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1. Medan listrik oleh muatan terdistribusi. Analogi dari medan listrik oleh muatan titik.

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\hat{R}_0$$

Maka medan listrik oleh muatan kontinue dapat dihitung dengan 2 cara, yaitu:

a. Hukum Coulomb dalam penerapannya menggunakan integral dalam sistem koordinat kartesian, bola dan silinder.

1). Untuk integral garis

$$\vec{E} = \int_{line} \frac{1}{4\pi\epsilon_0}\frac{dq}{R^2}\hat{R}_0 \quad \text{dengan } dq = \lambda\, dl$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda\, dl}{R^2}\hat{R}_0$$

2). Untuk luas permukaan

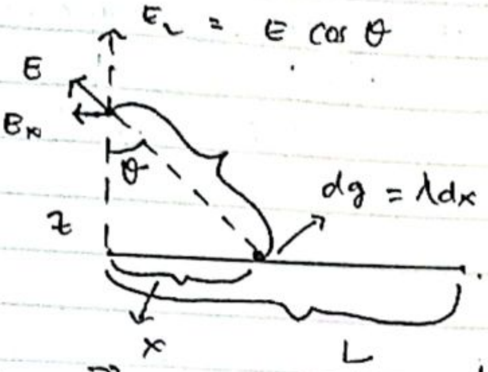
$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\iint \frac{\sigma\, d\vec{s}}{R^2}\hat{R}_0}$$

3). Untuk volume :

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\iiint \frac{\rho\, dv}{R^2}\hat{R}_0}$$

Date :

1.



$$\vec{E} = \vec{E}_z + \vec{E}_x$$

$$\vec{E}_z = \frac{1}{4\pi\epsilon_0}\int_0^L d\vec{E}\cos\theta$$

Ambil $\vec{E}_z = \dfrac{1}{4\pi\epsilon_0}\displaystyle\int_0^L \frac{dq}{r^2} = \frac{1}{\square}\int_0^L \frac{dx}{\rho^2}\cos\theta$

$\Rightarrow \cos\theta = \dfrac{z}{\rho}$

$$\vec{E}_z = \frac{1}{\square}\int_0^L \frac{z\lambda dx}{\rho^3} = \frac{1}{\square}\int_0^L \frac{z\lambda dx}{(z^2+x^2)^{3/2}}$$

$$= \frac{z\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{(z^2+x^2)^{3/2}}$$

lihat di daftar integral

$$\boxed{\begin{aligned} &\int_0^L \frac{dx}{(z^2+x^2)^{3/2}} = \frac{1}{z^2}\frac{x}{\sqrt{z^2+x^2}}\Bigg|_0^L \\ &\vec{E}_z = \frac{1}{4\pi\epsilon_0}\lambda z\left[\frac{1}{z^2}\frac{x}{\sqrt{z^2+x^2}}\right|_0^L \end{aligned}}$$

$$\vec{E}_x = -\frac{1}{4\pi\epsilon_0}\int_0^L \frac{\lambda dx}{\rho^2}\sin\theta = -\frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{x\,dx}{(z^2+x^2)^{3/2}}$$

arah x ke kiri

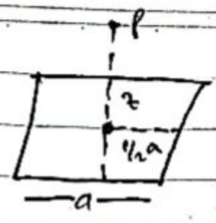
$$\boxed{\begin{aligned} &= -\frac{\lambda}{4\pi\epsilon_0}\cdot\frac{1}{2}\int_0^L \frac{dx^2}{(z^2+x^2)^{3/2}} \\ &= -\frac{\lambda}{4\pi\epsilon_0}\cdot\frac{1}{2}\left[-2\frac{1}{(z^2+x^2)^{1/2}}\right|_0^L \\ &= -\frac{\lambda}{4\pi\epsilon_0}\left[-\left(\frac{1}{\sqrt{z^2+x^2}} - \frac{1}{z}\right)\right] \end{aligned}}$$

$$\vec{E}_x = -\frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+x^2}}\right]$$

Jadi $\vec{E} = \vec{E}_z + \vec{E}_x$

PAPERLINE

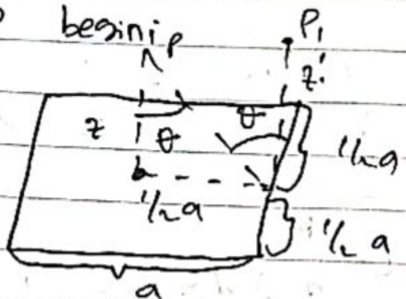
2.

