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"KELISTRIKAN & KEMAGNETAN"

* Problem 1.11

Find the gradients of the following functions.

a) $f(x,y,z) = x^2 + y^3 + z^4$

b) $f(x,y,z) = x^2 y^3 z^4$

c) $f(x,y,z) = e^x \sin(y) \ln(z)$

Jawab :

a) $\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$

$= \frac{\partial(x^2+y^3+z^4)}{\partial x}\hat{i} + \frac{\partial(x^2+y^3+z^4)}{\partial y}\hat{j} + \frac{\partial(x^2+y^3+z^4)}{\partial z}\hat{k}$

$\vec{\nabla} f = 2x\hat{i} + 3y^2\hat{j} + 4z^3\hat{k}$

b) $\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$

$= \frac{\partial(x^2y^3z^4)}{\partial x}\hat{i} + \frac{\partial(x^2y^3z^4)}{\partial y}\hat{j} + \frac{\partial(x^2y^3z^4)}{\partial z}\hat{k}$

$\vec{\nabla} f = 2xy^3z^4\hat{i} + 3x^2y^2z^4\hat{j} + 4x^2y^3z^3\hat{k}$

c) $\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$

$= \frac{\partial(e^x \sin(y)\ln(z))}{\partial x}\hat{i} + \frac{\partial(e^x \sin(y)\ln(z))}{\partial y}\hat{j} + \frac{\partial(e^x \sin(y)\ln(z))}{\partial z}\hat{k}$

$\vec{\nabla} f = e^x \sin(y)\ln(z)\hat{i} + e^x \cos(y)\ln(z)\hat{j} + e^x \sin(y)\left(\frac{1}{z}\right)\hat{k}$

* Problem 1.12

The height of a certain hill (in feet) is given by

$h(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$.

Where y is the distance (in miles) north, x the distance east of South Hadley

a) where is the top of the hill located?

b) How high is the hill?

c) How steep is the slope (in feet per mile) at a point 1 mile north and one mile east of South Hadley? In what direction is the slope steepest, at that point?

Jawab :

a) $\vec{\nabla} h = \frac{\partial h}{\partial x}\hat{i} + \frac{\partial h}{\partial y}\hat{j}$

$= \frac{\partial(10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12))}{\partial x}\hat{i} + \frac{\partial(10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12))}{\partial y}\hat{j}$

$\vec{\nabla} h = 10(2y - 6x - 18)\hat{i} + 10(2x - 8y + 28)\hat{j}$

$2y - 6x - 18 = 0$
$2x - 8y + 28 = 0$ $\Big\}$ $(x,y) = (-2,3)$

$\Rightarrow$ Jadi, puncak bukit berada pada 2 mil sebelah barat dan 3 mil sebelah utara.

b) subtitusi $(x,y) = (-2,3)$ pada $h(x,y)$

$h(-2,3) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$

$= 10(2.(-2).3 - 3.(-2)^2 - 4.(3)^2 - 18(-2) + 28(3) + 12)$

$= 10(-12 \cancel{-12} \cancel{-36} \cancel{+36} + 84 \cancel{+12})$

$= 10(72)$

$h(-2,3) = 720$ mil

c) subtitusikan $(x,y) = (1,1)$ pada $\vec{\nabla} h$

$\vec{\nabla} h(1,1) = 10(2y - 6x - 18)\hat{i} + 10(2y - 8y + 28)\hat{j}$

$= 10(2.1 - 6.1 - 18)\hat{i} + 10(2.1 - 8.1 + 28)\hat{j}$

$= 10(2 - 6 - 18)\hat{i} + 10(2. - 8 + 28)\hat{j}$

$= 10(-22)\hat{i} + 10(22)\hat{j}$

$\vec{\nabla} h(1,1) = -220\hat{i} + 220\hat{j}$

$= 220(-\hat{i} + \hat{j})$

$= 220(-\hat{x} + \hat{y})$

$|\vec{\nabla} h| = 220\sqrt{2}$

$|\vec{\nabla} h| \approx 311$ kaki/mil di arah barat laut.

Problem 1.13

Let $\mathfrak{r}$ be the separation vector from a fixed point (x', y', z') to the point (x, y, z) and let π be is length. Show that :

a) $\nabla(\mathfrak{r}^2) = 2\mathfrak{r}$

b) $\nabla(1/\mathfrak{r}) = -\hat{\mathfrak{r}}/\mathfrak{r}^2$

c) What is the general formula for $\nabla(\mathfrak{r}^n)$?

