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Problem 1.11

Find the gradient of the following functions:

a) $F(x, y, z) = x^2 + y^3 + z^4$

$$\nabla F = \frac{\partial F}{\partial x} \hat{x} + \frac{\partial F}{\partial y} \hat{y} + \frac{\partial F}{\partial z} \hat{z} = 2x\hat{x} + 3y^2\hat{y} + 4z^3\hat{z} = 2xi + 3y^2\hat{j} + 4z^3\hat{k}$$

b) $F(x, y, z) = x^2 y^3 z^4$

$$\begin{aligned} \nabla F &= \frac{\partial F}{\partial x} \hat{x} + \frac{\partial F}{\partial y} \hat{y} + \frac{\partial F}{\partial z} \hat{z} \\ &= \frac{\partial}{\partial x} (x^2 y^3 z^4) \hat{x} + \frac{\partial}{\partial y} (x^2 y^3 z^4) \hat{y} + \frac{\partial}{\partial z} (x^2 y^3 z^4) \hat{z} \\ &= (2xy^3z^4)\hat{x} + (3x^2y^2z^4)\hat{y} + (4x^2y^3z^3)\hat{z} \end{aligned}$$

Problem 1.12

a) $\nabla h = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$

$\nabla h = 0$ ada di ~~pusat~~ puncak

$$2y - 6x - 18 = 0$$

$$2x - 8y + 28 = 0$$

$$2y - 6x - 18 = 0$$

$$6x - 24y + 84 = 0$$

$$-22y + 66 = 0$$

$$y = 3$$

$$2y - 6x - 18 = 0$$

$$2(3) - 6x - 18 = 0$$

$$-6x = 12$$

$$x = -2$$

b) Substitusikan $x = -2, y = 3 \rightarrow h = 10(2(3)(-2) - 3(-2)^2 - 4(3)^2 - 18(-2) + 28(3) + 12)$

$$h = 10(-12 - 12 - 36 + 36 + 84 + 12)$$

$$h = 720 \text{ kaki}$$

c) Subs = $x = 1, y = 1$

$$\nabla h = 10((2 - 6 - 18)\hat{x} + (2 - 8 + 28)\hat{y})$$

$$\sim 10(-22\hat{x} + 22\hat{y})$$

$$= 220(-\hat{x} + \hat{y})$$

$$|\nabla h| = 220\sqrt{2} = 311 \text{ kaki/mil}$$

3) calculate the divergence of the follow Vector functions =

a) $\vec{V}(1) = 2\vec{i}$

$$r = (x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}$$

$$r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\nabla(r) = \frac{\partial}{\partial x} \left(\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \right) + \frac{\partial}{\partial y} \left(\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \right) + \frac{\partial}{\partial z} \left(\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \right)$$

$$= \frac{1}{r} \left((x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z} \right)$$

$$= \frac{1}{r} \left((x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z} \right) = 2$$

Problem 1.14.

$$\vec{y} = y \cos \theta + z \sin \theta$$

$$\vec{y} \sin \theta = y \sin \theta \cos \theta + z \sin^2 \theta$$

$$\vec{z} = -y \sin \theta + z \cos \theta$$

$$z \cos \theta - \vec{z} \sin \theta = y$$

Sehingga

$$\frac{\partial y}{\partial y} = \cos \theta, \quad \frac{\partial y}{\partial z} = -\sin \theta; \quad \frac{\partial z}{\partial y} = \sin \theta; \quad \frac{\partial z}{\partial z} = \cos \theta$$

$$(\nabla F)_y = \frac{\partial F}{\partial y} = \frac{\partial F}{\partial y} \frac{\partial y}{\partial y} + \frac{\partial F}{\partial z} \frac{\partial z}{\partial y} = \cos \theta (\nabla F)_y + \sin \theta (\nabla F)_z$$

$$(\nabla F)_z = \frac{\partial F}{\partial z} = \frac{\partial F}{\partial y} \frac{\partial y}{\partial z} + \frac{\partial F}{\partial z} \frac{\partial z}{\partial z} = -\sin \theta (\nabla F)_y + \cos \theta (\nabla F)_z$$