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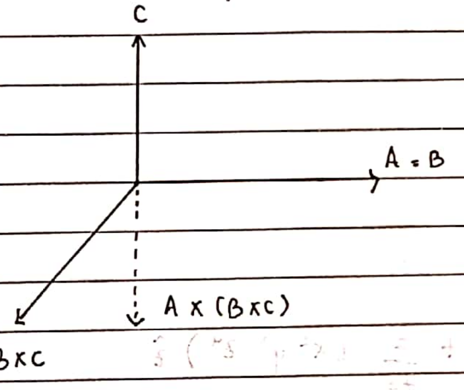
Ayo Berlatih !

•) Problem 1.2 (Halaman 4)

Is the cross product associative ? $(A \times B) \times C = A \times (B \times C)$
If so, prove it ; If not, provide a counter example (the simples the better)

Penyelesaian :

Perkalian cross product tidak bersifat asosiatif .



Hasil kali silang rangkap tiga umumnya tidak bersifat asosiatif. Misalnya , A = B dan C tegak lurus terhadap A. Kemudian (B x C) menuju keluar, dan $A \times (B \times C)$ menuju ke bawah dan memiliki besar ABC. $(A \times B) = 0$
Maka,
$(A \times B) \times C = 0 \quad A \times (B \times C)$

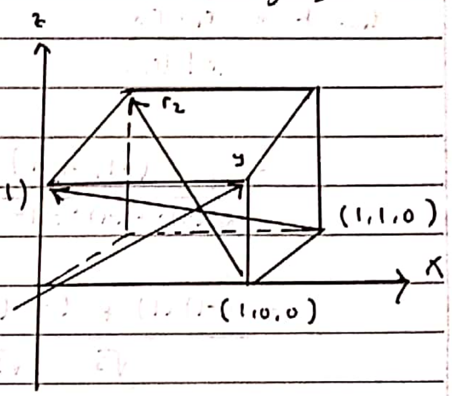
Misalnya :
$A = (-1, 3, 4)$, $B = (3, -2, -8)$, $C = (4, 4, 4)$
Hasilnya adalah :
$A \times (B \times C) = (140, 116, -52)$
$(A \times B) \times C = (-12, 36, 80)$.

•) Problem 1.3

Find the angle between the body diagonals of a cube.

Penyelesaian :

•) Untuk r_1 dan r_2 menjadi vektor perpindahan untuk diagonal tubuh yang memiliki titik awal pada simpul yang berdekatan dalam kubus satuan. Misalnya yang ditentukan pada gambar berikut ini :



$r_1 = (0,0,1) - (1,1,0) = \langle -1,-1,1 \rangle$

$r_2 = (0,1,1) - (1,0,0) = \langle -1,1,1 \rangle$

•) Gunakan definisi produk titik antara r_1 dan r_2

$$r_1 \cdot r_2 = |r_1||r_2| \cos\theta$$

•) Kemudian untuk sudut antara r_1 dan r_2

$$\cos\theta = \frac{r_1 \cdot r_2}{|r_1|\,|r_2|}$$

$$= \frac{(-1,-1,1)\cdot(-1,1,1)}{\sqrt{(-1)^2+(-1)^2+1^2}\ \sqrt{(-1)^2+1^2+1^2}}$$

$$= \frac{(-1)(-1)+(-1)(1)+(1)(1)}{\sqrt{3}\ \sqrt{3}} = \frac{1}{3}.$$

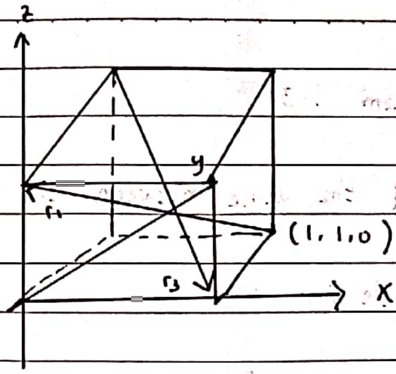
Maka, sudut antara diagonal yang berdekatan :

$$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 70{,}5^\circ.$$

•) Pada r_1 dan r_3 menjadi vektor perpindahan untuk diagonal yang memiliki titik awal pada simpul yang berlawanan dalam kubus satuan, misalnya yang ditunjukkan seperti pada gambar berikut :