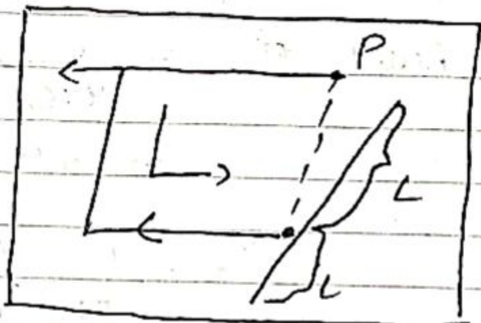


Solusi: untuk memperluas pg soal diatas gunakan contoh no. 3 yaitu kawat yang panjangnya 2L, dengan $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z'^2}$.

$$\vec{E}' = \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z'^2}$$

konsep beginip





$P = P'\cos\theta$; karena ada 4 sisi, maka:
$P = 4P'\cos\theta$;

$$\vec{E}' = \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z'^2} = 4\cdot\frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z'^2}\cos\theta$$

karena $L = \frac{1}{2}a$, dan $z' = P = \sqrt{z^2+(\frac{1}{2}a)^2}$

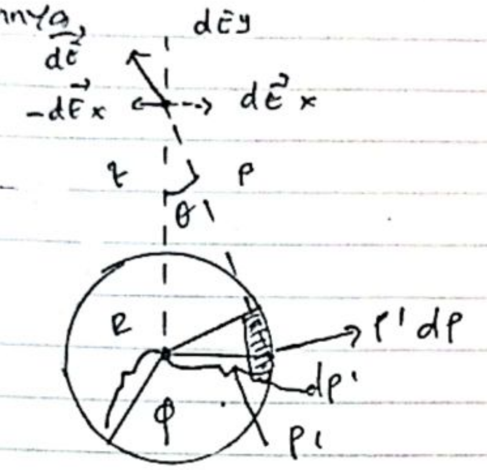
dan $\cos\theta = \frac{z}{\sqrt{z^2+(\frac{1}{2}a)^2}}$ maka:

$$\boxed{\begin{aligned} \vec{E}' &= 4\cdot\frac{1}{4\pi\epsilon_0}\frac{2\lambda(\frac{1}{2}a)\cdot z}{\left(z^2+(\frac{1}{2}a)^2\right)\sqrt{z^2+(\frac{1}{2}a)^2}} \\ \vec{E}' &= \frac{1}{4\pi\epsilon_0}\frac{4\lambda a z}{\left(z^2+\frac{a^2}{a}\right)\sqrt{z^2+\frac{a^2}{4}}} \end{aligned}}$$

3. Tentukan medan listrik berjarak z di atas pusat permukaan berbentuk lingkaran dengan jari" R yang membawa permukaan seragam :

Cara perolehannya,

1.



<u>elemen luar.</u>

$ds = da = p \times l$

$= d\rho' \cdot \rho' d\phi$

$da = \rho' dr' d\phi$

$d\vec{E}' = d\vec{E}y + d\vec{E}x$

$= dE \cos\theta$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_s \frac{\sigma da}{\rho^2}\cos\theta$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_0^R\int_0^{2\pi}\frac{\sigma}{\rho^2}\cos\theta\,\underbrace{r'dr'd\phi}_{da}$$

cos θ dengan ρ² menjadi

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\sigma z\int_0^R\int_0^{2\pi}\frac{\rho' d\rho' d\phi}{(\rho'^2+z^2)^{3/2}}$$

$$= \frac{\cancel{2}\pi\sigma z}{\cancel{4}\pi\epsilon_0}\int_0^R\frac{\rho' dr'}{(\rho'^2+z^2)^{3/2}}$$

ρ' dibuat diferensial tidak mengubah makna

$$= \frac{\sigma z}{2\epsilon_0}\int_0^R\frac{1}{2}\frac{d(r'^2+z^2)}{(\rho'^2+z^2)^{3/2}}$$