Kelistrikan dan Kemagnetan.

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Problem 1.11 (hal 15)

- find the gradien of the following functions
  a.) $f(x,y,z) = x^2 + y^3 + z^4$
  b.) $f(x,y,z) = x^2 y^3 z^4$
  c.) $f(x,y,z) = e^x \sin(y) \ln(z)$

- Solution
  a.) $f(x,y,z) = x^2 + y^3 + z^4$
  $\Rightarrow \vec{\nabla} f = 2x\,\hat{i} + 3y^2\,\hat{j} + 4z^3\,\hat{k}$

  b.) $f(x,y,z) = x^2 y^3 z^4$
  $\Rightarrow \vec{\nabla} f = 2xy^3 z^4\,\hat{i} + 3x^2 y^2 z^4\,\hat{j} + 4x^2 y^3 z^4\,\hat{k}$

  c.) $f(x,y,z) = e^x \sin(y) \ln(z)$
  $\Rightarrow \vec{\nabla} f = e^x \sin(y) \ln(z)\,\hat{i} + e^x \cos(y) \ln(z)\,\hat{j} + e^x \sin(y) \left(\frac{1}{z}\right)\hat{k}$

Problem 1.12

- The height of a certain hill (in feet) is given by
  $h(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$
  where y is distance (in miles) north, x the distance east of south hadley.

a.) Where is the top of the hill located?
b.) How high is the hill
c.) How steep is the slope (in feet permile) at a point 1 mile with and one mile east of south hadley? In what direction is the slope steepes, at that point?

• Solution

a.) Dimanakah puncak bukit tersebut berada ?

=> Tentukan fungsi gradien terlebih dahulu

$\vec{\nabla} b = 10 [(2y - 6x - 18) \hat{i} + (2x - 8y + 28) \hat{j}]$

untuk menentukan puncak bukit, gunakan syarat $\vec{\nabla} b = 0$ (puncak bukit merupakan salah satu jenis stationer).

$\vec{\nabla} b = 10 [(2y - 6x - 18) i + (2x - 8y + 28) j] = 0$, menghasilkan sistem persamaan linear dua peubah.

$2y - 6x - 18 = 0$

$2x - 8y + 28 = 0$ } solusi dari sistem persamaan $\Rightarrow (x, y) = (-2, 3)$

Dengan demikian, puncak bukit tersebut berada pada 2 km sebelah barat dan 3 km utara Bandung.

b.) Berapa ketinggian bukit tersebut ?

=> Substitusikan $(x, y) = (-2, 3)$ pada $b(x, y)$:

$b = (-2, 3) = 10 (-12 - 12 - 36 + 36 + 84 + 12) = 720$ m

Jadi, ketinggiannya adalah 720 m.

c.) Seberapa curam kemiringan (dalam satuan m/km) pada sebuah titik 1 km utara dan 1 km timur kota Bandung ? Pada arah manakah kemiringan tercuram di titik tersebut ?

=> Substitusikan $(x, y) = (1, 1)$ pada $\vec{\nabla} b$.

$\vec{\nabla} b (1, 1) = 10 [(2 - 6 - 18) \hat{i} + (2 - 8 - 28) \hat{j}] = 220 (\hat{i} + \hat{j})$

$|\vec{\nabla} b| = 220\sqrt{2} \approx 311$ m/km, arahnya ke barat laut (135° dari sumbu x positif).

Problem 1.13

Let $\mathbf{r}$ be the separation vector from a fixed point (x', y', z') to the point (x, y, z), and let r be its length. Show that:

a.) $\nabla (r^2) = 2\mathbf{r}$

b.) $\nabla (1/r) = -\mathbf{r}/r^3$

c.) What is the general formula for $\nabla (r^n)$?

Solution :

(x, y, z) $\quad \mathbf{r} = (x - x_0)\hat{i} + (y - y_0)\hat{i} + (z - z_0)\hat{k}$

$\quad r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$

(x_0, y_0, z_0) $\quad r^2 = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$

a.) $\vec{\nabla} r^2 = \frac{\partial}{\partial x}\left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2\right]\hat{i} + \frac{\partial}{\partial y}\left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2\right]\hat{j} + \frac{\partial}{\partial z}\left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2\right]\hat{k}$

$= 2(x - x_0)\hat{i} + 2(y - y_0)\hat{j} + 2(z - z_0)\hat{k} = 2\vec{r}$ (terbukti)

c) $\frac{\partial}{\partial x}(r^n) = n r^{n-1} \frac{dr}{dx} = n r^{n-1}\left(\frac{1}{2}\,\frac{1}{r}\,2x\right) = n r^{n-1} i_x$,

$(2x = x - x_0)$

$\frac{\partial}{\partial y}(r^n) = n r^{n-1} i_y$, $\frac{\partial}{\partial z}(z^n) = n r^{n-1} i_z$; sehingga

$\vec{\nabla}(r^n) = n r^{n-1} i$