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Problem 1.11 Find the gradients of the following functions :

(a) $f(x,y,z) = x^2 + y^3 + z^4$

$$\vec{\nabla} f = 2x\hat{i} + 3y^2\hat{j} + 4z^3\hat{k}$$

(b) $f(x,y,z) = x^2 y^3 z^4$

$$\vec{\nabla} f = 2xy^3z^4\hat{i} + 3x^2y^2z^4\hat{j} + 4x^2y^3z^3\hat{k}$$

(c) $f(x,y,z) = e^x \sin(y)\ln(z)$

$$\vec{\nabla} f = e^x \sin(y)\ln(z)\hat{i} + e^x\cos(y)\ln(z)\hat{j} + e^x \sin(y)\left(\frac{1}{z}\right)\hat{k}$$

Problem 1.12 The height of a certain hill (in feet) is given by $h(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$, Where y is the distance (in miles) north, x the distance east of south hadley.

(a) Where is the top of the hill located ?

Tentukan fungsi gradien terlebih dahulu

$$\vec{\nabla} b = 10[(2y - 6x - 18)\hat{i} + (2x - 8y + 28)\hat{j}]$$

Untuk menentukan puncak bukit, gunakan syarat $\vec{\nabla} b = 0$ (puncak bukit merupakan salah satu jenis stasioner)

$\vec{\nabla} b = 10[(2y - 6x - 8)\hat{i} + (2x - 8y + 28)\hat{j}] = 0$ menghasilkan sistem persamaan linear dua peubah :

$$\left.\begin{array}{l} 2y - 6x - 18 = 0 \\ 2x - 8y + 28 = 0 \end{array}\right\}$$ Solusi dari sistem persamaan ini = $(x,y) = (-2,3)$

Dengan demikian, puncak bukit tersebut berada pada 2 km sebelah barat dan 3 km utara bandung.

(b) How high is the hill ?

Substitusikan $(x,y) = (-2,3)$ pada $b(x,y) = b(-2,3) = 10(-12-12-36+36+84+84+12) = 720$ m

(c) How steep is the slope (in feet per mile) at a point 1 mile north and 1 mile east of South hadley ? In what direction is the slope steepest, at the point ?

Subtitusikan $(x,y) = (1,1)$ pada $\vec{\nabla} b$

$$\vec{\nabla} b(1,1) = 10[2-6-18]\hat{i} + (2-8+28)\hat{j} = 220(-\hat{i}+\hat{j})$$

Date :

$|\vec{b}| = 220\sqrt{2} \approx 311$ m, arahnya ke barat laut (135° dari sumbu x positif)

Problem 1.13 Let $\mathfrak{r}$ be the separation vector from a fixed point (x', y', z') to the point (x, y, z), and let $\mathfrak{r}$ be its length. Show that

(a) $\nabla(\mathfrak{r}^2) = 2\mathfrak{r}$

(b) $\nabla\left(\dfrac{1}{\mathfrak{r}}\right) = -\dfrac{\hat{\mathfrak{r}}}{\mathfrak{r}^2}$

(c) What is the general formula for $\nabla(\mathfrak{r}^n)$?

$$(x, y, z)\ \hat{\mathfrak{r}} = (x - x_0)\hat{i} + (y - y_0)\hat{j} + (z - z_0)\hat{k}$$

$$\mathfrak{r} = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$$

$$\mathfrak{r}^2 = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$$

(a) $$\vec{\nabla}\mathfrak{r}^2 = \frac{\partial}{\partial x}\left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2\right]\hat{i} + \frac{\partial}{\partial y}\left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2\right]\hat{j}$$

$$+ \frac{\partial}{\partial z}\left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2\right]\hat{k}$$

$$= 2(x - x_0)\hat{i} + 2(y - y_0)\hat{j} + 2(z - z_0)\hat{k}$$

$$= 2\hat{\mathfrak{r}}$$

(c) $$\frac{\partial}{\partial x}(\hat{\mathfrak{r}}) = n_2^{\,n-1}\frac{\partial n}{\partial x} = n_2^{\,n-1}\left(\frac{1}{2}\frac{1}{\mathfrak{r}}2\mathfrak{r}_x\right) = n_1^{\,n-1}\mathfrak{r}_x, \quad (\mathfrak{r}_x = x - x_0)$$

$$\frac{\partial}{\partial y}(\mathfrak{r}^n) = n_2^{\,n-1}\mathfrak{r}_y, \quad \frac{\partial}{\partial z}(2^n) = n_2^{\,n-1}\mathfrak{r}_z$$

Sehingga $\vec{\nabla}(\mathfrak{r}^m) = n_2^{\,n-1}\,\mathfrak{r}.$