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" Resume Materi "

2.4. Potensial Listrik

Sifat dari muatan listrik $\vec{E}$, yaitu :

$\nabla \times \vec{E} = 0$, ini berarti medan $\vec{E}$ merupakan gradien suatu skalar.

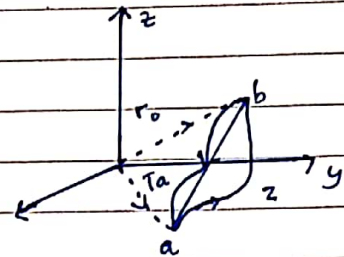
Gradien suatu skalar, jadi :

$$\vec{E} \propto \nabla \phi, \text{ maka } \nabla \times \vec{E} = 0$$

Menurut teorema Stokes

$$\oint \vec{E} \cdot d\vec{r} = \int \nabla \times \vec{E} \cdot d\vec{r} = 0$$

Artinya, $\oint_a^b \vec{E} \cdot d\vec{r}$



Hal ini berlaku untuk medan $\vec{E}$ dari suatu muatan listrik atau setiap konfigurasi muatan.

Bila lintasan a ke b tertutup melalui jalan 1 $\oint \vec{E} \cdot d\vec{r} \neq 0$

Selanjutnya : $\vec{E} = -\nabla V$

$$\nabla \cdot \vec{E} = -\nabla \cdot \nabla V$$

$$\nabla \cdot \vec{E} = -\nabla^2 V$$

$$\rho/\epsilon_0 = -\nabla^2 V$$

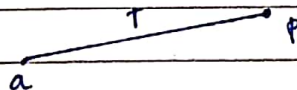
$$\nabla V \cdot d\vec{r} = dV$$

$$\vec{E} = -\nabla V$$

$$\int \vec{E} \cdot d\vec{r} = -\int \nabla V \cdot d\vec{r} = -\int dV$$

$$V_b - V_a = \int_a^b \vec{E} \cdot d\vec{r} \rightarrow V(p) = -\int \vec{E} \cdot d\vec{r} \text{ (Rumus umum)}$$

Misal :



$$V(p) = -\int_a^r \vec{E} \cdot d\vec{r}$$

$$= -\int_a^r \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} dr$$

$$V(p) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

Potensial oleh distribusi muatan

1). Untuk Kawat

$$V(p) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda dr}{r}$$

2). Untuk distribusi permukaan

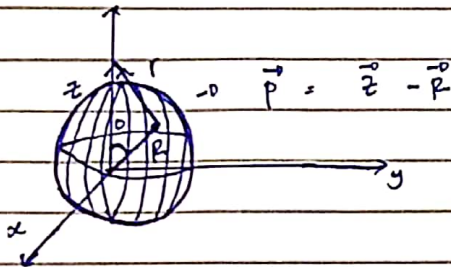
$$V(p) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma d\vec{s}}{r}$$

3). Distribusi oleh Volume

$$V(p) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho d\vec{s}}{r}$$

Contoh.

1. Potensial oleh muatan permukaan bersambung



bola konduktor bermuatan homogen dengan rapat muatan σ .

Berapa potensial di titik P.

Solusi: distribusi muatan pada permukaan bola, maka:

$$\begin{aligned} V(p) &= \int \frac{1}{4\pi\epsilon_0} \frac{\sigma d\vec{s}}{r} \\ &= \frac{\sigma}{4\pi\epsilon_0} \int \frac{R^2 \sin \theta d\theta d\phi}{(z^2 + R^2 - 2Rz \cos \theta)^{1/2}} \\ &= \frac{\sigma R^2}{4\pi\epsilon_0} \int_0^\pi \int_0^{2\pi} \frac{d\phi \sin \theta d\theta}{(z^2 + R^2 - 2Rz \cos \theta)^{1/2}} \\ &= \frac{\sigma R^2}{4\pi\epsilon_0} \frac{1}{2} \int_0^\pi \frac{d(-2Rz \cos \theta)}{Rz (z^2 + R^2 - 2Rz \cos \theta)^{1/2}} \\ &= \frac{\sigma R}{2\epsilon_0 z} \left[(z^2 + R^2 + 2Rz)^{1/2} - (z^2 + R^2 - 2Rz)^{1/2} \right] \end{aligned}$$

$$\therefore V(p) = \frac{\sigma R}{2\epsilon_0 z} \left\{ [(R+z)^2]^{1/2} - [(R-z)^2]^{1/2} \right\}$$

