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Kelas : B

21.1). a). $U = Cxyz + C$

$$\vec{F} = -\vec{\nabla} U = -\hat{i}\frac{\partial U}{\partial x} - \hat{j}\frac{\partial U}{\partial y} - \hat{k}\frac{\partial U}{\partial z} = -C(\hat{i}yz + \hat{j}xz + \hat{k}xy)$$

b). $U = \alpha x^2 + \beta y^2 + \gamma z^2 + C$

$$\vec{F} = -\vec{\nabla} U = -\hat{i}2\alpha + \hat{j}2\beta y + \hat{k}2 \cdot \gamma z$$

c). $U = Ce^{-(\alpha x + \beta y + \gamma z)}$

$$\vec{F} = -\vec{\nabla} U = Ce^{-(\alpha x + \beta y + \gamma z)}(\hat{i}\alpha + \hat{j}\beta + \hat{k}\gamma)$$

d). $U = cr^n$ dalam koordinat bola

$$\vec{F} = -\vec{\nabla} U = -\hat{e_r}\frac{\partial U}{\partial x} - \hat{e} \cdot \frac{1}{r}\frac{\partial U}{\partial \theta} - \hat{e}_\phi \frac{1}{r\sin}\frac{\partial U}{\partial \theta}$$

4.2). a). $F = \hat{i}x + \hat{j}y + \hat{k}z$

$$\rightarrow \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0 \rightarrow \text{konservatif}$$

b). $F = \hat{i}x - \hat{j}y + \hat{k}z^2$

$$\rightarrow \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -y & z^2 \end{vmatrix} = \hat{k}(-1-1) \neq 0 \rightarrow \text{tdk konservatif}$$

c). $\vec{F} = \hat{i}x + \hat{j}y + \hat{k}z^2$

$$\rightarrow \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z^2 \end{vmatrix} = \hat{k}(1-1) = 0 \rightarrow \text{konservatif}$$

d). $F = -kr^{-n} e_r$ dalam koordinat bola

$$\rightarrow \vec{\nabla} \times \vec{F} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{e_r} & \hat{e}_\theta r & \hat{e}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ -kr & 0 & 0 \end{vmatrix} = 0 \rightarrow \text{konservatif}$$

4.2). a). E = konstan $= U(x,y,z) + \frac{1}{2}mv^2$

diasalnya ; $E = 0 + \frac{1}{2}mv^2$

Pada $(1,1,1)$ $E = \alpha + \beta + \gamma + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

b). $v_0 = 2/m(\alpha - \beta + \gamma) = 0$

$v_0 = [2/m(\alpha + \beta + \gamma)]^{1/2}$

c). $m\ddot{x} = F_x = -\partial V/\partial x$

$m\ddot{x} = -\alpha$

$m\ddot{y} = -\partial V/\partial y = -2\beta y$

$m\ddot{z} = -\partial V/\partial z = 3yz^2$

4.5). a). $F = \hat{i}x + \hat{j}y$

di $x = y$ i $\quad \vec{dr} = \hat{i}\,dx + \hat{j}\,dy$

$\int_{(0,0)}^{(1,1)} \vec{F}\cdot\vec{dr} = \int_0^1 F_x\,dx + \int_0^1 F_y\,dy = \int_0^1 x\,dx + \int_0^1 y\,dy = -1$

Pada lintasan sepanjang sb x $= \vec{dr} = \hat{i}\,dx$

$\vec{dr} = \hat{i}\,dy$

dan pada garis $x=1$

$\int_{(0,0)}^{(1,1)} \vec{F}\cdot\vec{dr} - \int_0^1 F_x\,dx + \int_0^1 F_y\,dy - 1$, $\vec{F}$ = konservatif

b). $\vec{F} = \hat{i}y - \hat{j}x$

di $x = y$ - $\int_{(0,0)}^{(1,1)} \vec{F}\cdot\vec{dr} = \int_0^1 F_x\,dx + \int_0^1 F_y\,dy = \int_0^1 y\,dx - \int_0^1 x\,dy$

dan dg $x = y$ $\int_{(0,0)}^{(1,1)} \vec{F}\cdot\vec{dr} = \int_0^1 x\,dx - \int_0^1 y\,dy = 0$