$r_1 = (0,0,1) - (1,1,0) = \langle -1,-1,1 \rangle$

$r_2 = (1,0,0) - (0,1,1) = \langle 1,-1,-1 \rangle$



•) Gunakan definisi produk antara r_1 + r_3

$r_1 \cdot r_3 = |r_1||r_3| \cos \gamma$

•) Untuk γ, sudut antara r_1 dan r_3

$$\cos \gamma = \frac{r_1 \cdot r_3}{|r_1|\,|r_3|}$$

$$= \frac{(-1,-1,1) \cdot (1,-1,-1)}{\sqrt{(-1)^2+(-1)^2+1^2}\ \sqrt{1^2+(-1)^2+(-1)^2}}$$

$$= \frac{(-1)(1) + (-1)(-1) + (1)(-1)}{\sqrt{3}\ \sqrt{3}} = -\frac{1}{3}$$

Maka, sudut antara diagonal terbentuk yang berlawanan adalah :

$$\gamma = \cos^{-1}\left(-\frac{1}{3}\right) = 109{,}5^\circ$$

Jadi, $\gamma + \theta = 109{,}5^\circ + 70{,}5^\circ = 180^\circ$ //

•) Problem 1.11 (Halaman 15)

Find the gradients of the following functions :

a.) $f(x,y,z) = x^2 + y^3 + z^4$

Penyelesaian :

$$\nabla f(x,y,z) = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$= \hat{x}\frac{\partial}{\partial x}(x^2+y^3+z^4) + \hat{y}\frac{\partial}{\partial y}(x^2+y^3+z^4) + \hat{z}\frac{\partial}{\partial z}(x^2+y^3+z^4)$$

$$= (2x)\hat{x} + (3y^2)\hat{y} + (4z^3)\hat{z} \;.\; //$$

b.) $f(x,y,z) = x^2 y^3 z^4$

Penyelesaian :

$$\nabla f(x,y,z) = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$= \frac{\partial}{\partial x}(x^2y^3z^4)\hat{x} + \frac{\partial}{\partial y}(x^2y^3z^4)\hat{y} + \frac{\partial}{\partial z}(x^2y^3z^4)\hat{z}$$

$$= (2xy^3z^4)\hat{x} + (3x^2y^2z^4)\hat{y} + (4x^2y^3z^3)\hat{z} \;.\; //$$

c.) $f(x,y,z) = e^x \sin(y) \ln(z)$

Penyelesaian :

$$\nabla f(x,y,z) = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$= \frac{\partial}{\partial x}(e^x\sin(y)\ln(z))\hat{x} + \frac{\partial}{\partial y}(e^x\sin(y)\ln(z))\hat{y} + \frac{\partial}{\partial z}(e^x\sin(y)\ln(z))\hat{z}$$

$$= (e^x\sin(y)\ln(z))\hat{x} + (e^x\cos(y)\ln(z))\hat{y} + \left(\frac{e^x\sin(y)}{z}\right)\hat{z} \;.\; //$$

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•) Problem 1.12 (Halaman 15)

The height of a certain hill (in feet) is given by
$h(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$
Where y is the distance (in miles) north, x the distance east of south hadley

a.) Where is the top of the hill located ?

Penyelesaian :
$\nabla h = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$
$\nabla h = 0$, di puncak.

$2y - 6x - 18 = 0$
$2x - 8y + 28 = 0$.

$-6x + 2y - 18 = 0$
$6x - 24y + 84 = 0$
$-22y + 66 = 0$
$-22y = -66$
$y = 3$.

$2y - 6x - 18 = 0$
$2(3) - 6x - 18 = 0$
$-6x = 12$
$x = -2$.

Sehingga letak puncak itu terletak di 3 mil utara dan 2 mil barat dari Hadley selatan. //

b.) How high is the hill ?

Penyelesaian :
Subtitusikan $x = -2$, $y = 3$
$h = 10(-12 - 12 - 36 + 36 + 84 + 12)$
$h = 720$ Kaki.

Jadi, tinggi bukit tersebut adalah 720 Kaki. //

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c.) How steep is the slope (in feet per mile) at a point 1 mile north and 1 mile east of South Hadley? In what direction is the slope steepest, at that point?

Penyelesaian :

Subtitusikan $x = 1$, $y = 1$

$$\nabla h = 10\left(((2-6-18))\hat{x} + ((2-8+28))\hat{y}\right)$$
$$= 10\left((-22)\hat{x} + 22\hat{y}\right)$$
$$= -220\hat{x} + 220\hat{y}$$
$$= 220(-\hat{x} + \hat{y})$$

$|\nabla h| = 220\sqrt{2} \simeq 311$ kaki/mil

Jadi, kemiringan pada titik 1 mil di utara dan 1 mil di timur South hadley adalah $220\sqrt{2} \simeq 311$ kaki/mil dengan arah kemiringan paling curam di arah barat laut. //

•) Problem 1.13 (Halaman 15)

