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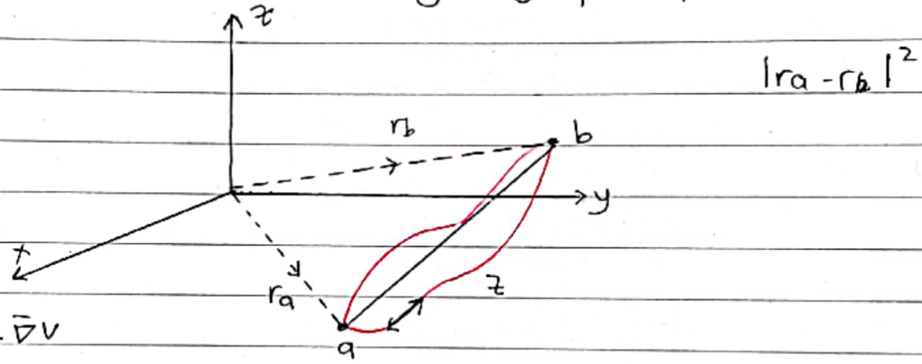
Nama = Alfia Rosa
NPM = 2013022040

Potensial listrik

Sifat khusus dari medan listrik $\vec{E}$ yaitu
$\vec{\nabla} \times \vec{E} = 0$, ini artinya medan $\vec{E}$ merupakan gradien suatu skalar
Jadi, $\vec{E} \sim \nabla \phi$, maka : $\vec{\nabla} \times \vec{E} = 0$
Menurut teorema stokes
$\oint \hat{E}\, d\vec{p} = \int \nabla \times \vec{E}\, d\vec{s} = 0$
artinya $\oint_a^b \vec{E}\, d\vec{p}$ tidak bergantung pada pilihan jalan a ke b



$\vec{E} = -\vec{\nabla} V$
$E\phi = -\nabla V \cdot d\vec{p}$
$= -dV$
$\int \vec{E}\, d\vec{p} = \int_a^b -dV$
$\int \vec{E}\, dp = V_b - V_a$

V, lintasan tertutup
$V_b = V_a$
$V_b - V_a = 0$
sehingga
$\int \vec{E} \cdot d\vec{p} = 0$

$\int E\, dp = \int \nabla \times E\, d\vec{p}$
$d\vec{x} \neq 0$
$\vec{\nabla} \times \vec{E} = 0$

Hal ini berlaku untuk medan E dari suatu muatan titik dititik asal atau dari setiap konfigurasi muatan sebagai status.

Bila lintasan a ke b tertutup, melalui jalan 1 ada jalan 2 harga $\oint \vec{E}\, d\vec{p} \neq 0$ karena $(r_1 \neq r_2)$

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Dipindai dengan CamScanner

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Selanjutnya : $\vec{E} = -\nabla V$ ($\vec{E}$ merupakan gradien potensial)

$$\vec{E} = -\nabla V$$
$$\nabla \vec{E} = -\nabla . \nabla V$$
$$\nabla \vec{E} = -\nabla^2 V$$
$$\rho / \varepsilon_0 = -\nabla^2 V$$
$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} V . d\rho = dV$$
$$E = -\nabla V$$
$$\int \vec{E}\, d\vec{\rho} = -\int \nabla V\, d\vec{\rho}$$
$$= -\int dV$$

Jadi, persamaan diatas

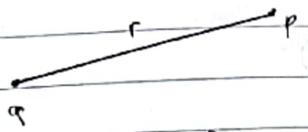
$$\left.\begin{aligned} V(a) &= -\int_{\infty}^{a} \vec{E}\, d\vec{\rho} \\ V(b) &= -\int_{a}^{b} E\, d\vec{\rho} \end{aligned}\right] V_{ab} = V_b - V_a$$
$$= -\int_{\infty}^{b} \vec{E}\, d\rho + \int_{\infty}^{a} \vec{E}\, d\rho$$
$$= -\int_{a}^{b} \vec{E}\, d\vec{\rho}$$
$$= -\int_{a}^{b} E\, d\rho - \int_{a}^{\infty} E\, d\vec{\rho}$$
$$= -\int_{\infty}^{p} \vec{E}\, d\vec{\rho}$$

$$V_b - V_a = -\int_{a}^{b} \vec{E} . d\rho$$
$$\boxed{V(P) = -\int \vec{E}\, d\rho}$$

