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2.4 Potensial Listrik

Sifat khusus dari medan listrik $\vec{E}$ yaitu $\nabla \times \vec{E} = 0$, medan $\vec{E}$ merupakan gradien suatu skalar. Jadi

$$\vec{E} \sim \nabla \phi \quad \text{maka} \quad \nabla \times \vec{E} = 0$$

Teorema stokes

$$\oint \vec{E}\, d\vec{P} = \int \nabla \times \vec{E}\, d\vec{s} = 0$$

artinya $\oint_a^b \vec{E} \cdot d\vec{P}$ tidak bergantung

pada pilihan a ke b

$$\vec{E} = -\nabla V$$

$$E \cdot dP = -\nabla V \cdot d\vec{P}$$

$$\int E \cdot dP = \int_a^b -dV$$

$$\int \vec{E} \cdot d\vec{P} = V_b - V_a$$

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Dipindai dengan CamScanner

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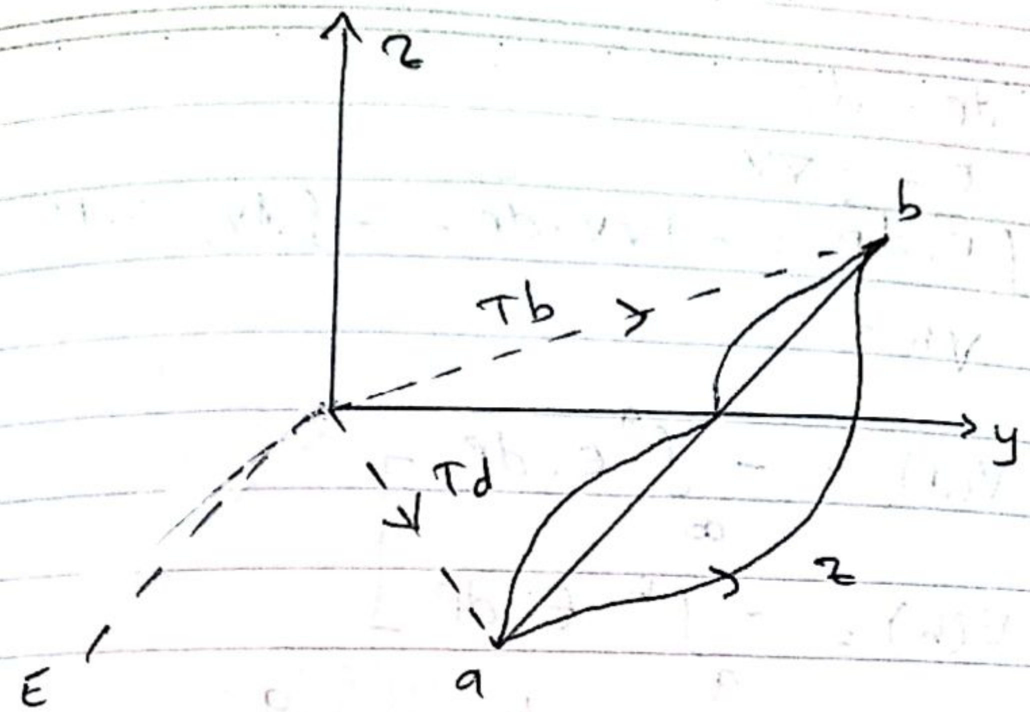

Hal ini berlaku untuk medan $\vec{E}$ dari suatu titik di titik asal atau dari setiap konfigurasi muatan yang statis.

Bila lintasan a ke b tertutup melalui jalan 1 dan jalan 2 harga $\oint \vec{E} \, d\vec{P} \neq 0$ karena $(r_1 + r_2)$

Selanjutnya : $\vec{E} = -\nabla V$ ($\vec{E}$ mrp. gardien Potential).

$$E = -\nabla V$$
$$\nabla \vec{E} = -\nabla \cdot \nabla V$$
$$\nabla \vec{E} = -\nabla^2 V$$
$$\rho / \epsilon_0 = -\nabla^2 V$$
$$= \nabla^2 V = \frac{-\rho}{\epsilon_0}$$

$$E = -\nabla V$$
$$E \cdot dP = -\nabla V \, dP$$
$$\int_a^b E \, dP = \int_a^b dV$$
$$= 0 \quad \int_a^b dV = V_b - V_a$$
$$V_B = V_a$$

