

NO

DATE

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Prodi : Pendidikan Fisika (A)

Problem 1.11

Find the gradion of the following function

a. $F(x, y, z) = x^2 + y^2 + z^4$

Jawab :

$$\vec{\nabla} = 2x\hat{i} + 2y\hat{j} + 4z^3\hat{k}$$

b. $t(x, y, z) = x^2 y^3 z^4$

$$\vec{\nabla} = 2xy^3z^4\hat{i} + 3x^2y^2z^4\hat{j} + 4x^2y^3z^3\hat{k}$$

c. $t(x, y, z) = e^x \sin(y) \ln(z)$

$$= e^x \sin(y) \ln(z)\hat{i} + e^x \cos(y) \ln(z)\hat{j} + e^x \sin(y) \left(\frac{1}{z}\right)\hat{k}$$

Problem 1.12

The height of a certain hill (cintil) is given by

$$h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$$

Where y is distance (in miles) north, x the distance east of south Haddoy.

a). Where is the top of the hill located

b). How high is the hill

c). Low step is the slope (in feet per mile) at a point 1 mile north and one mile east of south Haddoy? In what direction is the slope steepest, at that point

Jawab :

a. Dimanakah puncak bukit tersebut berada?

$$\vec{\nabla} h = 10[(2y - 6x - 18)\hat{i} + (2x - 8y + 28)\hat{j}]$$

Syarat

$$\vec{\nabla} h = 0 \text{ (puncak bukit merupakan jenis statometer)}$$

$$\vec{\nabla} h = 10[(2y - 6x - 18)\hat{i} + (2x - 8y + 28)\hat{j}] = 0 \text{ menghasilkan sistem persamaan dua pilihan :}$$

$$\left. \begin{array}{l} 2y - 6x - 18 = 0 \\ 2x - 8y + 28 = 0 \end{array} \right\} \text{ Solusi dari persamaan}$$

$$\text{In } (x, y) = (12, 3)$$

Bantex

b. Berapa ketinggian bukit tersebut

→ Substitusi $(x, y) = (-2, 3)$ pada $b(x, y) =$

$$b(-2, 3) = 10(-12 - 12 - 36 + 36 + 0 + 12) = 720 \text{ m}$$

Jadi ketinggian 720 m

c. Substitusi $(x, y) = (1, 1)$ pada $\vec{\nabla} b$

$$\vec{\nabla} b(1, 1) = 10(2 - 6 - 18)\mathbf{i} + (2 - 8 + 20)\mathbf{j} = 220(\mathbf{i} + \mathbf{j})$$

$$|\vec{\nabla} b| = 220\sqrt{2} = 311 \text{ m/km}$$

arahnya ke barat laut (135° dan sumbu x positif)

Problem 1.13

Let the separation vector from a fixed point (x_0, y_0, z_0) to the point (x, y, z) and let be its length show that

a) $\vec{\nabla} (1/r) = -\vec{r}/r^3$

b) $\nabla (1/r) = -\vec{r}/r^3$

c) What is the general formula for $\nabla (1/r)$?

Jawab:

$$(x, y, z) \text{ } r = \frac{(x - x_0)\mathbf{i} + (y - y_0)\mathbf{j} + (z - z_0)\mathbf{k}}{r}$$

$$r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$$

$$(x_0, y_0, z_0) \text{ } r = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$$

$$a) \vec{\nabla} \frac{1}{r} = \frac{1}{2r} \left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \right] \mathbf{i} + \frac{1}{2r} \left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \right] \mathbf{j} + \frac{1}{2r} \left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \right] \mathbf{k}$$

$$= \frac{1}{2r} \left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \right] \mathbf{i} + \frac{1}{2r} \left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \right] \mathbf{j} + \frac{1}{2r} \left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \right] \mathbf{k}$$

$$= 2(x - x_0)\mathbf{i} + 2(y - y_0)\mathbf{j} + 2(z - z_0)\mathbf{k}$$

$$= 2\vec{r} \text{ (terbukti)}$$

$$c) \frac{1}{2r} (r^n) = r^{n-1} \cdot \frac{1}{2r} \frac{d}{dr} r^n = r^{n-1} \cdot \frac{1}{2} \frac{1}{r} \frac{d}{dr} r^n = r^{n-1} \cdot \frac{1}{2} \frac{1}{r} \cdot n r^{n-1} = \frac{n}{2} r^{n-2} \mathbf{i} \cdot (x - x_0)$$

$$\frac{1}{2r} (r^n) = r^{n-1} \cdot \frac{1}{2r} \frac{d}{dr} r^n = r^{n-1} \cdot \frac{1}{2} \frac{1}{r} \frac{d}{dr} r^n = r^{n-1} \cdot \frac{1}{2} \frac{1}{r} \cdot n r^{n-1} = \frac{n}{2} r^{n-2} \mathbf{j} \cdot (y - y_0)$$