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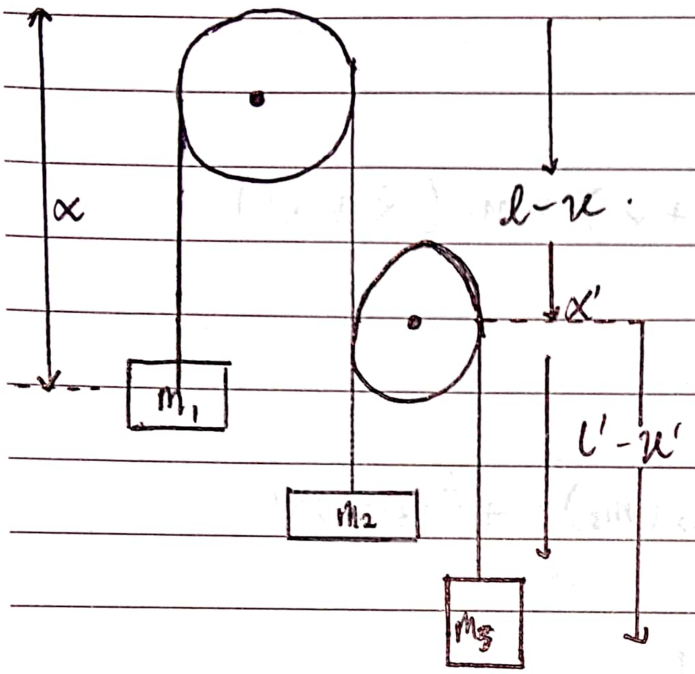
# Katrol Ganda.




•) Persamaan mekanika lagrangan

Diketahui sistem katrol ganda, dimana satu katrol bergerak bebas. Sistem ini jelas memiliki dua derajat kebebasan. Kita akan menentukan dengan koordinat $x$ dan $x'$. Dalam kasus ini, diabaikan massa dari katrol sehingga dapat menentukan Ek dan Ep sbb:

$$T = \frac{1}{2}m_1 x^2 + \frac{1}{2}m_2 (x'-x)^2 + \frac{1}{2}m_3 (x+x')^2.$$

$$V = -m_1 g x - m_2 g (l - x + x') - m_3 g (l - x + l' - x')$$

$$L = T - V = \frac{1}{2}m_1 x^2 + \frac{1}{2}m_2 (x^2 - 2xx' + x'^2) + \frac{1}{2}m_3 (x^2 + 2xx' + x'^2) +$$

$$m_1 g x + m_2 g (l - x + x') + m_3 g (l - x + l' - x').$$

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$$\frac{\partial L}{\partial x} = m_1\dot{x} + m_2(-\dot{x}' + \dot{x}) + m_3(x + x')$$

$$\frac{\partial L}{\partial x} = m_1 g - m_2 g - m_3 g$$

$$\left[\frac{\partial L}{\partial x}\right] = m_1\ddot{x} + m_2(-\ddot{x}' + \ddot{x}) + m_3(\ddot{x} + \ddot{x}')$$

$$\frac{\partial L}{\partial x} = m_1 g - m_2 g - m_3 g.$$

- $$\frac{d}{dt}\left[\frac{\partial L}{\partial x}\right] = \frac{\partial L}{\partial x} \Rightarrow (m_1 + m_2 + m_3)x + (m_3 - m_2)\ddot{x}'$$

$$= (m_1 - m_2 - m_3)g.$$

$$\frac{\partial L}{\partial x'} = m_2.\dot{x}' - m_2\dot{x} + m_3\dot{x} + m_3 x' \Rightarrow \frac{\partial L}{\partial x'}$$

$$= (m_2 - m_3)g.$$

- $$\frac{d}{dt}\left(\frac{\partial L}{\partial x}\right) = \frac{\partial L}{\partial x} \Rightarrow m_2(\ddot{x}' - \ddot{x}) + m_3(\ddot{x} + \ddot{x}')$$

