Date

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Mata Kuliah : Kelistrikan dan Kemagnetan

Ayo Berlatih

1.11 Find the gradients of the following functions :

(a) $f(x,y,z) = x^2 + y^3 + z^4$

$\Rightarrow$

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$= 2x\,\hat{x} + 3y^2\,\hat{y} + 4z^3\,\hat{z}$$

(b) $f(x,y,z) = x^2y^3z^4$

$\Rightarrow$

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$= 2xy^3z^4\,\hat{x} + 3x^2y^2z^4\,\hat{y} + 4x^2y^3z^3\,\hat{z}$$

(c) $f(x,y,z) = e^x \sin(y)\ln(z)$

$$\Rightarrow \nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$= 1.e^x \sin y \ln z\,\hat{x} + e^x \cos y \ln z\,\hat{y} + e^x \sin y\left(\frac{1}{z}\right)\hat{z}$$

$$= e^x \sin y \ln z\,\hat{x} + e^x \cos y \ln z\,\hat{y} + \frac{e^x \sin y}{z}\,\hat{z}$$

1.12 The height of a certain hill (in feet) is given by $h(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$, where y is the distance (in miles) north, x the distance east of south Hadley.

(a) Where is the top of the hill located ?

$\Rightarrow$ Tentukan gradien,

$\nabla h = 10[(2y - 6x - 18)\,\hat{x} + (2x - 8y + 28)\,\hat{y}]$. $\nabla h = 0$

$2y - 6x - 18 = 0$

$2x - 8y + 28 = 0 \Rightarrow 6x - 24y + 84 = 0$

$\}$ $2y - 18 - 24y + 84 = 0$

$22y = 66 \Rightarrow y = 3$

$2x - 24 + 28 = 0 \Rightarrow x = -2$

Jadi, puncaknya 2 km sebelah barat dan 3 km utara Bandung

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(b) How high is the hill ?

→ Substitusikan $x = -2$ dan $y = 3$ ke persamaan .

$h = 10\,(2(3)(-2) - 3(-2)^2 - 4(3)^2 - 18(-2) + 28(3) + 12)$

$= 10\,(-12 - 12 - 36 + 36 + 84 + 12)$

$= 720$

Jadi, ketinggiannya adalah 720 ft

(c) How steep is the slope at a point 1 mile north and one mile east of south Hadley ? In what direction is the slope steepest, at the point?

→ Subs. $x = 1, y = 1$

$\nabla h = 10[(2-6-18)\hat{x} + (2-8+28)\hat{y}]$

$= 10[-22\hat{x} + 22\hat{y}]$

$= 220(-\hat{x} + \hat{y})$

$|\nabla H| = 220\sqrt{2} \approx 311$ ft/mile arahnya barat laut .

1.15 Calculate the divergence of the following vector functions :

(a) $v_a = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$

⇒

$$\nabla \cdot v_a = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(3xz^2) + \frac{\partial}{\partial z}(-2xz)$$

$$= 2x + 0 - 2x$$

$$= 0$$

(b) $v_b = xy\hat{x} + 2yz\hat{y} + 3zx\hat{z}$

⇒

$$\nabla \cdot v_b = \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}(2yz) + \frac{\partial}{\partial z}(3zx)$$

$$= y + 2z + 3x$$

(c) $v_c = y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}$

⇒

$$\nabla \cdot v_c = \frac{\partial}{\partial x}(y^2) + \frac{\partial}{\partial y}(2xy + z^2) + \frac{\partial}{\partial z}(2yz)$$

$$= 0 + 2x + 2y$$

$$= 2(x+y)$$

1.18 Calculate the curls of the vector functions in prob 1.15

(a)

$$\nabla \times v_a = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 3xz^2 & -2xz \end{vmatrix} = \hat{x}(0 - 6xz) + \hat{y}(0 + 2z) + \hat{z}(3z^2 - 0)$$

$$= \underline{\underline{-6xz\,\hat{x} + 2z\hat{y} + 3z^2\hat{z}}}$$

Date

(b)

$$\nabla \times v_b = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix} = \hat{x}(0-2y) + \hat{y}(0-3z) + \hat{z}(0-x)$$

$$= -2y\hat{x} - 3z\hat{y} - x\hat{z}$$

(c)

$$\nabla \times v_c = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & (2xy+z^2) & 2yz \end{vmatrix} = \hat{x}(2z-2z) + \hat{y}(0-0) + \hat{z}(2y-2y)$$

$$= 0$$