Jadi, potensial di suatu titik P

$$V(P) = -\int_{\infty}^{P} \vec{E}\, d\vec{\rho}$$

misal



$$V(P) = -\int_{\infty}^{r} \vec{E}\, d\vec{\rho}$$
$$= -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}\, dr$$
$$V(P) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$

untuk muatan $Q_1, Q_2, Q_3, \ldots, Q_n$ masing masing pada jarak $r_1, r_2, r_3, r_4, \ldots, r_n$ maka

$$V(p) = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} + \ldots \frac{Q_n}{r_n}\right]$$

Potensial oleh distribusi muatan

1) untuk kawat

$$V(p) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\, dr}{r}$$

2) untuk distribusi oleh permukaan

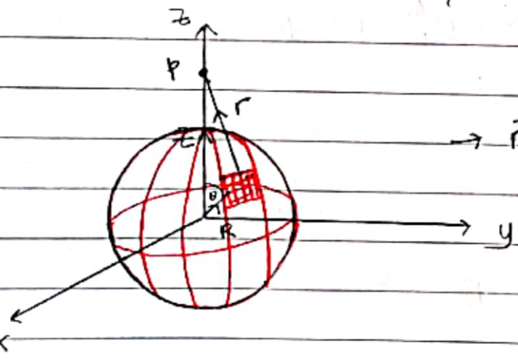
$$V(p) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\, ds}{r}$$

3) Distribusi oleh volume

$$V(p) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{r}$$

a. bola konduktor bermuatan homogen rapat muatan σ berapa potensial dititik p.

solusi : distribusi muatan pada permukaan bola, maka



$\rightarrow \vec{r} = \vec{z} - \vec{R}$

$$V(p) = \int \frac{1}{4\pi\varepsilon_0}\frac{\sigma\, ds}{r}$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\int_s \frac{R^2 \sin\theta\, d\theta\, d\phi}{(z^2+R^2-2Rz\cos\theta)^{1/2}}$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0}\int_0^{\pi}\int_0^{2\pi} \frac{d\phi \sin\theta\, d\theta}{(z^2+R^2-2Rz\cos\theta)^{1/2}}$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0}\cdot\frac{1}{2}\int_0^{\pi}\frac{1}{Rz}\frac{d(-2Rz\cos\theta)}{(z^2+R^2-2Rz\cos\theta)^{1/2}}$$

$$= \frac{\sigma R}{\varepsilon_0 z}\left(2(z^2+R^2-2Rz\cos\theta)^{\frac{1}{2}}\right)\Big|_0^{\pi}$$

$$= \frac{\sigma R}{2\varepsilon_0 z}\left[(z^2+R^2+2Rz)^{\frac{1}{2}} - (z^2+R^2-2Rz)^{\frac{1}{2}}\right]$$

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$$V(p) = \frac{\sigma R}{2\varepsilon_0 z}\left\{\left[(R+z)^2\right]^{1/2} - \left[(R-z)^2\right]^{1/2}\right\}$$

Bila $z > R$, maka

$\left[(R-z)^2\right]^{1/2} = z - R$, sehingga

$$V(p) = \frac{\sigma R}{2\varepsilon_0 z}\left[(R+z) - (z-R)\right]$$

$$= \frac{\sigma R^2}{\varepsilon_0 z}$$

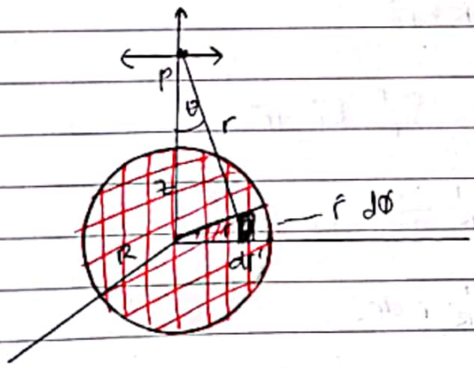
Bila $R > z$ maka

$\left[(R-z)^2\right]^{1/2} = R - z$

$$V(p) = \frac{\sigma R}{2\varepsilon_0 z}\left[(R+z) - R + z\right]$$

$$V(p) = \frac{\sigma R}{\varepsilon_0}$$

b.



$$V(p) = \frac{1}{4\pi\varepsilon_0}\int_s \frac{\sigma\, ds}{r}$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\int_s \frac{r'dr'd\phi}{(z^2+r'^2)^{1/2}}$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\int_0^R\int_0^{2\pi} \frac{d\phi\, r'dr'}{(z^2+r'^2)^{1/2}}$$

$$= \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^R \frac{1}{2}\,\frac{d(r'^2)}{(z^2+r'^2)^{1/2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\frac{1}{2}\cdot 2\,(z^2+r'^2)^{\frac{1}{2}}\right]_0^R$$

$$V(p) = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{z^2+R^2} - z\right]$$