Pada sumbu $x = \int_{(0,0)}^{(1,1)} \vec{F}\cdot\vec{dr} = \int_0^1 x\,dx = \int_0^1 y\,dx$

dan dg $y=0$ pd sb x $\int_{(0,0)}^{(1,0)} \vec{F}\cdot\vec{dr} = 0$

pada garis $x=1$ $\int_{(1,0)}^{(1,1)} \vec{F}\cdot\vec{dr} \cdot \int_0^1 F_y\,dy \cdot \int_0^1 x\,dy$

dan dg $x=1$ $\int_{(1,0)}^{(1,1)} \vec{F}\cdot\vec{dr} - \int_0^1 dy = 1$

Pada jalur ini $\int_{(0,0)}^{(1,1)} \vec{F}\cdot\vec{dr} = 0 + 1 = 1$, $\vec{F}$ = tidak konservatif

4.9). Dari contoh 2.3.2 $U(z) = -mg\,\dfrac{r_e^2}{(r_e+z)}$

$V(z) = -mg\,r_e\,(1 + z/r_e)^{-1}$

$V(z) = -mg\,r_e\,(1 - z/r_e + z^2/r_e + \dots)$

$V(z) = -mg\,r_e + mgz - mg^2/r_e + \dots$

dg $-mg\,r_e$ sebuah konstan

$V(z) \approx mgz\,(1 - z/r_e)$

$\vec{F} = -\vec{\nabla}V = -\hat{k}\,\partial/\partial z\, V(z)$

$= -\hat{k}mg\,[1 - \cancel{z}/r_e + z(-1/r_e)]$

$\vec{F} = -\hat{k}mg\,(1 - 2z/r_e)$

$m\ddot{x}\ \partial z/\partial z = -mg\,(1 - 2z/r_e)$

$\int_{v_z}^{0} \dot{z}\,dz = -g \cdot \int_0^h (1 - 2z/r_e)\,dz$

$-\tfrac{1}{2}v_0^2 z = -g\,(h - h^2/r_e)$

$h^2 - r_e h + \dfrac{r_e v_{0z}^2}{2g} = 0$

$h = r_e h + \dfrac{r_e v_{0z}^2}{2g} = 0$

$h - r_e h + \dfrac{r_e v_{0z}^2}{2g} = 0$

$h = r_e/2 - \tfrac{1}{2}\sqrt{r_e^2 - \dfrac{2r_e v_{0z}^2}{g r_e}}$ $(h, z << r_e)$

$h = r_e/2 - r_e/2\,\sqrt{1 - 2v_{0z}^2/g r_e}$

$(1+x)^{1/2} \approx 1 + x/2 - x^2/2 + \dots$

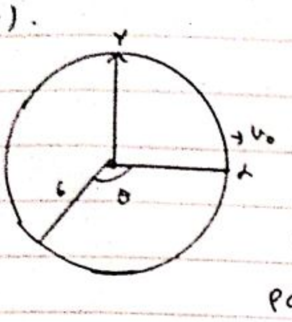
$h = r\epsilon/2 - r\epsilon/2 - v^2 o z/2g - v^4 o.z/4gr\epsilon + \dots$

$h \approx v^2 oz/2g \; (1 + v^2 oz/2gr\epsilon)$

dari contoh 2.3.2 $h = v_0^2/2g \; (1 - v_0^2/2gr\epsilon)^{-1}$

dan dg $(1-x)^{-1} \approx 1 + x$, $h \approx v_0^2/2g \; (1 + v_0^2/2gr\epsilon)$

1.7).



