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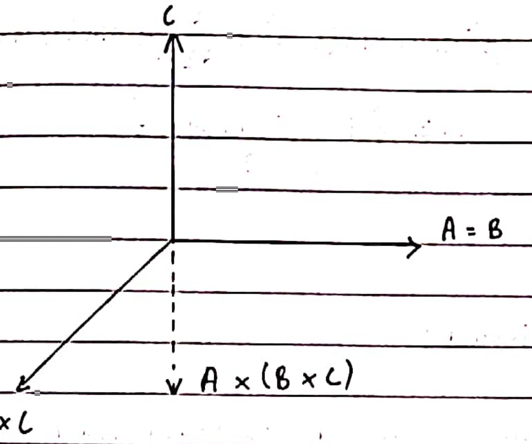
Ayo Berlatih

Problem 1. 2 (Halaman 4)
Is the cross product associative? $(A \times B) \times C = A \times (B \times C)$
If so, prove it; if not, provide a counterexample (the simpler the better).
Penyelesaian:
Perkalian cross product tidak bersifat asosiatif.



Hasil kali silang rangkap tiga umumnya tidak bersifat asosiatif. Misalnya, $A = B$ dan C tegak lurus terhadap A. Kemudian $(B \times C)$ menuju keluar, dan $A \times (B \times C)$ menuju ke bawah, dan memiliki besar ABC. $(A \times B) = 0$
Jadi, $(A \times B) \times C = 0$ $A \times (B \times C)$.

Contohnya : $A = (-1, 3, 4)$, $B = (3, -2, -8)$, $C = (4, 4, 4)$
Hasilnya adalah
$A \times (B \times C) = (140, 116, -52)$
$(A \times B) \times C : (-12, 36, 80)$

Problem 1.11 (Halaman 15)
Find the gradients of the following functions :
a) $f(x, y, z) = x^2 + y^3 + z^4$
Penyelesaian :
$$\nabla f(x, y, z) = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$
$$= \hat{x}\frac{\partial}{\partial x}(x^2 + y^3 + z^4) + \hat{y}\frac{\partial}{\partial y}(x^2 + y^3 + z^4) + \hat{z}\frac{\partial}{\partial z}(x^2 + y^3 + z^4)$$
$$= (2x)\hat{x} + (3y^2)\hat{y} + (4z^3)\hat{z} \;//$$

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b) $f(x,y,z) = x^2 y^3 z^4$

Penyelesaian :

$$\nabla f(x,y,z) = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$= \frac{\partial}{\partial x}(x^2y^3z^4)\hat{x} + \frac{\partial}{\partial y}(x^2y^3z^4)\hat{y} + \frac{\partial}{\partial z}(x^2y^3z^4)\hat{z}$$

$$= (2xy^3z^4)\hat{x} + (3x^2y^2z^4)\hat{y} + (4x^2y^3z^3)\hat{z} \quad //$$

c) $f(x,y,z) = e^x \sin(y) \ln(z)$

Penyelesaian :

$$\nabla f(x,y,z) = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$= \frac{\partial}{\partial x}(e^x \sin(y) \ln(z))\hat{x} + \frac{\partial}{\partial y}(e^x \sin(y) \ln(z))\hat{y} + \frac{\partial}{\partial z}(e^x \sin(y) \ln(z))\hat{z}$$

$$= (e^x \sin(y) \ln(z))\hat{x} + (e^x \cos(y) \ln(z))\hat{y} + \left(\frac{e^x \sin(y)}{z}\right)\hat{z} \quad //$$

Problem 1.12 (Halaman 15)

The height of a certain hill (in feet) is given by
$h(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$
where y is the distance (in miles) north, x the distance east of South Hadley

a) Where is the top of the hill located?

Penyelesaian :

$\nabla h = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$

$\nabla h = 0$ di puncak

$2y - 6x - 18 = 0$

$2x - 8y + 28 = 0$

$-6x + 2y - 18 = 0$

$6x - 24y + 84 = 0$

$-22y + 66 = 0$

$-22y = -66$

$y = 3$

$2y - 6x - 18 = 0$

$2(3) - 6x - 18 = 0$

$-6x = 12$

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$x = -2$

Sehingga letak puncak itu terletak di 3 mil utara dan 2 mil barat dari Hadley selatan //

b) How high is the hill?
Penyelesaian :
Substitusikan $x = -2$, $y = 3$
$h = 10(-12-12-36+36+84+12)$
$= 720$ kaki
Jadi, tinggi bukit tersebut adalah 720 kaki //

c) How steep is the slope (in feet per mile) at a point 1 mile north and 1 mile east of South Hadley? In what direction is the slope steepest, at that point?
Penyelesaian :
Substitusikan $x = 1$, $y = 1$

$$\nabla h = 10\left((2-6-18)\hat{x} + (2-8+28)\hat{y}\right)$$
$$= 10\left((-22)\hat{x} + 22\hat{y}\right)$$
$$= -220\hat{x} + 220\hat{y}$$
$$= 220(-\hat{x} + \hat{y})$$
$$|\nabla h| = 220\sqrt{2} \approx 311 \text{ kaki/mil}$$