Let $\boldsymbol{\mathscr{r}}$ be the separation vector from a fixed point (x', y', z') to the point (x, y, z), and let $\mathscr{r}$ be its length. Show that :

a.) $\nabla(\mathscr{r}^2) = 2\boldsymbol{\mathscr{r}}$

Penyelesaian :

$$\boldsymbol{\mathscr{r}} = (x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}$$
$$\mathscr{r} = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\nabla(\mathscr{r})^2 = \frac{\partial}{\partial x}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)\hat{x} + \frac{\partial}{\partial y}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)\hat{y}$$
$$+ \frac{\partial}{\partial z}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)\hat{z}$$
$$= 2(x-x')\hat{x} + 2(y-y')\hat{y} + 2(z-z')\hat{z}$$
$$= 2\boldsymbol{\mathscr{r}}. \quad //$$

b.) $\nabla\left(\frac{1}{\eta}\right) = -\frac{\hat{\eta}}{\eta^2}$

Penyelesaian :

$$= \frac{\partial}{\partial x}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-\frac{1}{2}}\hat{x} + \frac{\partial}{\partial y}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-\frac{1}{2}}\hat{y}$$

$$+ \frac{\partial}{\partial z}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-\frac{1}{2}}\hat{z}$$

$$= \frac{1}{2}\left[\frac{\partial}{\partial x}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-\frac{1}{2}}\right]^{-\frac{3}{2}} 2(x-x')\hat{x} - \frac{1}{2}\left[\frac{\partial}{\partial y}\left((x-x')^2+\right.\right.$$

$$\left.\left.(y-y')^2+(z-z')^2\right)^{-\frac{1}{2}}\right]^{-\frac{3}{2}} 2(y-y')\hat{y} - \frac{1}{2}\left[\frac{\partial}{\partial z}\left((x-x')^2+(y-y')^2+(z-z')^2\right)\right.$$

$$\left.^{-\frac{1}{2}}\right]^{\frac{3}{2}} 2(z-z')\hat{z}$$

$$= \left[-\left(\frac{2}{2}(x-x')^2+(y-y')^2+(z-z')^2\right)^{-\frac{1}{2}}\right]^{-\frac{3}{2}}\left((x-x')\hat{x}+(y-y')\hat{y}+(z-z')\hat{z}\right)$$

$$= -\frac{1}{\eta^3}\eta = -\frac{1}{\eta^2}\hat{\eta} = -\frac{\hat{\eta}}{\eta^2} \quad //$$

c.) What is the general formula for $\nabla(\eta^n)$

Penyelesaian :

$$\frac{\partial}{\partial x}(\eta^n) = n\eta^{n-1}\frac{\partial \eta}{\partial x}$$

$$= n\eta^{n-1}\left(\frac{1}{2}\frac{1}{\eta}2\eta x\right)$$

$$= n\eta^{n-1}\hat{\eta}x$$

Jadi, $\nabla(\eta^n) = n\eta^{n-1}\hat{\eta}$. //

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•) Problem 1.14 (Halaman 16)

Suppose that f is a function of two variables (y and z) only. Show that the gradien ∇f, $(\partial f/\partial y)\hat{y} + (\partial f/\partial z)\hat{z}$ transforms as a vector under rotations, fq. 1.29. [Hint: $\partial f/\partial \bar{y} = (\partial f/\partial y)(\partial y/\partial \bar{y}) + (\partial f/\partial z)(\partial z/\partial \bar{y})$, and the anologous formula for $\partial f/\partial \bar{z}$. We know that $\bar{y} = y\cos\theta + z\sin\theta$ and $\bar{z} = -y\sin\theta + z\cos\theta$; "Solve" these quations for y and z (as functions of $\bar{y}$ and $\bar{z}$), and compute the needed derivatives $\partial y/\partial \bar{y}$, $\partial z/\partial \bar{y}$, etc]

Penyelesaian :

$\bar{y} = y\cos\theta + z\sin\theta$

$\bar{y}\sin\theta = y\sin\theta\cos\theta + z\sin^2\theta$

$\bar{z} = -y\sin\theta + z\cos\theta$

$\bar{z}\cos\theta = -y\sin\theta\cos\theta + z\cos^2\theta$

$\bar{y}\sin\theta + \bar{z}\cos\theta = z(\sin^2\theta + \cos^2\theta) = z$

$\bar{y}\cos\theta - \bar{z}\sin\theta = y$

Sehingga, $\frac{\partial y}{\partial \bar{y}} = \cos\theta$; $\frac{\partial y}{\partial \bar{z}} = -\sin\theta$; $\frac{\partial z}{\partial \bar{y}} = \sin\theta$; $\frac{\partial z}{\partial \bar{z}} = \cos\theta$

$$(\overline{\nabla f})_y = \frac{\partial f}{\partial \bar{y}} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial \bar{y}} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \bar{y}} = \cos\theta\,(\nabla f)_y + \sin\theta\,(\nabla f)_z$$

$$(\overline{\nabla f})_z = \frac{\partial f}{\partial \bar{z}} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial \bar{z}} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \bar{z}} = -\sin\theta\,(\nabla f)_y + \cos(\theta)\,(\nabla f)_z \quad //$$

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