No.

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Problem 1.11 (Page 15)

Find the gradient of the Following Functions:

a). $F(x,y,z) = x^2 + y^3 + z^4$

Penyelesaian :

$$\vec{\nabla} F = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) F$$
$$= \left(\frac{\partial F}{\partial x}\hat{i} + \frac{\partial F}{\partial x}\hat{j} + \frac{\partial F}{\partial z}\hat{k}\right)$$

$\vec{\nabla} F = \ldots$

$\Rightarrow \frac{\partial F}{\partial x} = 2x$

$\Rightarrow \frac{\partial F}{\partial y} = 3y^2$

$\Rightarrow \frac{\partial F}{\partial z} = 4z^3$

$\therefore \vec{\nabla} F = (2x)\hat{i} + (3y^2)\hat{j} + (4z^3)\hat{k}$

b). $F(x,y,z) = x^2 y^3 z^4$

Penyelesaian :

$$\vec{\nabla} F = \left(\frac{\partial F}{\partial x}\hat{i} + \frac{\partial F}{\partial y}\hat{j} + \frac{\partial F}{\partial z}\hat{k}\right)$$

$\vec{\nabla} F = \ldots$

$\Rightarrow \frac{\partial F}{\partial x} = 2xy^3z^4$

$\Rightarrow \frac{\partial F}{\partial y} = 3y^2x^2z^4$

$\Rightarrow \frac{\partial F}{\partial z} = 4z^3x^2y^3$

$\therefore \vec{\nabla} F = (2xy^3z^4)\hat{i} + (3x^2y^2z^4)\hat{j} + (4x^2y^3z^3)\hat{k}$

c). $F(x,y,z) = e^x \sin(y) \ln(z)$

Penyelesaian :

$$\vec{\nabla} F = \left(\frac{\partial F}{\partial x}\hat{i} + \frac{\partial F}{\partial y}\hat{j} + \frac{\partial F}{\partial z}\hat{k}\right)$$

$\vec{\nabla} F = \ldots$

$\Rightarrow \frac{\partial F}{\partial x} = e^x \sin(y) \ln(z)$

$\Rightarrow \frac{\partial F}{\partial y} = e^x \cos(y) \ln(z)$

$\Rightarrow \frac{\partial F}{\partial z} = e^x \sin(y) \left(\frac{1}{z}\right)$

$\therefore \vec{\nabla} F = (e^x \sin(y)\ln(z))\hat{i} + (e^x \cos(y)\ln(z))\hat{k} + (e^x \sin(y)\frac{1}{z})\hat{k}$

Problem 1.12

The height of a certain hill (in Feet) is given by $h(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$ where y is the distance (in miles) north, x the distance east of South Hadley.

(a). where is the top of the hill located ?

Penyelesaian :

$\Rightarrow$ Tentukan Fungsi gradien terlebih dahulu

$$\vec{\nabla} h = \left(\frac{\partial h}{\partial x}\hat{i} + \frac{\partial h}{\partial y}\hat{j}\right)$$

$\Rightarrow \frac{\partial h}{\partial x} = 10(2y - 6x - 18)$

$\Rightarrow \frac{\partial h}{\partial y} = 10(2x - 8y + 28)$

$\vec{\nabla} h = 10[(2y - 6x - 18)\hat{i} + (2x - 8y + 28)\hat{j}]$

$\Rightarrow$ menentukan puncak, syarat $\vec{\nabla} h = 0$

$\vec{\nabla} h = 10[(2y - 6x - 18)\hat{i} + (2x - 8y + 28)\hat{j}] = 0$

$$\left.\begin{aligned} &\Rightarrow 2y - 6x - 18 = 0 \\ &2x - 8y + 28 = 0 \end{aligned}\right\} \text{Solusi} \Rightarrow (x,y) = (-2,3)$$

$\therefore$ Maka dari itu, Puncak bukit berada pada 2 mil sebelah barat dan 3 mil sebelah utara

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(b). How high is the hill?

Penyelesaian :

Sub $(x,y) = (-2,3)$ pada $h(x,y)$:

$h(-2,3) = 10(2(-2)(3) - 3(-2)^2 - 4(3)^2 - 18(-2) + 28(3) + 12)$

$= 10(-12 - 12 - 36 + 36 + 84 + 12)$

$= 10(72)$

$= 720$ mil

(c). How steep is the slope (in feet per mile) at a point 1 mile north and one mile east of South Hadley? in what direction is the slope steepest at that point?

Penyelesaian:

Subsitusikan $(x,y) = (1,1)$ pada $\vec{\nabla} h$

$\vec{\nabla} h(1,1) = 10[(2-6-18)\hat{i} + (2-8+28)\hat{j}] = 220(-\hat{i}+\hat{j})$

$|\vec{\nabla} h| = 220\sqrt{2} = 311$ m/km ke arah barat laut.

