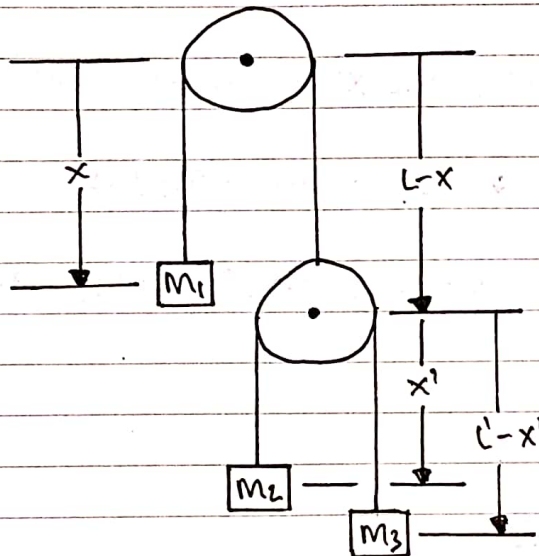


Kelompok 5

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Persamaan mekanika Lagrange dan Newton pada katrol ganda.



• Persamaan mekanika Lagrange

Diketahui sistem katrol ganda, dimana 1 katrol bergerak bebas. Sistem ini jelas memiliki 2 kebebasan. Kita akan menentukan dengan koordinat x dan x' . Dalam kasus ini diabaikan massa ke katrol sehingga kita dapat menentukan E_k dan E_p . Sebagai berikut.

$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 (\dot{x}' - \dot{x})^2 + \frac{1}{2} m_3 (\dot{x} + \dot{x}')^2$$

$$V = -m_1 g x - m_2 g (l - x + x') - m_3 g (l - x + l' - x')$$

$$L = T - V$$

$$= \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 (\dot{x}'^2 - 2\dot{x}\dot{x}' + \dot{x}^2) + \frac{1}{2} m_3 (\dot{x}^2 + 2\dot{x}\dot{x}' + \dot{x}'^2)$$

$$+ m_1 g x + m_2 g (l - x + x') + m_3 g (l - x + l' - x')$$

$$\frac{\partial L}{\partial x} = m_1 \ddot{x} + m_2 (-\dot{x}' + \ddot{x}) + m_3 (\ddot{x} + \dot{x}')$$

$$\frac{\partial L}{\partial x} = m_1 g - m_2 g - m_3 g$$

$$\left(\frac{\partial L}{\partial x} \right) = m_1 \ddot{x} + m_2 (-\dot{x}' + \ddot{x}) + m_3 (\ddot{x} + \dot{x}')$$

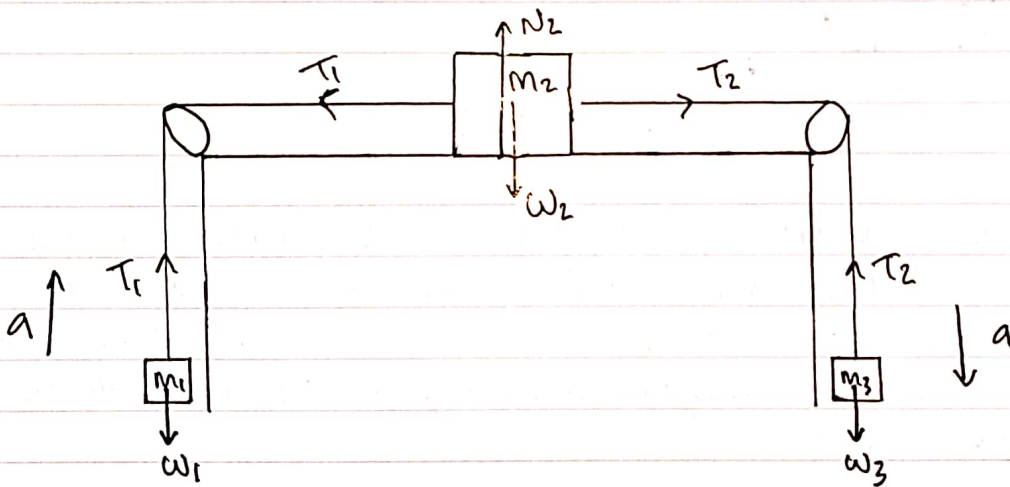
$$\frac{\partial L}{\partial x} = m_1 g - m_2 g - m_3 g$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = \frac{\partial L}{\partial x} \Rightarrow (m_1 + m_2 + m_3) \ddot{x} + (m_3 - m_2) \ddot{x}' - (m_1 - m_2 - m_3) g$$

