No.
Date :

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Problem 1.11 (halaman 15)

a) $f(x,y,z) = x^2 + y^3 + z^4$
b) $f(x,y,z) = x^2 y^3 z^4$
c) $f(x,y,z) = e^x \sin(y) \ln(z)$

Solution

a) $f(x,y,z) = x^2 + y^3 + z^4$
$\vec{\nabla} f = 2x\hat{i} + 3y^2\hat{j} + 4z^3 k$

b) $f(x,y,z) = x^2 y^3 z^4$
$\vec{\nabla} f = 2xy^3 z^4 \hat{i} + 3x^2 y^2 z^4 \hat{j} + 4x^2 y^3 z^3 k$

c) $f(x,y,z) = e^x \sin(y) \ln(z)$
$\vec{\nabla} f = e^x \sin(y) \ln(z)\hat{i} + e^x \cos(y) \ln(z)\hat{j} + e^x \sin(y)(1/z) k$

Problem 1.12

- The height of a certain hill (in feet) is given by.
$h(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$
Where is distance (in miles) north x the distance east of south hadiy.

a) Where is the top of the hill located?
→ Menentukan fungsi gradien
$\vec{\nabla} h = 10(2y - 6x - 18)\hat{i} + (2x - 8y + 28)\hat{j}$

Untuk menentukan puncak bukit, menggunakan $\vec{\nabla} h = 0$
$\vec{\nabla} h = 10(2y - 6x - 18)\hat{i} + (2x - 8y + 28)\hat{j} = 0$
$= 2y - 6x - 18 = 0$
$= 2x - 8y + 28 = 0$ } Solusi dari sistem persamaan ini $(x,y) = (-2,3)$

b) How hight is the hill?
→ subsitusi $(x,y) = (-2,3)$ pada (x,y)
$h(-2,3) = 10(-12 - 12 - 36 + 36 + 84 + 12)$
$= 720$ m.
Jadi ketinggiannya adalah 720 m.

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c) How steep is the slope (in feet permile) at a point 1 mile with and one mile east of south Hadley? In what direction is the slope steepest, at that point?

$\Rightarrow$ subsitusikan $(x, y) = (1,1)$ pada $\vec{\nabla} b$.

$\vec{\nabla} b\ (1,1) = 10\,(2-6-18)\,i + (2-8+28)\,j = 220\,(-i+j)$

$|\vec{\nabla} b| = 220\sqrt{2} \approx 311$ m/km, arahnya kebarat laut (135° dan sumbu x positif)

Problem 1.13

Letibe the separation Vektor from a fixed point (x', y', z') to the point (x, y, z) and let be its length show that:

a) $\nabla(\imath) = 2\imath$

b) $\nabla(1/\imath) = -\imath/\imath^2$

c) What is the general formula for $\nabla(\imath^n)$?

Soleetion :

$(x,y,z)\ \imath = (x-x_0)i + (y-y_0)j + (z-z_0)k$

$\imath = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$

$(x_0, y_0, z_0)\ \imath = (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^0$

a) $$\vec{\nabla}\imath^2 = \frac{\partial}{\partial x}\left[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2\right] i + \frac{\partial}{\partial y}\left[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2\right] j + \frac{\partial}{\partial z}\left[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2\right] k$$

$$= 2(x-x_0)i + 2(y-y_0)j + 2(z-z_0)k$$

$$= 2\imath \text{ (terbukti)}$$

c) $$\frac{\partial}{\partial x}(\imath^n) = n\imath^{n-1}\frac{\partial \imath}{\partial x} = n\imath^{n-1}\left(1/2\,\imath^{-1/2}\,2x\right)$$

$$= n\imath^{n-1}\,\imath x, \quad (\imath = x - x_0)$$

$$\frac{\partial}{\partial y}(\imath^n) = n\imath^{n-1}\,\imath y, \quad \frac{\partial}{\partial z}(\imath^n) = n\imath^{n-1}\,\imath z, \text{ sehingga } \vec{\nabla}(\imath^n) = n\imath^{n-1}\,\hat{\imath}$$