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$$\nabla V \cdot d\rho = dV$$

$$E = -\nabla V$$

$$\int \vec{E} \cdot d\vec{\rho} = -\int \nabla V \cdot d\vec{\rho} = -\int dV,$$ Jadi

$$V(a) = -\int_{\infty}^{a} E \cdot d\rho$$

$$V(b) = -\int_{a}^{b} E \cdot d\rho$$

$$V_{ab} = V_B - V_a = -\int_{a}^{b} E \cdot d\rho + \int_{b}^{a} E \cdot d\rho = -\int_{a}^{b} E \cdot d\rho$$

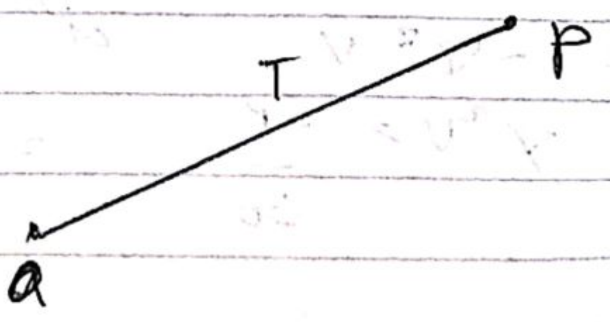
$$V_b - V_a = -\int_{a}^{b} \vec{E} \cdot d\vec{r}$$

$$\boxed{V(P) = -\int E \cdot d\rho}$$

Jadi potensial di suatu titik P.

$$V(P) = -\int_{\infty}^{P} \vec{E} \cdot d\vec{r}$$

misal.



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$$V(P) = -\int_{\infty}^{T} \vec{E} \cdot d\vec{P}$$

$$= -\int_{\infty}^{T} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} dr.$$

$$V(P) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$

untuk sejumlah muatan $q_1, q_2, q_3 \dots q_n$ masing-masing pada jarak $r_1, r_2, r_3 \dots r_n$, maka.

$$V(P) = \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \dots \frac{q_n}{r_n} \right]$$

Potential oleh distribusi muatan

1). untuk kawat

$$V(P) = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda dr}{r}$$

2). untuk distribusi oleh permukaan

$$V(P) = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma ds}{r}$$

3). distribusi oleh volume

$$V(P) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho d\tau}{r}$$

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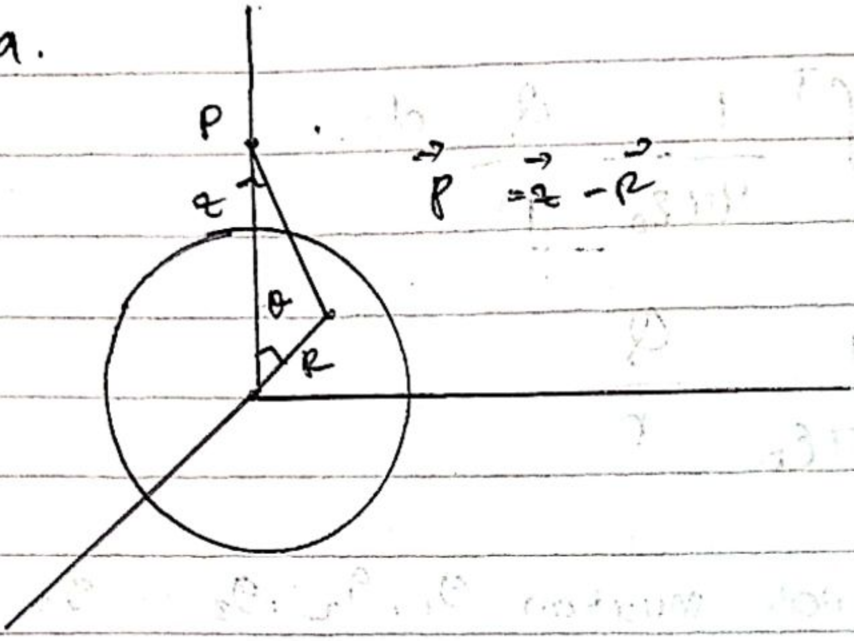
1. Potensial oleh muatan Permukaan bersambung...

a.



$\vec{p} = \vec{z} - \vec{R}$

bola konduktor bermuatan homogen dengan rapat muatan σ

Berapa Potensial di titik P.