$$= (m_2 - m_3)g.$$

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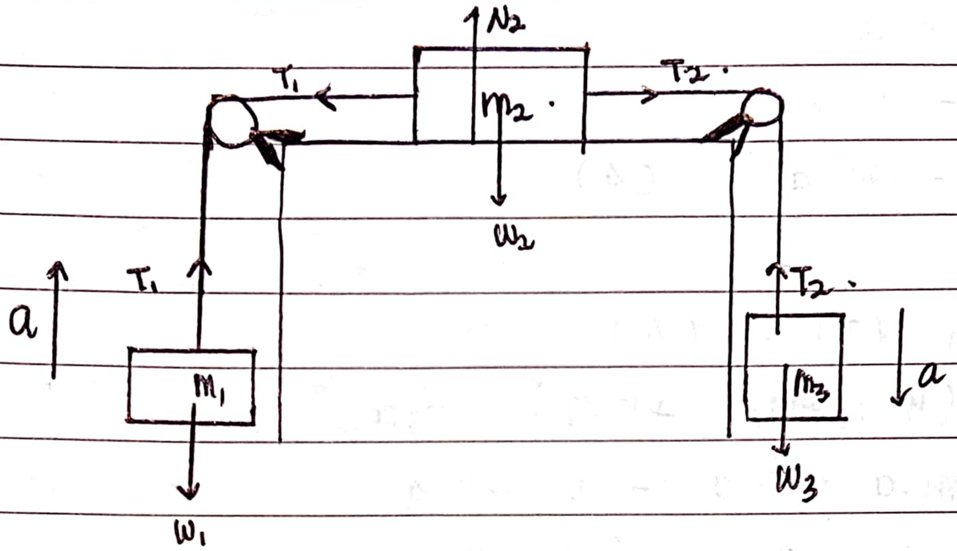
•) Persamaan Mekanika Newtonian.




→ $m_1 + m_2 < m_3$ ( bidang licin )

•) Resultan gaya balok 1

$$\Sigma F_y = m a$$

$$T_1 - W_1 = m_1 a$$

$$T_1 = m_1 (a + g) \quad \ldots (1)$$

•) Resultan gaya balok 2

$$\Sigma F_x = m \cdot a$$

$$T_2 - T_1 = m_2 \cdot a \quad (\ldots 2).$$

•) Subsitutis (1)---(2)

$$T_2 - (m_1 (a + g)) = m_2 . a$$

$$T_2 = m_1 a + m_2 a + m_1 g \quad \ldots (3)$$

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•) Resultan gaya balok 3

$$\Sigma F_y = m \cdot a$$

$$W_3 - T_2 = m_3 a$$

$$m_3 g - T_2 = m_3 a \quad \ldots (4)$$

→ Substitusi (3) ... (4)

$$m_3 g - (m_1 a + m_2 a + m_1 g) = m_3 a$$

$$m_1 a + m_2 a + m_3 a = m_3 g - m_1 g$$

$$(m_1 + m_2 + m_3) a = (m_3 - m_1) g$$

$$a = \frac{(m_3 - m_1) g}{(m_1 + m_2 + m_3)}$$

•) Tegangan tali antar balok 1 dan balok 2

(5) → (1)

$$T_1 = m_1 a + m_1 g$$

$$= m_1 \left[ \frac{(m_3 - m_1) g}{(m_1 + m_2 + m_3)} \right] + m_1 g$$

$$= \left[ \frac{(m_1 m_3 g - m_1^2 g)}{(m_1 + m_2 + m_3)} \right] + m_1 g$$

$$= \frac{(m_1 m_3 g - m_1^2 g + m_1^2 g + m_1 m_2 g + m_1 m_3 g)}{(m_1 + m_2 + m_3)}$$

$$T_1 = \frac{m_1 m_2 g + 2 m_1 m_3 g}{(m_1 + m_2 + m_3)} \quad \ldots (6)$$

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•) Tegangan tali antar balok 2 dan balok 3

pers (5) → (3)

$$T_2 = m_1 a + m_2 a + m_1 g = (m_1 + m_2) a + m_1 g.$$

$$= \left[ \frac{(m_1 + m_2)(m_3 - m_1) g}{(m_1 + m_2 + m_3)} \right] + m_1 g.$$

$$= \left[ \frac{(m_1 m_3 g - m_1^2 g + m_2 m_3 g - m_1 m_2 g + m_1^2 g + m_1 m_2 g + m_1 m_3 g)}{(m_1 + m_2 + m_3)} \right.$$

$$T_2 = \frac{(m_2 m_3 g + 2 m_1 m_3 g)}{(m_1 + m_2 + m_3)}$$

$$T_2 = \frac{(m_2 m_3 + 2 m_1 m_3 g)}{(m_1 + m_2 + m_3)}$$