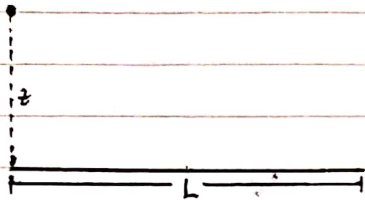
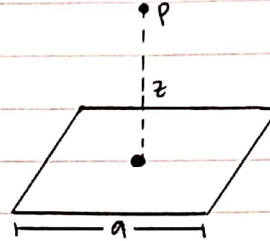


1. Hitung medan listrik pada salah satu ujung kawat yang panjangnya L (Gambar 2-10) yang membawa muatan seragam λ . Bagaimana besar E , jika $z \gg L$

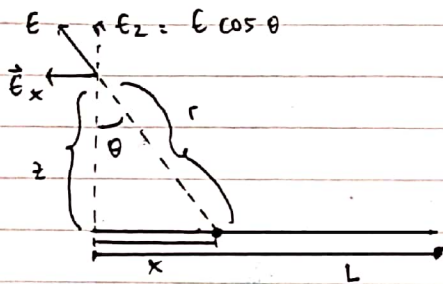


Gambar 2-10



Gambar 2-4

Penyelesaian:



$$\vec{E} = \vec{E}_z + \vec{E}_x$$

Ambil $\vec{E}_z = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda dx}{r^2} \cos \theta$, dimana $\cos \theta = \frac{z}{r}$

$$\vec{E}_z = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda z dx}{r^3} = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda z dx}{(z^2 + x^2)^{3/2}}$$

$$= \frac{\lambda z}{4\pi\epsilon_0} \int_0^L \frac{dx}{(z^2 + x^2)^{3/2}}$$

diintegrasikan $\int_0^L \frac{dx}{(z^2 + x^2)^{3/2}} = \frac{1}{z^2} \left[\frac{x}{\sqrt{z^2 + x^2}} \right]_0^L$

$$= \frac{1}{4\pi\epsilon_0} \lambda z \left[\frac{1}{z^2} \frac{x}{\sqrt{z^2 + x^2}} \right]_0^L$$

$$\vec{E}_z = \frac{\lambda z}{4\pi\epsilon_0} \frac{L}{z^2 \sqrt{z^2 + x^2}} = \frac{\lambda}{4\pi\epsilon_0} \frac{L}{z \sqrt{z^2 + x^2}}$$

$$\vec{E}_x = - \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda dx}{r^2} \sin \theta = - \frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{x dx}{(z^2 + x^2)^{3/2}}$$

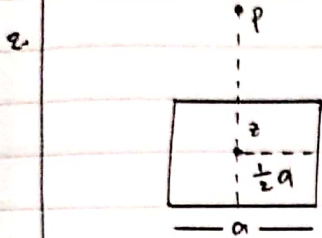
arah $\vec{E}_x$ ke kiri: $- \frac{\lambda}{4\pi\epsilon_0} \cdot \frac{1}{2} \int_0^L \frac{dx^2}{(z^2 + x^2)^{3/2}}$

$$= - \frac{\lambda}{4\pi\epsilon_0} \cdot \frac{1}{2} \cdot \left[-2 \frac{1}{(z^2 + x^2)^{1/2}} \right]_0^L$$

$$= -\frac{\lambda}{4\pi\epsilon_0} \left[-\left(\frac{1}{\sqrt{z^2+x^2}} - \frac{1}{z} \right) \right]$$

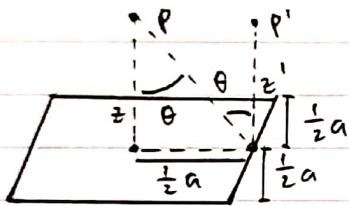
$$\vec{E}_x = -\frac{\lambda}{4\pi\epsilon_0} \left[\frac{1}{z} - \frac{1}{\sqrt{z^2+x^2}} \right]$$

Jadi $\vec{E} = \vec{E}_z + \vec{E}_x$



Solusi: Untuk menyelesaikan soal tersebut gunakan contoh no. 2 yaitu kawat yang panjangnya $2L$, dengan $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2 \times L}{z^2}$

Maka $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z^2}$ Hasil contoh 2



$P = P' \cos \theta$, Karena ada 4 sisi, maka:

$P = 4P' \cos \theta$, sehingga:

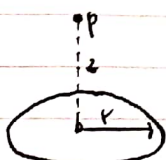
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z'^2} = 4 \cdot \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z'^2} \cos \theta$$

Karena $L = \frac{1}{2} a$, dan $z' = \sqrt{z^2 + (\frac{1}{2}a)^2}$, dan $\cos \theta = \frac{z}{\sqrt{z^2 + (\frac{1}{2}a)^2}}$, maka:

$$\vec{E} = 4 \cdot \frac{1}{4\pi\epsilon_0} \frac{2\lambda (\frac{1}{2}a) \cdot z}{(z^2 + (\frac{1}{2}a)^2) \sqrt{z^2 + (\frac{1}{2}a)^2}}$$

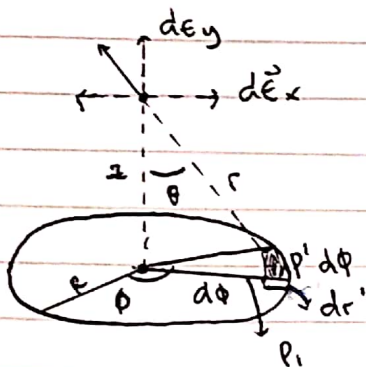
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{4\lambda a z}{(z^2 + \frac{a^2}{4}) \sqrt{z^2 + \frac{a^2}{4}}}$$

3. Hitung medan listrik di atas pusat kawat melingkar sejauh z yang membawa muatan seragam (Gambar 2.12)



Gambar 2.12.

Penyelesaian



elemen luas

$$\begin{aligned} ds &= da = r' d\phi \cdot dr' \\ &= dr' \cdot r' d\phi \\ da &= r' dr' d\phi \end{aligned}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma da}{r^2} \cos \theta$$

$$\begin{aligned} d\vec{E} &= d\vec{E}_y + d\vec{E}_x = 0 \\ &= dE \cos \theta \end{aligned}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_0^R \int_0^{2\pi} \frac{r'}{r^2} \cos \theta \cdot r' dr' d\phi$$

Tukar $\cos \theta$ dengan $\frac{z}{r}$. Sehingga menjadi :

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{1}{z} \int_0^R \int_0^{2\pi} \frac{r' dr' d\phi}{(r'^2 + z^2)^{3/2}}$$

$$= \frac{2\pi\sigma z}{4\pi\epsilon_0} \int_0^R \frac{r' dr'}{(r'^2 + z^2)^{3/2}}$$

$$= \frac{\sigma z}{2\epsilon_0} \int_0^R \frac{1}{2} \frac{d(r'^2 + z^2)}{(r'^2 + z^2)^{3/2}}$$