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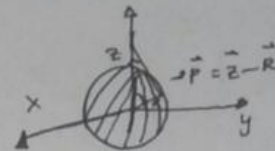
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Kelas : B

a. Bola konduktor bermuatan homogen dengan rapat muatan σ berapa potensial di titik P.

solusi : distribusi muatan pada permukaan bola, maka

$$\begin{aligned} V(r) &= \int \frac{1}{4\pi\epsilon_0} \frac{\sigma d\vec{s}}{r} \\ &= \frac{\sigma}{4\pi\epsilon_0} \int \frac{R^2 \sin\theta d\theta d\phi}{(z^2 + R^2 - 2Rz \cos\theta)^{\frac{1}{2}}} \\ &= \frac{\sigma R^2}{4\pi\epsilon_0} \int_0^\pi \int_0^{2\pi} \frac{d\phi \sin\theta d\theta}{(z^2 + R^2 - 2Rz \cos\theta)^{\frac{1}{2}}} \\ &= \frac{\sigma R^2}{4\pi\epsilon_0} \frac{1}{2} \int_0^\pi \frac{1}{Rz} \frac{\sigma (-2Rz \cos\theta)}{(z^2 + R^2 - 2Rz \cos\theta)^{\frac{1}{2}}} \\ &= \frac{\sigma R^2}{4\pi\epsilon_0 z} \left[(z^2 + R^2 + 2Rz)^{\frac{1}{2}} - (z^2 + R^2 - 2Rz)^{\frac{1}{2}} \right] \end{aligned}$$



$$* V(P) = \frac{\sigma R}{2\epsilon_0 z} \left\{ \left[(R+z)^2 \right]^{\frac{1}{2}} - \left[(R-z)^2 \right]^{\frac{1}{2}} \right\}$$

Bila $z > R$, maka

$$\left[(R-z)^2 \right]^{\frac{1}{2}} = z - R, \text{ sehingga.}$$

$$\begin{aligned} V(P) &= \frac{\sigma R}{2\epsilon_0 z} \left[(R+z) - (z-R) \right] \\ &= \frac{\sigma R^2}{\epsilon_0 z} \end{aligned}$$

Bila $R > z$, maka

$$\left[(R-z)^2 \right]^{\frac{1}{2}} = R - z$$

$$\begin{aligned} V(P) &= \frac{\sigma R}{2\epsilon_0 z} \left[(R+z) - R + z \right] \\ &= \frac{\sigma R}{\epsilon_0 z} \end{aligned}$$

Bila $R > z$, maka

$$\begin{aligned} \left[(R-z)^2 \right]^{\frac{1}{2}} &= R - z \quad \rightarrow \quad V(P) = \frac{\sigma R}{2\epsilon_0 z} \left[(R+z) - R + z \right] \\ V(P) &= \frac{\sigma R}{\epsilon_0 z} \end{aligned}$$

b. persolan dengan rumus :

$$V(p) = - \int \vec{E}_z \cdot d\vec{z}$$

maka,

$$V(p) = \frac{1}{4\pi\epsilon_0} \int_S \frac{\sigma d\vec{s}}{r}$$

$$= \frac{\sigma}{4\pi\epsilon_0} \int \frac{r' dr' d\phi}{(z^2 + r'^2)^{\frac{1}{2}}}$$

$$= \frac{\sigma}{4\pi\epsilon_0} \int_0^R \int_0^{2\pi} \frac{d\phi r' dr'}{(z^2 + r'^2)^{\frac{1}{2}}}$$

$$= \frac{2\pi\sigma}{4\pi\epsilon_0} \int_0^R \frac{1}{2} \frac{dr(r'^2)}{(z^2 + r'^2)^{\frac{1}{2}}}$$

$$= \frac{\sigma}{2\epsilon_0} \left[\frac{1}{2} \cdot x (z^2 + r'^2)^{\frac{1}{2}} \right]_0^R$$

$$V(p) = \frac{\sigma}{2\epsilon_0} \left[\sqrt{z^2 + R^2} - z \right]$$