Jawab :

a) $\mathfrak{r} = (x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}$

$\mathfrak{r} = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$

$$\nabla \mathfrak{r} = \frac{\partial}{\partial x}\left((x-x')^2+(y-y')^2+(z-z')^2\right)\hat{x} + \frac{\partial}{\partial y}\left((x-x')^2+(y-y')^2 + (z-z')^2\right)\hat{y} + \frac{\partial}{\partial z}\left((x-x')^2+(y-y')^2+(z-z')^2\right)\hat{z}$$

$$= 2(x-x')\hat{x} + 2(y-y')\hat{y} + 2(z-z')\hat{z}$$

$\nabla \mathfrak{r} = 2\mathfrak{r}$

b) $\nabla(1/\mathfrak{r}) = -\hat{\mathfrak{r}}/\mathfrak{r}^2$

$$= \frac{\partial}{\partial x}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-1/2}\hat{x} + \frac{\partial}{\partial y}\left((x-x')^2+(y-y')^2 + (z-z')^2\right)^{-1/2}\hat{y} + \frac{\partial}{\partial z}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-1/2}\hat{z}$$

$$= -\frac{1}{2}\left[\frac{\partial}{\partial x}\left((x-x')^2+(y-y')^2+(z-z')^2\right]^{-3/2} 2(x-x')\hat{x} - \frac{1}{2}\left[\frac{\partial}{\partial y}\left((x-x')^2+(y-y')^2+(z-z')^2\right]^{-3/2} 2(y-y')\hat{y} - \frac{1}{2}\left[\frac{\partial}{\partial z}\left((x-x')^2+(y-y')^2+(z-z')^2\right]^{-3/2} 2(z-z')\hat{z}$$

$$= -\frac{1}{\mathfrak{r}^3}\cdot\mathfrak{r}$$

$$= -\frac{1}{\mathfrak{r}^2}\hat{\mathfrak{r}}$$

$$\nabla(1/\mathfrak{r}) = -\frac{\hat{\mathfrak{r}}}{\mathfrak{r}^2}$$

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c) $\frac{\partial}{\partial x}(r^n) = n r^{n-1} \frac{\partial r}{\partial x}$

$= n r^{n-1} \left(\frac{1}{2} \cdot \frac{1}{r} 2 r x \right)$

$\frac{\partial}{\partial x}(r^n) = n r^{n-1} \hat{r} x$

$\Rightarrow$ Jadi $\nabla (r^n) = n r^{n-1} \hat{r}$

Problem 1.14

Suppose that f is a function of two variables (y and z) only. Show that the gradient $\nabla f = (\partial f/\partial y)\hat{y} + (\partial f/\partial z)\hat{z}$ transforms as a vector under reaction. Eq. 1.29. [Hint: $(\partial f/\partial \bar{y}) = (\partial f/\partial y)(\partial y/\partial \bar{y}) + (\partial f/\partial z)(\partial z/\partial \bar{y})$, and the analogous formula for $\partial f/\partial \bar{z}$. We know that $\bar{y} = y \cos\phi + z \sin\phi$ and $\bar{z} = -y \sin\phi + z \cos\phi$; "solve" these equations for y and z (as functions of $\bar{y}$ and $\bar{z}$) and computer the needed derivatives $\partial y/\partial \bar{y}$, $\partial z/\partial \bar{y}$, etc.].

Jawab:

$\bar{y} = y \cos\phi + z \sin\phi$

$\bar{y} \sin\phi = y \sin\phi \cos\phi + z \sin^2\phi$

$\bar{z} = -y \sin\phi + z \cos\phi$

$\bar{z} \cos\phi = -y \sin\phi \cos\phi + z \cos^2\phi$

$\bar{y} \sin\phi + \bar{z} \cos\phi = z(\sin^2\phi + \cos^2\phi) z$

$\bar{y} \cos\phi - \bar{z} \sin\phi = y$

$\Rightarrow$ Sehingga:

$\frac{\partial y}{\partial \bar{y}} = \cos\phi$; $\frac{\partial y}{\partial \bar{z}} = -\sin\phi$; $\frac{\partial z}{\partial \bar{y}} = \sin\phi$; $\frac{\partial z}{\partial \bar{z}} = \cos\phi$

$(\overline{\nabla F})_y = \frac{\partial F}{\partial \bar{y}} = \frac{\partial F}{\partial y}\frac{\partial y}{\partial \bar{y}} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial \bar{y}} = \cos\phi (\nabla F)_y + \sin\phi (\nabla F)_z$

$(\overline{\nabla F})_z = \frac{\partial F}{\partial \bar{z}} = \frac{\partial F}{\partial y}\frac{\partial y}{\partial \bar{z}} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial \bar{z}} = -\sin\phi (\nabla F)_y + \cos\phi (\nabla F)_z$