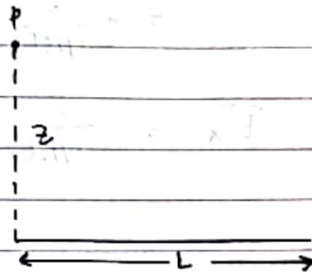


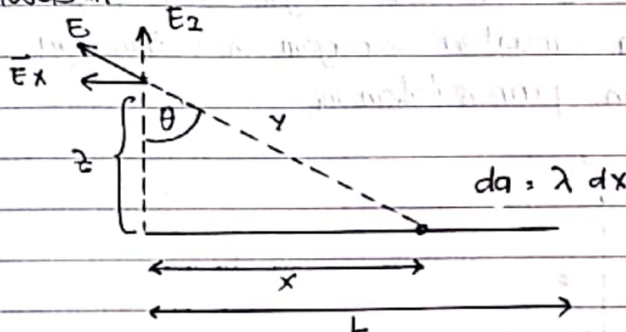
Nama = Alfia Rosa

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- 1) Hitung medan listrik pada salah satu ujung kawat yang panjangnya L yang membawa muatan seragam λ . Bagaimana besar E , jika $z \gg L$



Jawaban:



$$\vec{E} = \vec{E}_z + \vec{E}_x$$

$$\vec{E}_z = \frac{1}{4\pi\epsilon_0} \int_0^L dE \cos \theta$$

$$\vec{E}_z = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda dx}{r^2} \cos \theta$$

$$\Rightarrow \cos \theta = \frac{z}{r}$$

$$\vec{E}_z = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{z \lambda dx}{r^3} = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{z \lambda dx}{(z^2 + x^2)^{3/2}}$$

$$= \frac{z \lambda}{4\pi\epsilon_0} \int_0^L \frac{dx}{(z^2 + x^2)^{3/2}}$$

$$\text{lihat di daftar integral } \int_0^L \frac{dx}{(z^2 + x^2)^{3/2}} = \frac{1}{z^2} \left[\frac{x}{\sqrt{z^2 + x^2}} \right]_0^L$$

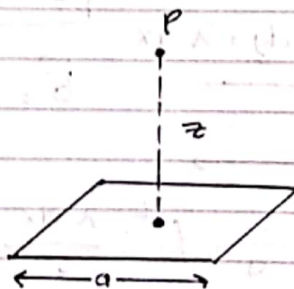
$$= \frac{1}{4\pi\epsilon_0} \lambda z \left[\frac{1}{z^2} \frac{x}{\sqrt{z^2 + x^2}} \right]_0^L$$

$$\vec{E}_z = \frac{\lambda z}{4\pi\epsilon_0} \frac{L}{z^2 \sqrt{z^2 + x^2}} = \frac{\lambda}{4\pi\epsilon_0} \frac{L}{z \sqrt{z^2 + x^2}}$$

$$\begin{aligned}
 \vec{E}_x &= -\frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda dx}{r^2} \sin \theta = -\frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{x dx}{(z^2+x^2)^{3/2}} \\
 &= -\frac{\lambda}{4\pi\epsilon_0} \cdot \frac{1}{2} \int_0^L \frac{dx^2}{(z^2+x^2)^{3/2}} \\
 &= -\frac{\lambda}{4\pi\epsilon_0} \cdot \frac{1}{2} \left[-2 \frac{1}{(z^2+x^2)^{1/2}} \right]_0^L \\
 &= -\frac{\lambda}{4\pi\epsilon_0} \left[-\left(\frac{1}{\sqrt{z^2+x^2}} \right) - \frac{1}{z} \right] \\
 \vec{E}_x &= -\frac{\lambda}{4\pi\epsilon_0} \left[\frac{1}{z} - \frac{1}{\sqrt{z^2+x^2}} \right]
 \end{aligned}$$

