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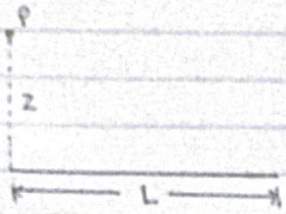
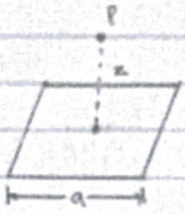
Kelas : B

Tugas

1) Hitung medan listrik pada salah satu ujung kawat yg panjangnya L (gambar 2.10) yg membawa muatan seragam λ. Bagaimana besar E, jika z >> L



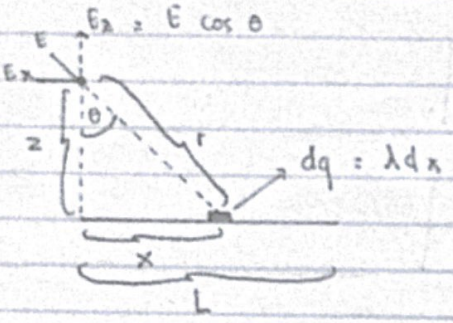
gambar 2.10



gambar 2.11

penyelesaian

1)



$E_z = E\cos\theta$

$$\vec{E}_z = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{\lambda dx}{r^2}\cos\theta$$

$$\Rightarrow \cos\theta = \frac{z}{r}$$

$$\vec{E}_z = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{z\lambda dx}{r^3} = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{z\lambda dx}{(z^2+x^2)^{3/2}}$$

$$= \frac{z\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(z^2+x^2)^{3/2}}$$

lihat di daftar integral $\int_0^L \frac{dx}{(z^2+x^2)^{3/2}} = \frac{1}{z^2}\frac{x}{\sqrt{z^2+x^2}}\Big|_0^L$

$$\vec{E}_z = \frac{1}{4\pi\varepsilon_0}\lambda z\left[\frac{1}{z^2}\frac{x}{\sqrt{z^2+x^2}}\right]_0^L$$

$$\vec{E}_z = \frac{\lambda z}{4\pi\varepsilon_0} \frac{L}{z^2\sqrt{z^2+x^2}}$$

$$\vec{E}_z = \frac{\lambda}{4\pi\varepsilon_0} \frac{L}{z\sqrt{z^2+x^2}}$$

$$\vec{E}_x = -\frac{1}{4\pi\varepsilon_0} \int_0^L \frac{\lambda\, dx}{r^2} \sin\theta$$

$$= \frac{-\lambda}{4\pi\varepsilon_0} \int_0^L \frac{x\, dx}{(z^2+x^2)^{3/2}}$$

$$= \frac{-\lambda}{4\pi\varepsilon_0} \cdot \frac{1}{2} \int_0^L \frac{dx^2}{(z^2+x^2)^{3/2}}$$

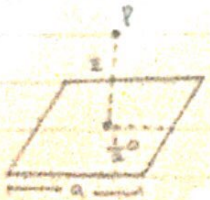
$$= \frac{-\lambda}{4\pi\varepsilon_0} \cdot \frac{1}{2} \left[-2 \cdot \frac{1}{(z^2+x^2)^{1/2}} \right]_0^L$$

$$= \frac{-\lambda}{4\pi\varepsilon_0} \left[-\left(\frac{1}{\sqrt{z^2+x^2}} - \frac{1}{z} \right) \right]$$

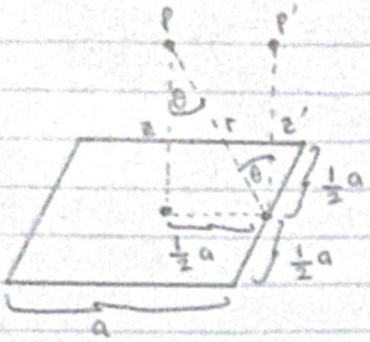
$$\vec{E}_x = \frac{-\lambda}{4\pi\varepsilon_0} \left[\frac{1}{z} - \frac{1}{\sqrt{z^2+x^2}} \right]$$

jadi $\vec{E} = \vec{E}_z + \vec{E}_x$

2) Hitung medan listrik sejauh z dari titik tengah loop segi empat panjang (gambar 2.11) yg membawa muatan seragam λ. Gunakan contoh no 3 untuk menyelesaikan permasalahan ini.



solusi : $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda L}{z^2}$



$P = P' \cos\theta$

$P = 4P' \cos\theta$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda L}{z'^2} = 4 \frac{1}{4\pi\varepsilon_0} \frac{2\lambda L}{z'^2} \cos\theta$$

karena $L = \frac{1}{2}a$; $z' = r = \sqrt{z^2 + \left(\frac{1}{2}a\right)^2}$; $\cos\theta = \frac{z}{\sqrt{z^2 + \left(\frac{1}{2}a\right)^2}}$

maka

$$\vec{E} = 4 \cdot \frac{1}{4\pi\varepsilon_0} \frac{2\lambda\left(\frac{1}{2}a\right) \cdot z}{\left(z^2 + \left(\frac{1}{2}a\right)^2\right)\sqrt{z^2 + \left(\frac{1}{2}a\right)^2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{4\lambda a z}{\left(z^2 + \frac{a^2}{4}\right)\sqrt{z^2 + \frac{a^2}{4}}} \cdot \hat{z}$$