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Mata Kuliah : Kelistrikan dan Kemagnetan

Problem 1.11 (halaman 15)

Find the gradient of the following functions :

a) $f(x, y, z) = x^2 + y^3 + z^4$

b) $f(x, y, z) = x^2 y^3 z^4$

c) $f(x, y, z) = e^x \sin(y) \ln(z)$

Solution

a) $f(x, y, z) = x^2 + y^3 + z^4$

$\rightarrow \vec{\nabla} f = 2x\hat{i} + 3y^2\hat{j} + 4z^3\hat{k}$

b) $f(x, y, z) = x^2 y^3 z^4$

$\rightarrow \vec{\nabla} f = 2xy^3z^4\hat{i} + 3x^2y^2z^4\hat{j} + 4x^2y^3z^3\hat{k}$

c) $f(x, y, z) = e^x \sin(y) \ln(z)$

$\rightarrow \vec{\nabla} f = e^x \sin(y) \ln(z)\hat{i} + e^x \cos(y) \ln(z)\hat{j} + e^x \sin(y) \left(\frac{1}{z}\right)\hat{k}$

Problem 1.12

The height of a certain hill (in feet) is given by

$$h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$$

Where y is distance (in miles) north, x the distance east of South Hadley

a) Where is the top of the hill located?

b) How high is the hill

c) How steep is the slope (in feet per mile) at a point 1 mile north and one mile east of South Hadley? In what direction is the slope steepest, at that point?

Solution

a) Di manakah puncak bukit tersebut berada?

$\rightarrow$ Tentukan fungsi gradien terlebih dahulu

$$\vec{\nabla} h = 10[(2y - 6x - 18)\hat{i} + (2x - 8y + 28)\hat{j}]$$

Untuk menentukan puncak bukit, gunakan syarat $\vec{\nabla} b = 0$ (puncak bukit merupakan salah satu jenis stationer):

$\vec{\nabla} b = 10 [(2y - 6x - 18)\hat{i} + (2x - 8y + 28)\hat{j}] = 0$ menghasilkan sistem persamaan linear dua peubaru

$$\begin{cases} 2y - 6x - 18 = 0 \\ 2x - 8y + 28 = 0 \end{cases} \quad \text{Solusi dari sistem persamaan ini} = (x, y) = (-2, 3)$$

Dengan demikian puncak bukit tersebut berada pada 2 km sebelah barat dan 3 km utara Bandung.

b) Berapa ketinggian bukit tersebut?

~ Substitusikan $(x, y) = (-2, 3)$ pada $b(x, y)$.

$$b(-2, 3) = 10 (-12 - 12 - 36 + 36 + 84 + 12) = 720 \text{ km}$$

Jadi, ketinggiannya adalah 720 m

c) Seberapa curam kemiringan (dalam satuan m/km) pada sebuah titik: 1 km utara dan 1 km timur kota Bandung? Pada arah manakah kemiringan tercuram di titik tersebut?

~ Substitusikan $(x, y) = (1, 1)$ pada $\vec{\nabla} b$.

$$\vec{\nabla} b(1, 1) = 10 [(2 - 6 - 18)\hat{i} + (2 - 8 + 28)\hat{j}] = 220 (-\hat{i} + \hat{j})$$

$$|\vec{\nabla} b| = 220\sqrt{2} \approx 311 \text{ m/km, arahnya ke barat laut (135° dari sumbu x positif)}$$

Problem 1.13

Let $\vec{r}$ be the separation vector from a fixed point (x', y', z') to the point (x, y, z) , and let r be its length. Show that

a.) $\nabla(r^2) = 2\vec{r}$

b.) $\nabla(1/r) = -\vec{r}/r^3$

c.) What is the general formula for $\nabla(r^n)$?

.) Solution

$$(x, y, z) \quad \vec{r} = (x - x_0)\hat{i} + (y - y_0)\hat{j} + (z - z_0)\hat{k}$$

$$r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$$

$$(x_0, y_0, z_0)$$

$$r^2 = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$$

$$\begin{aligned}
 a) \vec{\nabla} \kappa^2 &= \frac{\partial}{\partial x} [(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2] \hat{i} + \frac{\partial}{\partial y} [(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2] \hat{j} \\
 &\quad + \frac{\partial}{\partial z} [(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2] \hat{k} \\
 &= 2(x-x_0) \hat{i} + 2(y-y_0) \hat{j} + 2(z-z_0) \hat{k} = 2\vec{r} \text{ (terbukti)}
 \end{aligned}$$

$$c.) \frac{\partial}{\partial x} (\kappa^n) = n \kappa^{n-1} \frac{\partial \kappa}{\partial x} = n \kappa^{n-1} \left(\frac{1}{2} \frac{1}{\kappa} 2x \right) = n \kappa^{n-1} x, \quad (\kappa_x = x - x_0)$$

$$\frac{\partial}{\partial y} (\kappa^n) = n \kappa^{n-1} y, \quad \frac{\partial}{\partial z} (\kappa^n) = n \kappa^{n-1} z, \quad \text{sehingga } \vec{\nabla} (\kappa^n) = n \kappa^{n-1} \vec{r}.$$