$\vec{r} = \hat{i} b \cos\theta - \hat{j}\, b \sin\theta$

$\theta = \omega t = v_0 t/b$, jadi $\vec{r} = -\hat{i}\, v_0 \sin\theta$
$- \hat{j}\, v_0 \cos\theta$

relatif ke tanah, $\vec{v} = \hat{i}\, v_0 (1 - \sin\theta) - \hat{j}\, v_0 \cos\theta$

Partikel keluar tepi : $y_0 = -b\sin\theta$ dan $v_{0y} = -v_0 \cos\theta$

jadi, $v_y = v_{0y} - gt = -v_0 \cos\theta - gt$ dan $y = -b \sin\theta - v_0 t \cos\theta - 1/2\, gt^2$

tinggi maks $v_y = 0$

$t = \dfrac{-v_0 \cos\theta}{}$

$h = -b \sin\theta - v_0 \left(\dfrac{-v_0 \cos\theta}{g}\right) \cos\theta - 1/2\, g \left(\dfrac{-v_0 \cos\theta}{g}\right)^2$

$h = -h \sin\theta - \dfrac{v_0^2 \cos^2\theta}{2g}$

h maks terjadi untuk $dh/d\theta = 0 = -b\cos\theta - \dfrac{2v_0^2 \cos\theta \sin\theta}{2g}$

$\sin\theta = \dfrac{-gb}{v_0^2}$

$\cos^2\theta = 1 - \sin\theta = \dfrac{v_0^4 - g^2 b^2}{v_0^4}$

$h_{max} = gb^2/v_0^2 + v_0^4 g^2 b^2/2g v_0^2 = gb^2/2v_0^2 + v_0^2/2g$

lumpur meninggalkan roda di $\theta = \sin^{-1}(-gb/v_0^2)$

1.8). $x = R\cos\phi$ dan $x = v_{0x} t \cdot (v_0 \cos\alpha) t$

jadi $t = \dfrac{R\cos\phi}{v \cos\alpha}$

$y = R\sin\phi$ dan $y = v_y t = -1/2\, gt^2 = (v_0 \sin\alpha) t - 1/2\, gt^2$

$z\sin\theta = (v_0 \sin\alpha) \dfrac{R\cos\theta}{v_0 \cos\alpha} - 1/2\, g \left(\dfrac{R\cos\phi}{v_0 \cos\alpha}\right)^2$

$\sin\phi = \tan\alpha \cos\theta - \dfrac{gR \cos^2\phi}{2v_0^2 \cos^2\alpha}$

$R = \dfrac{2v_0^2 \cos^2\alpha}{g\cos\phi} (\tan\alpha \cos\alpha - \sin\phi) = \dfrac{2v_0^2 \cos\alpha}{g\cos^2\phi} \left(\begin{matrix}\sin\alpha\cos\phi \\ -\cos\alpha \sin\phi\end{matrix}\right)$

$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$$R = \frac{2v_0^2\cos\alpha\,\sin(\alpha-\theta)}{g\cos^2\phi}$$

R_{maks} untuk $\frac{dR}{dx} = 0 = \frac{2v_0^2}{g\cos^2\phi}\left[-\sin\alpha\sin(\alpha-\phi) + \cos\alpha\cos(\alpha-\phi)\right]$

$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$ jadi $\cos(2x-\phi) = 0$

$2\alpha - \phi = \pi/2$

$$R_{maks} = \frac{2v\alpha}{2\cos\theta}\cos\left(\pi/4 + \frac{\phi}{2}\right)\sin\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$

$$\sin\left(\frac{\pi}{2} - \frac{\phi}{2}\right) = \cos\left[\frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{\phi}{2}\right)\right] = \cos\left(\frac{\pi}{2} + \frac{\phi}{2}\right)$$

$$R_{max} = \frac{2v_0^2}{g\cos^2\phi}\cos^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