Problem 1.13

let $\mathcal{r}$ be the separation vektor from fixed point (x',y',z') to the point (x,y,z) and let $\mathcal{r}$ be its lenght. Show that

(a). $\nabla(\mathcal{r}^2) = 2\mathcal{r}$

Penyelesaian :

$\mathcal{r} = (x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}$

$\mathcal{r} = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$

$$\nabla(\mathcal{r}^2) = \frac{\partial}{\partial x}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)\hat{x} + \frac{\partial}{\partial y}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)\hat{y}$$

$$+ \frac{\partial}{\partial z}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)\hat{z}$$

$$= 2(x-x')\hat{x} + 2(y-y')\hat{y} + 2(z-z')\hat{z}$$

$$= 2\mathcal{r}$$

(b). $\nabla\left(\frac{1}{\mathcal{r}}\right) = -\frac{\hat{\mathcal{r}}}{\mathcal{r}^2}$

$$= \frac{\partial}{\partial x}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)^{-1/2}\hat{x} + \frac{\partial}{\partial x}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)^{-1/2}\hat{y}$$

$$+ \frac{\partial}{\partial z}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)^{-1/2}\hat{z}$$

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$$= -\frac{1}{2}\left[\frac{\partial}{\partial x}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-1/2}\right]^{3/2} 2(x-x')\hat{x} - \frac{1}{2}\left[\frac{\partial}{\partial y}\left((x-x')^2+(y-y')^2+(z-z')^2\right)\right]^{-3/2} 2(y-y')\hat{y}$$

$$-\frac{1}{2}\left[\frac{\partial}{\partial y}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-1/2}\right]^{-3/2} 2(z-z')\hat{z}$$

$$= -\left(\frac{\partial}{\partial}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-1/2}\right)^{-3/2}\left((x-x')\hat{x}+(y-y')\hat{y}+(z-z')\hat{z}\right)$$

$$= -\frac{1}{\mathscr{r}^3}\,\mathscr{r}$$

$$= -\frac{1}{\mathscr{r}^2}\,\hat{\mathscr{r}}$$

$$= -\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

(c) what is the general formula for $\nabla(\mathscr{r}^n)$

Penyelesaian:

$$\frac{\partial}{\partial x}(\mathscr{r}^n) = n\mathscr{r}^{n-1}\frac{\partial \mathscr{r}}{\partial x}$$

$$= n\mathscr{r}^{n-1}\left(\frac{1}{2}\frac{1}{\mathscr{r}}\,2\mathscr{r}x\right)$$

$$= n\mathscr{r}^{n-1}\hat{\mathscr{r}}\,x$$

Jadi, $\nabla(\mathscr{r}^n) = n\mathscr{r}^{n-1}\hat{\mathscr{r}}$

Problem 1.14 (halaman 16)

Suppose that F is a function of two variables (y and z) only. Show that the gradient $\nabla F = (\partial F/\partial y)\hat{y} + (\partial F/\partial z)\hat{z}$ transforms as a vector under rotations, Eq 1.29 [Hint: $(\partial F/\partial \bar{y}) = (\partial F/\partial y)(\partial y/\partial \bar{y}) + (\partial F/\partial z)(\partial z/\partial \bar{y})$, and the anologous formula for $\partial F/\partial \bar{z}$. we know that $\bar{y} = y\cos\theta + z\sin\theta$ and $\bar{z} = -y\sin\theta + z\cos\theta$; solve these quations for y and z (as functions of $\bar{y}$ and $\bar{z}$) and compute the needed derivatves $\partial y/\partial \bar{y}$, $\partial z/\partial \bar{y}$, etc]

Penyelesaian:

$\bar{y} = y\cos\theta + z\sin\theta$

$\bar{y}\sin\theta = y\sin\theta\cos\theta + z\sin^2\theta$

$\bar{z} = -y\sin\theta + z\cos\theta$

$\bar{z}\cos\theta = -y\sin\theta\cos\theta + z\cos^2\theta$

$\bar{y}\sin\theta + \bar{z}\cos\theta = z(\sin^2\theta + \cos^2\theta)z$

$\bar{y}\cos\theta - \bar{z}\sin\theta = y$

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sehingga, $\frac{\partial y}{\partial \bar{y}} = \cos\theta$; $\frac{\partial y}{\partial \bar{z}} = -\sin\theta$; $\frac{\partial z}{\partial \bar{y}} = \sin\theta$; $\frac{\partial z}{\partial \bar{z}} = \cos\theta$

$$\overline{(\nabla F)}_y = \frac{\partial F}{\partial \bar{y}} = \frac{\partial F}{\partial y}\frac{\partial y}{\partial \bar{y}} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial \bar{y}} = \cos\theta(\nabla F)_y + \sin\theta(\nabla F)_z$$

$$\overline{(\nabla F)}_z = \frac{\partial F}{\partial \bar{z}} = \frac{\partial F}{\partial y}\frac{\partial y}{\partial \bar{z}} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial \bar{z}} = -\sin\theta(\nabla F)_y + \cos\theta(\nabla F)_z$$

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