Persoalan diatas dapat juga dihitung dengan rumus $V(p) = -\int \vec{E}_z \cdot d\vec{z}$

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Potensial oleh muatan volume

9.

bola dengan rapat muatan
ρ = konstan, dengan Jari-jari R
Hitung potensial pada
$r < R$
$r > R$

Solusi

$r < R$, $E_1 \rightarrow \oint E_1 d\vec{s} = \frac{1}{\varepsilon_0} \rho V$

$E_1 (4\pi r^2) = \frac{1}{\varepsilon_0} \rho \left(\frac{4}{3}\pi r^3\right)$

$E_1 = \frac{1}{3\varepsilon_0} \rho r \hat{r_0}$

$r > R$, $E_2 = \oint E_2 d\vec{s} = \frac{1}{\varepsilon_0} \rho V$

$E_2 (4\pi r^2) = \frac{1}{\varepsilon_0} \rho \left(\frac{4}{3}\pi R^3\right)$

$\vec{E_2} = \frac{1}{3\varepsilon_0 r^2} \rho R^3 \hat{r_0}$

$V_1 = -\int \vec{E}\, d\vec{p} = -\int_\infty^R \vec{E_2}\, d\vec{r} - \int_R^r \vec{E_1}\, d\vec{r}$

$= -\int_\infty^R \frac{1}{3\varepsilon_0 r^2} \rho R^3 dr - \int_R^r \frac{1}{3\varepsilon_0} \rho r\, dr$

$= -\frac{\rho R^3}{3\varepsilon_0} \int_\infty^R \frac{dr}{r^2} - \frac{\rho}{3\varepsilon_0} \int_R^r r\, dr$

$= \frac{\rho R^2}{3\varepsilon_0} - \frac{\rho}{\varepsilon_0} (r^2 - R^2)$

$= \frac{\rho R^2}{3\varepsilon_0} + \frac{\rho R^2}{6\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$

$V_1 = \frac{\rho R^2}{\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}$

$V_1 = \frac{\rho}{\varepsilon_0} \left(R^2 - \frac{r^2}{6}\right)$

$V_2 = -\int_\infty^r E_2\, dr$

$= -\int_\infty^r \frac{1}{3\varepsilon_0 r^2} \rho R^3 dr$

$= -\frac{\rho R^3}{3\varepsilon_0} \int_\infty^r \frac{dr}{r^2} = \frac{\rho R^3}{3\varepsilon_0 r}$

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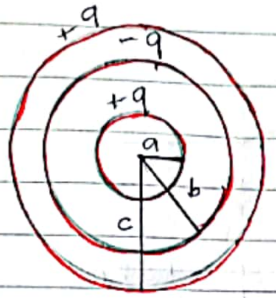
b. Bola konduktor konsentris seperti pada gambar



1) $r < a \rightarrow E = 0$

2) $a < r < b$

$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r_0}$

3) $b < r < c \rightarrow E = 0$

4) $r > c$

$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r_0}$

$V_1 = -\int_\infty^r \vec{E}\, d\vec{r}$

$= -\frac{q}{4\pi\varepsilon_0} \left[\int_a^c E_4\, dr + \int_c^{b=0} E_3\, dr + \int_b^a E_2\, dr + \int_a^{r=0} E_1\, dr \right]$

$= -\frac{q}{4\pi\varepsilon_0} \left[\int_\infty^c \frac{dr}{r^2} + \int_b^a \frac{dr}{r^2} \right]$

$= -\frac{q}{4\pi\varepsilon_0} \left[-\frac{1}{c} + \left(\frac{1}{a} + \frac{1}{b} \right) \right]$

$V_1 = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b} \right)$

$V_2 = \frac{-q}{4\pi\varepsilon_0} \left[\int_\infty^c \frac{dr}{r^2} + \int_c^{b=0} \frac{dr}{r^2} + \int_b^r \frac{dr}{r^2} \right]$

$= -\frac{q}{4\pi\varepsilon_0} \left[\int_\infty^c \frac{dr}{r^2} + \int_b^r \frac{dr}{r^2} \right]$

$= -\frac{q}{4\pi\varepsilon_0} \left[-\frac{1}{c} + \left(-\frac{1}{r} + \frac{1}{b} \right) \right]$

$V_2 = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{c} + \frac{1}{r} - \frac{1}{b} \right)$

$V_3 = -\frac{q}{4\pi\varepsilon_0} \left[\int_\infty^c \frac{dr}{r^2} + \int_c^r \frac{dr}{r^2} \right]$

$= -\frac{q}{4\pi\varepsilon_0} \left(-\frac{1}{c} + \left(-\frac{1}{r} + \frac{1}{c} \right) \right)$