Jadi $\vec{E} = \vec{E}_z + \vec{E}_x$

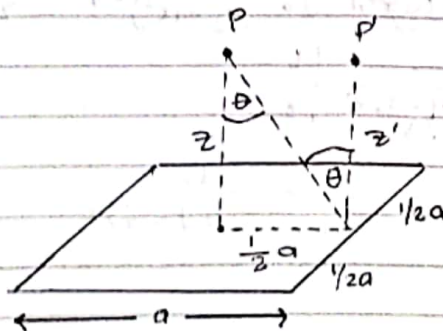
- 2) Hitung medan listrik sejauh z dari titik tengah loop segi empat panjang yang membawa muatan seragam λ . Gunakan contoh no.3 untuk menyelesaikan permasalahan ini.



Penyelesaian:

Untuk menyelesaikan soal diatas gunakan contoh nomor 3, yaitu kawat yang panjangnya $2L$, dengan $E = \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z^2}$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z^2}$$



$$P = p' \cos \theta \quad , \text{ karena ada 4 sisi , maka}$$

$$P = 4 p' \cos \theta$$

Selanjutnya :

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z'^2} = 4 \frac{1}{4\pi\epsilon_0} \frac{2\lambda L}{z'^2} \cos \theta$$

$$\text{Karena } L = \frac{1}{2} a \text{ dan } z' = r = \sqrt{z^2 + (\frac{1}{2}a)^2}$$

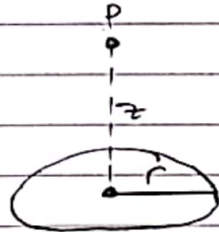
$$\text{dan } \cos \theta = \frac{z}{\sqrt{z^2 + (\frac{1}{2}a)^2}}$$

maka :

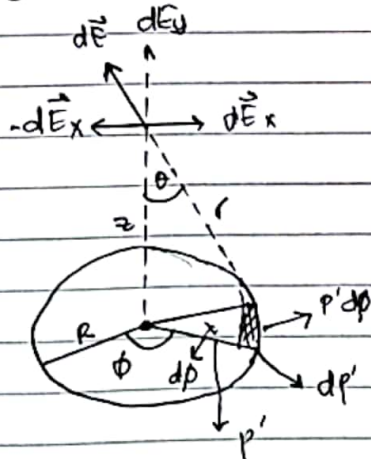
$$\vec{E} = 4 \frac{1}{4\pi\epsilon_0} \frac{2\lambda (\frac{1}{2}a) \cdot z}{(z^2 + (\frac{1}{2}a)^2) \sqrt{z^2 + (\frac{1}{2}a)^2}}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{4\lambda a z}{(z^2 + \frac{a^2}{4}) \sqrt{z^2 + \frac{a^2}{4}}}$$

- 3) Hitung medan listrik diatas pusat kawat melingkar sejauh z yang membawa muatan seragam λ



Penyelesaian :



$$ds = da = p \times L$$

$$= dp' \cdot p' d\phi$$

$$da = p' dr' d\phi$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_S \frac{\sigma da}{r^2} \cos \theta$$

$$d\vec{E} = d\vec{E}_y + d\vec{E}_x = 0$$

$$= dE \cos \theta$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_0^R \int_0^{2\pi} \frac{r}{r^2} \cos \theta \cdot r' dp' d\phi$$

$$r' dp' d\phi$$

buka $\cos \theta$ dengan p^2 , sehingga menjadi

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sigma z \int_0^R \int_0^{2\pi} \frac{p' dr' d\theta}{(p'^2 + z^2)^{3/2}}$$

$$= \frac{2\pi\sigma z}{4\pi\epsilon_0} \int_0^R \frac{p' dp'}{(p'^2 + z^2)^{3/2}}$$

$$= \frac{\sigma z}{2\epsilon_0} \int_0^R \frac{1}{2} \frac{d(p'^2 + z^2)}{(p'^2 + z^2)^{3/2}}$$

p' dibuat dalam bentuk diferensial, tetapi tidak mengubah makna.