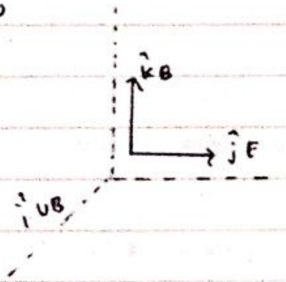
$$R_{max} = \frac{2v_0^2}{g\cos^2\phi}\left[\tfrac{1}{2}\cos(\pi/2+\phi) + \tfrac{1}{2}\right] \frac{v_0^2}{g\cos^2\phi}\left[\cos\left(\frac{\pi}{2}+\phi\right) + 1\right]$$

menggunakan $\cos(\pi/2 + \phi) = -\sin\theta$

$$R_{max} = \frac{v_0^2}{g(1-\sin\theta)}(1-\sin\theta)$$

$$= \frac{v_0^2}{g(1+\sin\theta)}$$

4.20



$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

$\vec{v}\times\vec{B} = (\hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z}) \times kB = \hat{i}\dot{y}B - \hat{j}\dot{x}B$

$\vec{F} = \hat{i}\,q\dot{y}B + \hat{j}\,q(E - \dot{x}B)$

$m\ddot{x} = F_x = q\dot{y}B$

$\dot{x} - \dot{x}_0 = \frac{qB}{m}y$

$m\ddot{y} = F_y = qE - q\dot{x}B = qE - qB\left(\dot{x}_0 + \frac{qB}{m}y\right)$

$$\ddot{y} = \frac{qE}{m} - \frac{qB\dot{x}}{m} - \left(\frac{qB}{m}\right)^2 y = \frac{eE}{m} + \frac{eBx}{m} - \left(\frac{eB}{m}\right)^2 y$$

$$\ddot{y} + \omega^2 y = -\frac{eE}{m} + \omega\dot{x}_0$$

$$y = \frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}\right) + A\cos(\omega t + \theta_0)$$

$\dot{y} = -A\omega\sin(\omega t + \theta)$

$y_0 = 0$, jadi $\theta_0 = 0$, $A = -\frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}_0\right)$

$y = a(1-\cos\omega t),\ a = \frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}_0\right)$

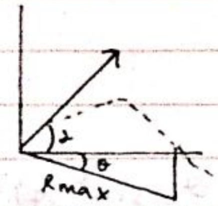
$\dot{x} = \dot{x}_0 = \frac{aB}{m}\,y = x - \omega y = \dot{x}_0 - \omega a(1-\cos\omega t)$

$x = (\dot{x}_0 - \omega a)t + a\sin\omega t$

$x = a\sin\omega t + bt \quad b = \dot{x}_0 - \omega a$

$m\ddot{z} = F_z = 0 \quad z = \dot{z}_0 t + z_0 = 0$

4.9). a).



$R = \frac{2V_0^2}{g}\frac{\cos\alpha\sin(\alpha+\phi)}{\cos^2\phi}$

$-b = \dots R\,max = V_0^2(1+\sin\phi)$ dan $\dots 2\alpha = \pi/2 - \phi$

$R_{max} = \frac{h}{\sin\gamma} = \frac{V_0^2}{g}\frac{(1+\sin 6)}{\cos^2\gamma} = \frac{V_0^2}{g(\sin\gamma)}$

$\sin 6 = \frac{gh}{V_0^2}\Big/\left(1+\frac{gh}{V_0^2}\right)$

$\sin\gamma = \sin(\pi/2 - 2\alpha) = \cos 2\alpha = 1 - 2\sin^2\alpha$

$1 - 2\sin^2\alpha = \frac{gh}{V_0^2}\Big/\left(1+\frac{gh}{V_0^2}\right)$

$2\sin^2\alpha = 2/cx^2\alpha = 1 - gh/V_0^2 \Big/ \left(1+\frac{gh}{V_0^2}\right) = \frac{1}{1+gh/V_0^2}$

akhirnya $= cs^2\alpha = 2(1 + gh/V_0^2)$

b). $R_{max} = \frac{h}{\sin 6} = \frac{h}{1-2\sin^2\alpha} = \frac{h}{1-2/(sc^2\alpha)}$