$= \frac{q}{4\pi\varepsilon_0 r}$; $b < r < c$

$V_4 = -\int_a^r \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}\, dr$

$V_4 = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

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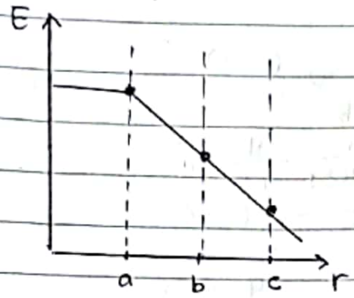
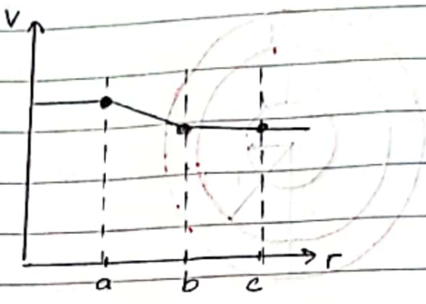
Grafiknya





Potensial oleh muatan pada kawat

a) Kawat panjang sekali

$$E_p = \frac{2\lambda}{4\pi\varepsilon_0}$$

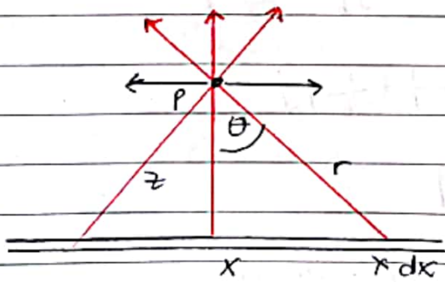
$$V_p = -\int_{\infty}^{a} E \cdot ap$$

$$= -\frac{2\lambda}{4\pi\varepsilon_0} \int_{\infty}^{a} \frac{dz}{z}$$

$$V_p = \frac{-2\lambda}{4\pi\varepsilon_0} \ln z$$

b) Kawat dengan panjang tertentu; misalnya 2L Dari perhitungan didapat

$$E_p = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda L}{z\sqrt{z^2+L^2}}$$



$$V = \frac{2}{4\pi\varepsilon_0} \int_0^L \frac{\lambda\, dx}{r}$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(z^2+x^2)^{1/2}}$$

$$x = z\,tg\,\theta$$

$$dx = z \sec^2\theta \, d\theta$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \int_0^L \frac{z \sec^2\theta \, d\theta}{z(tg^2\theta+1)^{1/2}}$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \int_0^L \sec\theta \, d\theta$$

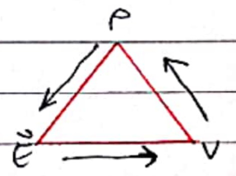
$$= \frac{2\lambda}{4\pi\varepsilon_0} \ln\left(\sec\theta + tg\,\theta\right) + c$$

$$= \frac{2\lambda}{2\pi\varepsilon_0} \ln\left(\frac{\sqrt{x^2+z^2}}{z} + \frac{x}{z}\right)\Big|_0^L$$

$$V = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{\sqrt{L^2+z^2}}{z} + \frac{L}{z}\right)$$

Hubungan antara $\vec{E}$, ρ dan V

Dalam elektronika hubungan antara kuat medan dengan rapat muatan volume dan V skalar adalah hubungan segitiga



$E = -\nabla V$ $\qquad$ $\nabla . E = 0$

$\bar{\nabla} V = -\frac{\rho}{\varepsilon_0}$ $\qquad$ $\nabla E = \frac{1}{\varepsilon_0}\rho$ $\Big\}$ Syarat elektrostatistika

$\rho = \varepsilon_0 \nabla E$

bentuk integralnya

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{dQ}{r^2} \; ; \; dQ = \rho \, d\tau$$

$$= \frac{1}{4\pi\varepsilon_0} \int \frac{\rho \, d\tau}{r^2}$$

$$V = -\int \vec{E} . d\vec{P}$$

Hubungan V dg ρ

$$V = \frac{1}{4\pi} \varepsilon_0 \int \frac{\rho \, d\tau}{r}$$

solusi hubungan segitiga diatas

1) misal E diketahui, V dan ρ dapat dihitung

$V = -\int dP$

$\rho = \varepsilon_0 \nabla . E$

2) misal: V diketahui E da ρ dapat dicari

$E = -\nabla V$

$\rho = -\frac{1}{\varepsilon_0} \nabla^2 V$

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3) misal ρ diketahui, E dan V dapat dihitung

$$E = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho\, d\tau}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho\, d\tau}{r}$$