Jadi, kemiringan pada titik 1 mil di utara dan 1 mil di timur South Hadley adalah $220\sqrt{2} \approx 311$ kaki/mil dengan arah kemiringan paling curam di arah barat laut //

Problem 1.13 (Halaman 15)
Let $\mathscr{r}$ be the separation vector from a fixed point (x', y', z') to the point (x, y, z), and let $\mathscr{r}$ be its length. Show that
a) $\nabla(\mathscr{r}^2) = 2\mathscr{r}$
Penyelesaian :
$$\mathscr{r} = (x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}$$
$$\mathscr{r} = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\nabla(\mathscr{r}^2) = \frac{\partial}{\partial x}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)\hat{x} + \frac{\partial}{\partial y}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)\hat{y} + \frac{\partial}{\partial z}\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)\hat{z}$$
$$= 2(x-x')\hat{x} + 2(y-y')\hat{y} + 2(z-z')\hat{z}$$
$$= 2\mathscr{r} \;//$$

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b) $\nabla\left(\frac{1}{\eta}\right) = -\frac{\hat{\eta}}{\eta^2}$

$$= \frac{\partial}{\partial x}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-\frac{1}{2}}\hat{x} + \frac{\partial}{\partial y}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-\frac{1}{2}}\hat{y}$$

$$+ \frac{\partial}{\partial z}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-\frac{1}{2}}\hat{z}$$

$$= \frac{1}{2}\left[\frac{\partial}{\partial x}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-\frac{1}{2}}\right]^{-\frac{3}{2}} 2(x-x')\hat{x} - \frac{1}{2}\left[\frac{\partial}{\partial y}\left((x-x')^2+\right.\right.$$

$$\left.\left.(y-y')^2+(z-z')^2\right)^{-\frac{1}{2}}\right]^{\frac{3}{2}} 2(y-y')\hat{y} - \frac{1}{2}\left[\frac{\partial}{\partial z}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-\frac{1}{2}}\right]^{-\frac{3}{2}}$$

$$2(z-z')\hat{z}$$

$$= -\left(\frac{\partial}{\partial}\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{-\frac{1}{2}}\right)^{-\frac{3}{2}}\left((x-x')\hat{x}+(y-y')\hat{y}+(z-z')\hat{z}\right)$$

$$= -\frac{1}{\eta^3}\eta$$

$$= -\frac{1}{\eta^2}\hat{\eta}$$

$$= -\frac{\hat{\eta}}{\eta^2}$$ //

c) What is the general formula for $\nabla(\eta^n)$

Penyelesaian:

$$\frac{\partial}{\partial x}(\eta^n) = n\eta^{n-1}\frac{\partial \eta}{\partial x}$$

$$= n\eta^{n-1}\left(\frac{1}{2}\frac{1}{\eta}2\eta x\right)$$

$$= n\eta^{n-1}\hat{\eta}_x$$

Jadi, $\nabla(\eta^n) = n\eta^{n-1}\hat{\eta}$ //

Problem 1.14 (Halaman 16)

Suppose that f is a function of two variables (y and z) only. Show that the gradien $\nabla f = (\partial f/\partial y)\hat{y} + (\partial f/\partial z)\hat{z}$ transforms as a vector under rotations, Eq. 129. [Hint : $(\partial f/\partial \bar{y}) = (\partial f/\partial y)(\partial y/\partial \bar{y}) + (\partial f/\partial z)(\partial z/\partial \bar{y})$, and the analogous formula for $\partial f/\partial \bar{z}$. We know that $\bar{y} = y\cos\theta + z\sin\theta$ and $\bar{z} = -y\sin\theta + z\cos\theta$; "solve" these quations for y and z (as functions of $\bar{y}$ and $\bar{z}$), and compute the needed derivatives $\partial y/\partial\bar{y}$, $\partial z/\partial\bar{y}$, etc]

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$\bar{y} = y\cos\theta + z\sin\theta$

$\bar{y}\sin\theta = y\sin\theta\cos\theta + z\sin^2\theta$

$\bar{z} = -y\sin\theta + z\cos\theta$

$\bar{z}\cos\theta = -y\sin\theta\cos\theta + z\cos^2\theta$

$\bar{y}\sin\theta + \bar{z}\cos\theta = z(\sin^2\theta + \cos^2\theta) = z$

$\bar{y}\cos\theta - \bar{z}\sin\theta = y$

Sehingga, $\frac{\partial y}{\partial y} = \cos\theta$; $\frac{\partial y}{\partial z} = -\sin\theta$; $\frac{\partial z}{\partial y} = \sin\theta$; $\frac{\partial z}{\partial z} = \cos\theta$

$$\overline{(\nabla f)y} = \frac{\partial f}{\partial y} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial y} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial y} = \cos\theta\,(\nabla f)y + \sin\theta\,(\nabla f)z$$

$$\overline{(\nabla f)z} = \frac{\partial f}{\partial z} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial z} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial z} = -\sin\theta\,(\nabla f)y + \cos\theta\,(\nabla f)z$$

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