Bila $z > R$, maka $[(R-z)^2]^{1/2} = z - R$, sehingga:

$$V(p) = \frac{\sigma R}{2\epsilon_0 z} [(R+z) - (z-R)]$$

$$= \frac{\sigma R^2}{\epsilon_0 z}$$

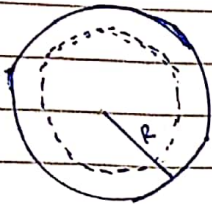
Bila $R > z$, maka $[(R-z)^2]^{1/2} = R - z$

$$V(p) = \frac{\sigma R}{2\epsilon_0 z} [(R+z) - (R-z)]$$

$$V(p) = \frac{\sigma R}{\epsilon_0}$$

2). Potensial oleh muatan volume

a.



bola dengan rapat muatan

 $\rho = \text{konstan}$, dengan jari-jari R .

Hitung potensial pada

i $r < R$, ii $r > R$

Solusi : $r < R$, $E_1 \rightarrow \oint E_1 d\vec{s} = \frac{1}{\epsilon_0} \rho V$

$$E_1 (4\pi r^2) = \frac{1}{\epsilon_0} \rho \left(\frac{4}{3} \pi r^3 \right)$$

$$\vec{E}_1 = \frac{1}{3\epsilon_0} \rho r \hat{r}_0$$

$$r > R, E_2 \rightarrow \oint E_2 d\vec{s} = \frac{1}{\epsilon_0} \rho V$$

$$E_2 (4\pi r^2) = \frac{1}{\epsilon_0} \rho \left(\frac{4}{3} \pi R^3 \right)$$

$$\vec{E}_2 = \frac{1}{3\epsilon_0 r^2} \rho R^3 \hat{r}_0$$

$$V_1 = - \int \vec{E} \cdot d\vec{s} = - \int_{\infty}^R E_2 \cdot d\vec{r} - \int_R^r \vec{E}_1 \cdot d\vec{r}$$

$$= - \int_{\infty}^R \frac{1}{3\epsilon_0 r^2} \rho R^3 dr - \int_R^r \frac{1}{3\epsilon_0} \rho r dr$$

$$= - \frac{\rho R^3}{3\epsilon_0} \int_{\infty}^R \frac{dr}{r^2} - \frac{\rho}{3\epsilon_0} \int_R^r r dr$$

$$= \frac{\rho R^2}{3\epsilon_0} + \frac{\rho R^2}{6\epsilon_0} - \frac{\rho r^2}{6\epsilon_0} = \frac{2\rho R^2}{6\epsilon_0} + \frac{\rho R^2}{6\epsilon_0} - \frac{\rho r^2}{6\epsilon_0}$$

$$V_1 = \frac{\rho R^2}{\epsilon_0} - \frac{\rho r^2}{6\epsilon_0} = \frac{3}{6} \frac{\rho R^2}{\epsilon_0} - \frac{\rho r^2}{6\epsilon_0}$$

$$\therefore V_1 = \frac{\rho}{\epsilon_0} \left(R^2 - \frac{r^2}{6} \right) = \frac{1}{6\epsilon_0} (3\rho R^2 - \rho r^2)$$

$$V_2 = \int_{\infty}^r E_2 \cdot dr = \int_{\infty}^r \frac{1}{3\epsilon_0} \rho R^3 \frac{dr}{r^2}$$

$$= - \int_{\infty}^r \frac{1}{3\epsilon_0} \rho R^3 \frac{dr}{r^2}$$

$$= \frac{\rho R^3}{3\epsilon_0 r}$$

3). Potensial oleh muatan pada kawat

a) Kawat panjang sekali.