Solusi : distribusi muatan pada Permukaan bola, maka

$$V(P) = \int \frac{1}{4\pi\varepsilon_0} \frac{\sigma \, d\vec{s}}{r}$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \int_s \frac{R^2 \sin\theta \, d\theta \, d\phi}{(z^2 + R^2 - 2Rz\cos\theta)^{1/2}}$$

$$\rightarrow |\vec{p}| \; (z^2 + R^2 - 2Rz\cos\theta)^{1/2}$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0} \int_0^{\pi} \int_0^{2\pi} \frac{d\phi \sin\theta \, d\theta}{(z^2 + R^2 - 2Rz\cos\theta)^{1/2}}$$

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$$= \frac{2\pi \sigma R^2}{4\pi \varepsilon_0} \frac{1}{2} \int_0^\pi \frac{1}{Rz} \frac{d(-2Rz\cos\theta)}{(z^2+R^2-2Rz\cos\theta)^{1/2}}$$

$$= \frac{\sigma R}{2\varepsilon_0 z} \left(2(z^2+R^2-2Rz\cos\theta)^{1/2} \right)_0^\pi$$

$$= \frac{\sigma R}{2\varepsilon_0 z} \left[(z^2+R^2+2Rz)^{1/2} - (z^2+R^2-2Rz)^{1/2} \right]$$

$$\therefore V(P) = \frac{\sigma R}{2\varepsilon_0 z} \left\{ \left[(R+z)^2\right]^{1/2} - \left[(R-z)^2\right]^{1/2} \right\}$$

Bila $z > R$, maka

$\left[(R-z)^2\right]^{1/2} = z - R$, sehingga:

$$V(P) = \frac{\sigma R}{2\varepsilon_0 z} \left[(R+z) - (z-R) \right]$$

$$= \frac{\sigma R^2}{\varepsilon_0 z}$$

Bila $R > z$, maka:

$$\left[(R-z)^2\right]^{1/2} = R - z$$

$$V(P) = \frac{\sigma R}{2\varepsilon_0 z} \left[(R+z) - R + z \right]$$

$$V(P) = \frac{\sigma R}{\varepsilon_0}$$

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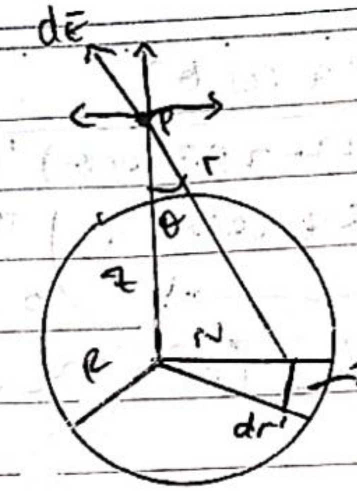
b.



$$V(P) = \frac{1}{4\pi\varepsilon_0}\int_s \frac{\sigma\, d\vec{s}}{r}$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\int_s \frac{r'dr'd\varphi}{(z^2+r'^2)^{1/2}}$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\int_0^R\int_0^{2\pi} \frac{d\varphi\, r'dr'}{(z^2+r'^2)^{1/2}}$$

$$= \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^R \frac{1}{2}\frac{d(r'^2)}{(z^2+r'^2)^{1/2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\frac{1}{2}\,2\,(z^2+r'^2)^{1/2}\right]_0^R$$

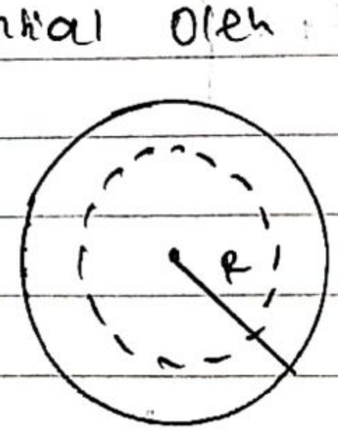
$$V(P) = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{z^2+R^2} - z\right]$$

Persoalan diatas dapat juga dihitung dengan rumus :

$$V(P) = -\int \vec{E}_z \cdot \vec{d}_z$$

2. Potential oleh muatan volume

a.



bola dengan rapat muatan ρ = constan dg. jari-jari R.