$$\frac{\partial L}{\partial x} = m_2 \cdot \ddot{x}' - m_2 \ddot{x} + m_3 \ddot{x} + m_3 \ddot{x}' \Rightarrow \frac{\partial L}{\partial x} = (m_2 - m_3) g$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = \frac{\partial L}{\partial x} \Rightarrow m_2 (\ddot{x}' - \ddot{x}) + m_3 (\ddot{x} + \ddot{x}') = (m_2 - m_3) g$$

→ Persamaan mekanika Newton



$m_1 + m_2 < m_3$ (bidang licin)

• Resultan gaya balok 1

$$\sum F_y = m \cdot a$$

$$T_1 - w_1 = m_1 \cdot a$$

$$T_1 = m_1 (a + g) \dots (1)$$

- Resultan gaya balok 2

$$\sum F_x = m \cdot a$$

$$T_2 - T_1 = m_2 \cdot a \quad \dots (2)$$

- substitusi (1) ke (2)

$$T_2 - (m_1(a + g)) = m_2 \cdot a$$

$$T_2 = m_1 a + m_2 a + m_1 g \quad \dots (3)$$

- Resultan gaya balok 3

$$\sum F_y = m \cdot a$$

$$m_3 g - T_2 = m_3 \cdot a$$

$$m_3 g - T_2 = m_3 \cdot a \quad \dots (4)$$

- substitusi (3) ke (4)

$$m_3 g - (m_1 a + m_2 a + m_1 g) = m_3 a$$

$$m_1 a + m_2 a + m_3 a = m_3 g - m_1 g$$

$$(m_1 + m_2 + m_3) a = (m_3 - m_1) g$$

$$a = \frac{(m_3 - m_1) g}{(m_1 + m_2 + m_3)} \quad \dots (5)$$

- tegangan tali antar balok 1 dan balok 2, (5) ke (1)

$$T_1 = m_1 a + m_1 g$$

$$T_1 = m_1 \left[\frac{(m_3 - m_1) g}{(m_1 + m_2 + m_3)} \right] + m_1 g$$

$$T_1 = \left[\frac{(m_1 m_3 g - m_1^2 g)}{(m_1 + m_2 + m_3)} \right] + m_1 g$$

$$T_1 = \frac{(m_1 m_3 g - m_1^2 g + m_1^2 g + m_1 m_2 g + m_1 m_3 g)}{(m_1 + m_2 + m_3)}$$

$$T_1 = \frac{m_1 m_2 g + 2 m_1 m_3 g}{(m_1 + m_2 + m_3)} \quad \dots (6)$$

- Tegangan tali antar balok 2 dan balok 3, substitusikan (1) ke (3)

$$T_2 = m_1 a + m_2 a + m_1 g$$

$$T_2 = (m_1 + m_2) a + m_1 g$$

$$T_2 = \left[\frac{(m_1 + m_2)(m_3 - m_1) g}{(m_1 + m_2 + m_3)} \right] + m_1 g$$

$$T_2 = \left[\frac{(M_1 M_3 g - M_1^2 g + M_2 M_3 g - M_1 M_2 g + M_1^2 g + M_1 M_2 g + M_1 M_3 g)}{(M_1 + M_2 + M_3)} \right]$$

$$T_2 = \frac{(M_2 M_3 g + 2 M_1 M_3 g)}{(M_1 + M_2 + M_3)}$$

$$T_2 = \frac{(M_2 M_3 + 2 M_1 M_3 g)}{(M_1 + M_2 + M_3)}$$