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Problem 1.11 (Hal 15)

- find the gradien of the following functions:

a). $f(x,y,z) = x^2 + y^3 + z^4$
b). $f(x,y,z) = x^2 y^3 z^4$
c) $f(x,y,z) = e^x \sin(y) \ln(z)$

~ Solution

a). $f(x,y,z) = x^2 + y^3 + z^4$
$\vec{\nabla} f = 2x\hat{i} + 3y^2\hat{j} + 4z^3\hat{k}$

b). $f(x,y,z) = x^2 y^3 z^4$
$\vec{\nabla} f = 2xy^3 z^4\hat{i} + 3x^2y^2 z^4\hat{j} + 4x^2y^3 z^4\hat{k}$

c). $f(x,y,z) = e^x \sin(y) \ln(z)$
$\vec{\nabla} f = e^x \sin(y) \ln(z)\hat{i} + e^x \cos(y) \ln(z)\hat{j} + e^x \sin(y)\left(\frac{1}{z}\right)\hat{k}$

Problem 1.12

- The height of a certain hill (in feer) is given by.

$h(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$

where y is distance (in miles) north, x the distance east of south hadley.

a). Where is the top of the hill located?
b). How high is the hill
c). How steep is the slope (in feet per mile) at a point 1 mil north and one mile east of south hadley? in what direction is the slope steepest, at the point?

~ Solution

a). Tentukan fungsi gradien terlebih dahulu

$\vec{\nabla} b = 10[(2y - 6x - 18)\hat{i} + (2x - 8y + 28)\hat{j}]$

untuk menentukan puncak bukit, gunakan syarat $\vec{\nabla} b = 0$ (puncak bukit merupakan salah satu jenis stasione):

$\vec{\nabla} b = 10[(2y-6x-18)\hat{i} + (2x-8y+28)\hat{j}] = 0$, menghasilkan sistem persamaan linear dua peubah :

$$\left.\begin{aligned} 2y-6x-18&=0 \\ 2x-8y+28&=0 \end{aligned}\right\}$$ solusi dari sistem persamaan ini $= (x,y) = (-2,3)$

Dengan demikian, puncak bukit tersebut pada 2km sebelah barat dan 3 km utara Bandung.

b). Subtitusikan $(x,y) = (-2,3)$ pada $b(x,y)$:

$b(-2,3) = 10(-12-12-36+36+84+12) = 720\,m$

Jadi ketinggiannya adalah 720 m.

c). Subtitusikan $(x,y) = (1,1)$ pada $\vec{\nabla} b$.

$\vec{\nabla} b(1,1) = 10[(2-6-18)\hat{i} + (2-8+28)\hat{j}] = 220(-i+j)$

$|\vec{\nabla} b| = 220\sqrt{2} \approx 311$ m/km, arahnya ke barat laut (135° dari sumbu x positif)

Problem 13

Let $\vec{r}$ be the separation vector from a fixed point (x', y', z') to the point (x,y,z) and let r be its length. Show that

a). $\nabla(r^2) = 2\vec{r}$

b) $\nabla(1/r) = -\hat{r}/r^2$

c) what is the general formula for $\nabla(r^n)$?

Solution :

(x,y,z) $\quad \vec{r} = (x-x_0)\hat{i} + (y-y_0)\hat{j} + (z-z_0)\hat{k}$

$r = \sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}$

(x_0,y_0,z_0) $\quad r^2 = (x-x_0)^2+(y-y_0)^2+(z-z_0)^2$

a). $\vec{\nabla} r^2 = \frac{\partial}{\partial x}[(x-x_0)^2+(y-y_0)^2+(z-z_0)^2]\hat{i} + \frac{\partial}{\partial y}[(x-x_0)^2+(y-y_0)^2+(z-z_0)^2]\hat{j} + \frac{\partial}{\partial z}[(x-x_0)^2+(y-y_0)^2+(z-z_0)^2]\hat{k}$

$= 2(x-x_0)\hat{i} + 2(y-y_0)\hat{j} + 2(z-z_0)\hat{k} = 2\vec{r}$ (terbukti

c). $\frac{\partial}{\partial x}(r^n) = n r^{n-1}\frac{\partial r}{\partial x} = n r^{n-1}\left(\frac{1}{2}\frac{1}{r}2r_x\right) = n r^{n-1}\hat{r}_x,\ (r_x = x-x_0)$

$\frac{\partial}{\partial y}(r^n) = n r^{n-1}\hat{r}_y,\ \frac{\partial}{\partial z}(r^n) = n r^{n-1}\hat{r}_z$; sehingga $\vec{\nabla}(r^n) = n r^{n-1}\hat{r}$