rz\cos\theta)^{1/2}$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0} \int_0^{\pi} \int_0^{2\pi} \frac{d\phi \sin\theta \, d\theta}{(z^2 + R^2 - 2Rz\cos\theta)^{1/2}}$$

$$= \frac{2\pi\sigma R^2}{4\pi\varepsilon_0} \cdot \frac{1}{2} \int_0^{\pi} \frac{1}{Rz} \frac{d(-2Rz\cos\theta)}{(z^2 + R^2 - 2Rz\cos\theta)^{1/2}}$$

$$= \frac{\sigma R}{2\varepsilon_0 z} \left\{ 2(z^2 + R^2 - 2Rz\cos\theta)^{1/2} \right\}_0^{\pi}$$

$$= \frac{\sigma R}{2\varepsilon_0 z} \left[(z^2 + R^2 - 2Rz\cos\theta)^{\frac{1}{2}} - (z^2 + R^2 - 2Rz)^{1/2} \right]$$

$$\therefore V(P) = \frac{\sigma R}{2\varepsilon_0 z} \left\{ \left[(R+z)^2\right]^{\frac{1}{2}} - \left[(R-z)^2\right]^{\frac{1}{2}} \right\}$$

Bila $z > R$ maka

$[(R-z)^2]^{\frac{1}{2}} = z - R$, sehingga

$$V(P) = \frac{\sigma R}{2\varepsilon_0 z} \left[(R+z) - (z-R) \right]$$

$$V(P) = \frac{\sigma R^2}{\varepsilon_0 z}$$

Bila $R > z$ maka

$$[(R+z) - R + z]$$

$$V(P) = \frac{\sigma R}{2\varepsilon_0 z} \left[(R+z) - R + z \right]$$

$$V(P) = \frac{\sigma R}{\varepsilon_0}$$

$$V_1 = -\int \vec{E}\, d\vec{r} = -\int_\infty^R E_2\, d\vec{r} - \int_R^r \vec{E}_1\, d\vec{r}$$

$$= -\int_\infty^R \frac{1}{3\varepsilon_0 r^2}\rho R^3\, dr - \int_R^r \frac{1}{3\varepsilon_0}\rho r\, dr$$

$$= \frac{-\rho R^3}{3\varepsilon_0}\int_\infty^R \frac{dr}{r^2} - \frac{\rho}{3\varepsilon_0}\int_R^r r\, dr$$

$$= \frac{\rho R^2}{3\varepsilon_0} - \frac{\rho}{\varepsilon_0}(r^2 - R^2)$$

$$= \frac{\rho R^2}{3\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$= \frac{2\rho R^2}{6\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$= \frac{3}{6}\frac{\rho R^2}{\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$= \frac{1}{6\varepsilon_0}(3e^2 - r^2)$$

$$V_1 = \frac{\rho R^2}{\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$\therefore V_1 = \frac{\rho}{\varepsilon_0}\left(R^2 - \frac{r^2}{6}\right)$$

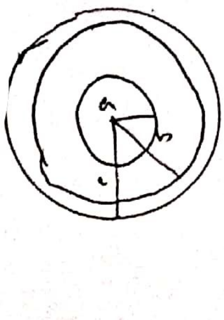
$$V_2 = -\int_a^r E_2\, dr$$

$$= -\int_\infty^r \frac{1}{3\varepsilon_0 r^2}\rho R^3\, dr$$

$$= \frac{-\rho R^3}{3\varepsilon_0}\int_\infty^r \frac{dr}{r^2}$$

$$\therefore V_2 = \frac{\rho R^2}{3\varepsilon_0 r}$$

b) bola konduktor konsentris seperti pada gambar



1. $r < a \rightarrow E = 0$

2. $a < r < b$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}_0$$

3. $b < r < c \rightarrow \vec{E} = 0$

4. $r > c$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}_0$$