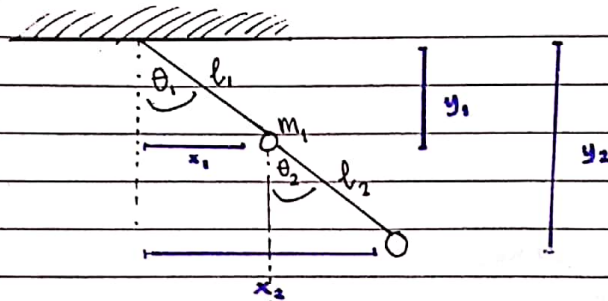


Kelompok 9

Sistem pendulum 2 massa menggunakan Mekanika Lagrangian



* Tinjau m_1

$$x_1 = l_1 \sin \theta_1$$

$$\dot{x}_1 = \frac{d}{dt} (l_1 \sin \theta_1) \frac{d\theta_1}{dt}$$

$$\dot{x}_1 = l_1 \frac{d\theta_1}{dt} \cdot \frac{d(\sin \theta_1)}{d\theta_1}$$

$$\dot{x}_1 = l_1 \dot{\theta}_1 \cos \theta_1$$

$$y_1 = -l_1 \cos \theta_1$$

$$\dot{y}_1 = \frac{d}{dt} (-l_1 \cos \theta_1) \frac{d\theta_1}{dt}$$

$$\dot{y}_1 = -l_1 \frac{d\theta_1}{dt} \cdot \frac{d(\cos \theta_1)}{d\theta_1}$$

$$\dot{y}_1 = -l_1 \dot{\theta}_1 (-\sin \theta_1)$$

$$\dot{y}_1 = l_1 \dot{\theta}_1 \sin \theta_1$$

* Tinjau m_2

$$x_2 = x_1 + l_2 \sin \theta_2$$

$$x_2 = l_1 \sin \theta_1 + l_2 \sin \theta_2$$

$$\dot{x}_2 = \frac{d}{dt} (l_1 \sin \theta_1 + l_2 \sin \theta_2)$$

$$\dot{x}_2 = l_1 \dot{\theta}_1 \cos \theta_1 + l_2 \dot{\theta}_2 \cos \theta_2$$

$$y_2 = y_1 - l_2 \cos \theta_2$$

$$y_2 = -l_1 \cos \theta_1 - l_2 \cos \theta_2$$

$$\dot{y}_2 = \frac{d}{dt} (-l_1 \cos \theta_1 - l_2 \cos \theta_2)$$

$$= l_1 \dot{\theta}_1 \sin \theta_1 + l_2 \dot{\theta}_2 \sin \theta_2$$

* Mencari Energi Potensial pada sistem

• persamaan energi potensial:

$$V = m_1 g y_1 + m_2 g y_2$$

$$V = m_1 g (-l_1 \cos \theta_1) + m_2 g (-l_1 \cos \theta_1 - l_2 \cos \theta_2)$$

$$V = -m_1 g l_1 \cos \theta_1 - m_2 g l_1 \cos \theta_1 - m_2 g l_2 \cos \theta_2$$

$$V = -(m_1 + m_2) g l_1 \cos \theta_1 - m_2 g l_2 \cos \theta_2$$

• persamaan energi kinetik

$$T = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$$

$$T = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2)$$

$$T = \frac{1}{2} m_1 [(l_1 \dot{\theta}_1 \cos \theta_1)^2 + (l_1 \dot{\theta}_1 \sin \theta_1)^2] + \frac{1}{2} m_2 [(l_1 \dot{\theta}_1 \cos \theta_1 + l_2 \dot{\theta}_2 \cos \theta_2)^2 + (l_1 \dot{\theta}_1 \sin \theta_1 + l_2 \dot{\theta}_2 \sin \theta_2)^2]$$

$$T = \frac{1}{2} m_1 [l_1^2 \dot{\theta}_1^2 \cos^2 \theta_1 + l_1^2 \dot{\theta}_1^2 \sin^2 \theta_1 + l_1^2 \dot{\theta}_1^2 \cos^2 \theta_1 + 2(l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos \theta_1 \cos \theta_2) +$$

$$l_2^2 \dot{\theta}_2^2 \cos^2 \theta_2 + l_2^2 \dot{\theta}_2^2 \sin^2 \theta_2 + 2(l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin \theta_1 \sin \theta_2) + l_2^2 \dot{\theta}_2^2 \sin^2 \theta_2]$$



$$T = \frac{1}{2} m_1 [l_1^2 \dot{\theta}_1^2 (\sin^2 \theta_1 + \cos^2 \theta_1)] + [l_1^2 \dot{\theta}_1^2 (\sin^2 \theta_1 + \cos^2 \theta_1) + l_2^2 \dot{\theta}_2^2 (\sin^2 \theta_2 + \cos^2 \theta_2) + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 (\cos \theta_1 \cos \theta_2 \sin \theta_1 \sin \theta_2)]$$

$$T = \frac{1}{2} m_1 [l_1^2 \dot{\theta}_1^2] + \frac{1}{2} m_2 [l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos (\theta_1 - \theta_2)]$$

* persamaan Lagrangian

$$L = T - V$$

$$= \frac{1}{2} m_1 (l_1^2 \dot{\theta}_1^2) + \frac{1}{2} m_2 (l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos (\theta_1 - \theta_2)) + (m_1 + m_2) g l_1 \cos \theta_1 - m_2 g l_2 \cos \theta_2$$