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Date :

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Ayo Berlatih:

1) Problem 1.11

• Find the gradien of the following functions.

a) $f(x,y,z) = x^2 + y^3 + z^4$.

$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} = 2x\hat{x} + 3y^2\hat{y} + 4z^3\hat{z}$ //

$2x\hat{i} + 3y^2\hat{j} + 4z^3\hat{k}$

b). $f(x,y,z) = x^2y^3z^4$

$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$

$= \frac{\partial}{\partial x}(x^2y^3z^4)\hat{x} + \frac{\partial}{\partial y}(x^2y^3z^4)\hat{y} + \frac{\partial}{\partial z}(x^2y^3z^4)\hat{z}$.

$= (2xy^3z^4)\hat{x} + (3x^2y^2z^4)\hat{y} + (4x^2y^3z^3)\hat{z}$.

c.) $f(x,y,z) = e^x \sin(y) \ln(z)$.

$\nabla f(x,y,z) = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$.

$= \frac{\partial}{\partial x}(e^x \sin(y)\ln(z))\hat{x} + \frac{\partial}{\partial y}(e^x \sin(y)\ln(z))\hat{y} + \frac{\partial}{\partial z}(e^x \sin(y)\ln(z))\hat{z}$.

$= (e^x \sin(y)\ln(z)x^2 + (e^x \cos(y)\ln(z))\hat{y} + \frac{e^x \sin(y)}{z})\hat{z}$ //

2. problem (1.12).

a. Where is the top of the hill located?

$\nabla h = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$

$\nabla h = 0$ ada di puncak

$2y - 6x - 18 = 0$ ... (1)
$2x - 8y + 28 = 0$ ... (2).

$2y - 6x - 18 = 0$.
$6x - 24y + 84 = 0$.
$-22y + 66 = 0$.
$y = 3$ //.

$2y - 6x - 18 = 0$
$2(3) - 6x - 18 = 0$
$-6x = 12$.
$x = -2$.

∴ maka letak puncak terletak di 3 km utara dan 2 km barat.

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b.) how high is the hill?

substitusikan $x = -2, y = 3$. $\Rightarrow h = 10(2(3)(-2) - 3(-2)^2 - 4(3)^2 - 18(-2) + 28(3) + 12)$

$h = 10(-12 - 12 - 36 + 36 + 84 + 12)$.

$h = 720$ kaki

c.) how steep is the slope at a point 1 mile ~~east~~ north and 1 mile east of south hadley? In what direction is the slope steepest, at the poin?

$\Rightarrow$ Sub. $x = 1, y = 1$.

$\nabla h = 10((2-6-18)\hat{x} + (2-8+28)\hat{y})$

$= 10(-22\hat{x} + 22\hat{y})$.

$= 220(-\hat{x} + \hat{y})$.

$|\nabla h| = 220\sqrt{2} \simeq 311$ kaki/mil arah barat laut.

3). Calculate the divergence of the following Vector functions :

a.) $\bar{v}(\vec{r}) = 2\hat{r}$

$\eta = (x-x')\hat{x} + (y-y')y^2 + (z-z')\hat{z}$

$\eta = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$

$\nabla(\vec{r}) = \frac{\partial}{\partial x}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right) + \frac{\partial}{\partial y}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)\hat{y}$

$+ \frac{\partial}{\partial z}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)\hat{z}$.

$= 2(x-x')\hat{x} + 2(y-y')\hat{y} + 2(z-z')\hat{z} = 2\eta$ //.

b.) $\nabla\left(\frac{1}{\eta}\right) = \frac{\hat{\eta}}{\eta^2}$

$= \frac{\partial}{\partial x}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)^{-1/2}\hat{x} + \frac{\partial}{\partial y}\left((x-x')^2 + (y-y')^2 + z - z')^2\right]^{1/2}\hat{y}$.

$+ \frac{\partial}{\partial z}\left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{-1/2}\hat{z}$.

$= \frac{1}{2}\left[\frac{\partial}{\partial x}\left((x-x')^2 + y-y')^2 + (z-z')^2\right)^{-1/2}\right]^{-3/2} 2(x-x')\hat{x} - \frac{1}{2}\left[\frac{\partial}{\partial y}\left((x-x')^2\right.\right.$

$\left.\left. + (y-y')^2 + (z-z')^2\right)^{-1/2}\right]^{-3/2} 2(y-y')\hat{y} - \frac{1}{2}\left[\frac{\partial}{\partial z}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)^{-\frac{1}{2}}\right]^{-3/2}$.

$2(z-z')\hat{z}$.

$= -\left(\frac{\partial}{2}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)^{-1/2}\right)^{-3/2}\left((x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}\right)$

$= -\frac{1}{\eta^2}\hat{\eta}$

$= -\frac{\hat{\eta}}{\eta^2}$ //.

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c.) what is the general formula for $\nabla(r^n)$?

$$\frac{\partial}{\partial x}(r^n) = n r^{n-1} \frac{\partial r}{\partial x}$$

$$= n r^{n-1} \left(\frac{1}{2}\frac{1}{r} 2rx\right) = n r^{n-1} r^{-1} x.$$

Sehingga $\bar{\nabla}(r^n) \cdot r$

$= n r^{n-1} \hat{r}$ //.

problem 1.14 (Halaman 16).

Suppose that f is a function of two variabel (y dan z) only. Show that the gradien $\nabla f = (\partial f / \partial y)\hat{y} + (\partial f / \partial z)\hat{z}$ transforms as a vektor under rotations, bg. bg.

penyelesaian :

$$\bar{y} = y \cos\theta + z \sin\theta$$

$$\bar{y} \sin\theta = y \sin\theta \cos\theta + z \sin^2\theta.$$

$$\bar{z} = -y \sin\theta + z \cos\theta$$

$$z \cos\theta = -y \sin\theta \cos\theta + z \cos^2\theta$$

$$y \sin\theta + \bar{z} \cos\theta = z(\sin^2\theta + \cos^2\theta) = z.$$

$$\bar{y} \cos\theta - \bar{z} \sin\theta = y.$$

Sehingga, $\frac{\partial y}{\partial y} = \cos\theta$; $\frac{\partial y}{\partial z} = -\sin\theta$; $\frac{\partial z}{\partial y} = \sin\theta$; $\frac{\partial z}{\partial z} = \cos\theta$.

$$\overline{(\nabla f)_y} \quad \frac{\partial f}{\partial y} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial y} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial y} = \cos\theta\,(\nabla f)_y + \sin\theta\,(\nabla f).$$

$$\overline{(\nabla f)_z} = \frac{\partial f}{\partial z} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial z} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial z} = -\sin\theta\,(\nabla f)_y + \cos\theta\,(\nabla f).$$

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