Hitung Potensial pada

i $r < R$

ii $r > R$

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Solusi

$r < R$, $E_1 \rightarrow \oint E_1 \, d\vec{s} = \frac{1}{\varepsilon_0} \rho V$

$$E_1 (4\pi r^2) = \frac{1}{\varepsilon_0} \rho \left(\frac{4}{3} \pi r^3\right)$$

$$\vec{E_1} = \frac{1}{3\varepsilon_0} \rho r \hat{r_0}$$

$r > R$, $E_2 \rightarrow \oint E_2 \, d\vec{s} = \frac{1}{\varepsilon_0} \rho V$

$$E_2 (4\pi r^2) = \frac{1}{\varepsilon_0} \rho \left(\frac{4}{3} \pi R^3\right)$$

$$\vec{E_2} = \frac{1}{3\varepsilon_0 r^2} \rho R^3 \hat{r_0}$$

$$V_1 = -\int \vec{E} \cdot d\vec{P} = -\int_{\infty}^{R} E_2 \, d\vec{r} - \int_{R}^{r} E_1 \cdot d\vec{r}$$

$$= -\int_{\infty}^{R} \frac{1}{3\varepsilon_0 r^2} \rho R^3 dr - \int_{R}^{r} \frac{1}{3\varepsilon_0} \rho r \, dr$$

$$= -\frac{\rho R^3}{3\varepsilon_0} \int_{\infty}^{R} \frac{dr}{r^2} - \frac{\rho}{3\varepsilon_0} \int_{R}^{r} r \, dr$$

$$= \frac{\rho R^2}{3\varepsilon_0} - \frac{\rho}{\varepsilon_0} (r^2 - R^2)$$

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$$= \frac{\rho R^2}{3\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0} = \frac{2\rho R^2}{6\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$= \frac{3}{6}\frac{\rho R^2}{\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$V_1 = \frac{\rho R^2}{\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$$

$$= \frac{1}{6\varepsilon_0}\left(3\rho R^2 - \rho r^2\right)$$

$$= \frac{\rho}{6\varepsilon_0}\left(3R^2 - r^2\right)$$

$$\therefore V_1 = \frac{\rho}{\varepsilon_0}\left(R^2 - \frac{r^2}{6}\right)$$

$$V_2 = -\int_\infty^r E_2\, dr$$

$$= -\int_\infty^r \frac{1}{3\varepsilon_0 r^2}\rho R^3\, dr$$

$$= -\frac{\rho R^3}{3\varepsilon_0}\int_\infty^r \frac{dr}{r^2}$$

$$\therefore V_2 = \frac{\rho R^3}{3\varepsilon_0 r}$$

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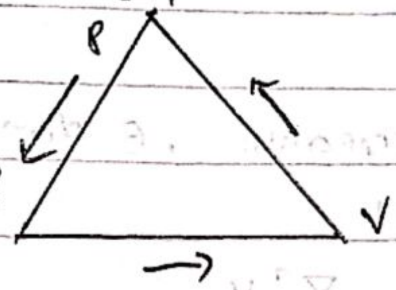
2.5. Hubungan antara $\vec{E}$, ρ dan V

Dalam elektrostatika hubungan antara kuat medan dengan rapat muatan volume dan V skalar adalah merupakan hubungan segitiga



$E = -\nabla V$ $\qquad$ $\nabla \times E = 0$ $\qquad$ syarat

$\nabla V = -\dfrac{\rho}{\varepsilon_0}$ $\qquad$ $\nabla \cdot E = \dfrac{1}{\varepsilon_0}\rho$ $\qquad$ elektrostatika

$\rho = \varepsilon_0 \nabla E$

Bentuk integralnya :

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ}{r^2} \;;\; dQ = \rho\, d\tau$$

$$= \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{r^2}$$

$$V = -\int E \cdot d\ell$$

hubungan V dengan ρ

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{r}$$

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Solusi hubungan segitiga diatas

1. misal E diketahui, V dan ρ dapat dihitung

$$V = -\int E \cdot d\rho$$

$$\rho = \varepsilon_0 \nabla \cdot E$$

2. Misal : V diketahui, E dan ρ dapat dicari

$$E = -\nabla V$$

$$\rho = -\frac{1}{\varepsilon_0} \nabla^2 V$$

3. Misal ρ diketahui E dan V dapat dihitung

$$E = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho \, d\tau_0}{\rho^2}$$

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho \, d\tau_0}{r}$$