$$E_p = \frac{2\lambda}{4\pi\epsilon_0 z}$$

$$V_p = - \int_{\infty}^z E \cdot dr$$

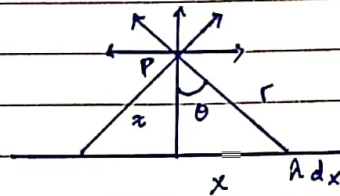
$$= - \frac{2\lambda}{4\pi\epsilon_0} \int_{\infty}^z \frac{dz}{z}$$

$$= - \frac{2\lambda}{4\pi\epsilon_0} \ln z$$

b). Kawat dengan panjang tertentu, misalnya $2L$

Dari perhitungan di depan

$$E_p = \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z\sqrt{z^2 + L^2}}$$



$$V = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda dx}{r}$$

$$= \frac{2\lambda}{4\pi\epsilon_0} \int_0^L \frac{dx}{(z^2 + x^2)^{1/2}}$$

$$x = z \tan \theta$$

$$dx = z \sec^2 \theta d\theta$$

$$= \frac{2\lambda}{4\pi\epsilon_0} \int_0^L \frac{z \sec^2 \theta d\theta}{z (\tan^2 \theta + 1)^{1/2}}$$

$$= \frac{2\lambda}{4\pi\epsilon_0} \int_0^L \sec \theta d\theta$$

$$= \frac{2\lambda}{4\pi\epsilon_0} \ln (\sec \theta + \tan \theta) + C$$

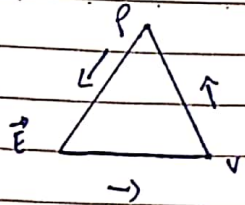
$$= \frac{2\lambda}{4\pi\epsilon_0} \ln \left(\frac{\sqrt{x^2 + z^2}}{z} + \frac{x}{z} \right) \Big|_0^L$$

$$\therefore V = \frac{\lambda}{2\pi\epsilon_0} \ln \left(\frac{\sqrt{L^2 + z^2}}{z} + \frac{L}{z} \right)$$

2.5. Hubungan antara $\vec{E}$, ρ dan V

Dalam elektronika hubungan antara kuat medan dengan rapat muatan volume dan V skalar.





$$\left. \begin{aligned} E &= -\nabla V \\ \nabla^2 V &= -\frac{\rho}{\epsilon_0} \end{aligned} \right\} \begin{aligned} \nabla \times \vec{E} &= 0 \\ \nabla \cdot \vec{E} &= \frac{1}{\epsilon_0} \rho \end{aligned} \quad \text{Syarat elektrostatis}$$

Bentuk Integralnya = $\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dQ}{r^2}$; $dQ = \rho d\sigma$

$$= \frac{1}{4\pi\epsilon_0} \int \frac{\rho d\sigma}{r^2}$$

$$V = - \int \vec{E} \cdot d\vec{p} \quad \text{hubungan } V \text{ dengan } \rho =$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho d\sigma}{r}$$

Solusi Segitiga diatas

1. Misal E diketahui, V dan ρ dapat dihitung

$$V = - \int E \cdot dp$$

$$\rho = \epsilon_0 \nabla \cdot E$$

2. Misal V diketahui, E dan ρ dapat dicari

$$E = -\nabla V$$

$$\rho = -\frac{1}{\epsilon_0} \nabla^2 V$$

3. Misal ρ diketahui, E dan V dapat dihitung

$$E = \frac{1}{4\pi\epsilon_0} \int \frac{\rho d\sigma}{r^2}$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho d\sigma}{r}$$

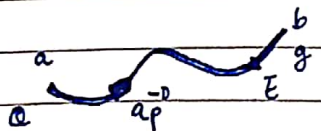
2.6. Usaha atau energi dalam Elektrostatika

$$W = \int \vec{F} \cdot d\vec{s}$$

Jika kerja yang dilakukan oleh gaya F melawan medan listrik yang ditimbulkan oleh muatan q , maka:

$$W = \int (-F) \cdot ds$$

Misal, kita memindahkan muatan Q dari a ke b



$$\vec{F} = \frac{Qq}{4\pi\epsilon_0 r^2} \hat{r}_0$$

$$F = -QE ; \quad \vec{E} = -\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}_0$$

$$W = \int_a^b (-F) \cdot d\rho$$

$$= - \int_a^b Q \vec{E} \cdot d\vec{\rho}$$

$$= -Q \int_a^b E \cdot d\vec{\rho} ; \quad - \int_a^b E \cdot a \vec{\rho} = V_b - V_a$$

$$\therefore W = Q (V_b - V_a)$$

$$W = -Q \int_{\infty}^P E \cdot a \rho$$

$$W = Q V_p$$

2.6.2 Distribusi muatan

a) Distribusi muatan listrik.