$R_{max} = \frac{V_0^2}{g}\left(1+\frac{gh}{V_0^2}\right)$

4.12). a). x dan z adalah posisi bola

$x = V_0 t\cos\frac{1}{2}\theta_0,\ z = V_0 t\cos\frac{1}{2}\theta_0\sin\theta_0 - \frac{1}{2}gt^2$

Sejak $V_x = V_0\cos\frac{1}{2}\theta_3$

$R = V_0^2/g\ \cos^2\frac{1}{2}\theta^2{}_0\sin 2\theta_0$

$max = \frac{\partial R}{\partial\theta} = 0$

$\frac{\partial R}{\partial\theta} = \frac{V_0^2}{g}(2\cos^2\frac{1}{2}\theta_0\cos 2\theta_0 - \cos\frac{1}{2}\theta_0\sin\frac{1}{2}\theta\sin 2\theta_0) = 0$

$2\cos^2\frac{1}{2}\theta_0\cos^2\theta_0 - \cos\frac{1}{2}\theta_0\sin\frac{1}{2}\theta_0\sin_2\theta_0$

$2\cos^2\theta_0 = 1 + \cos 2\theta_0$ dan $\sin 2\theta = 2\sin\theta_0\sin 2\theta_0$

didapat $1(+ + \cos\theta_0)(2\cos^2\theta_0 - 1) = \sin\theta_0\sin\theta_0\cos\theta_0 = (1-\cos^2\theta\cos\theta)$

atau $(1+\cos\theta_0)(3\cos^2\theta_0 - \cos\theta_0 - 1) = 0$

hanya akar positif yang berlaku adalah $0, 0 \leq 0 \leq \pi/2$

$\cos \theta_0 = 1/6\ (1 + \sqrt{13}) = 0{,}7676 \quad \theta_0 = 39^\circ\ 51$

b). Untuk $V_0 = 25\ m/s$ k max rr, 9m $\quad \theta_0 = 39{,}51$, max $\frac{dz}{dt} = 0$

$V_0 \cos 1/2\ \theta_0 \sin \theta_0 - gt$ atau $T = \frac{V_0 \cos 1/2\ \theta_0 \sin \theta_0}{g}$

atau $H = \frac{V_0^2}{2g} \cos^2 1/2 . \theta_0 \sin \theta_0$ x tetap di θ mak pada ketinggian

tetap terjadi $\frac{dH}{dx} = 0$

$\frac{dH}{dx} = \frac{V_0^2}{2g} (2\cos^2 1/2\ \theta = \sin \theta_0 \cos \theta_0 - \cos 1/2\ \theta_0 \sin 1/2\ \theta_0 \sin^2 \theta) = 0$

$(1 + \cos \theta_0) \sin \theta_0 \cos \theta_0 = \frac{1/2 \sin \theta_0 \sin^2 \theta_0 \quad 1/2 \sin \theta_0 (1 - \cos^2 \theta_0)}{\sin \theta_0 (1 + \cos \theta_0)(3 \cos \theta_0 - 1)} = 0$

$\sin \theta = 0$, $\cos \theta_0 = -1$

Jadi. $H_{max} = 18{,}9m \quad \theta_0 = \cos^{-1} 1/2 = 70{,}32^\circ$

4.16). r mengantikan x

$z = \frac{\dot{z}_0}{r_0} r - \frac{gr^2}{2\dot{r}\theta^2}$ dimana $r_0 = V_0 \cos \theta_0$ $\dot{z}_0 = V_0 \sin \theta_0$

$z = r \tan \theta_0 - \frac{gr^2}{2V_0^2} \sec^2 \theta$ sejam $\sec^2 \theta_0 = 1 + \tan^2 \theta$

$\frac{gr^2}{2V_0} \tan^2 \theta_0 - r \tan \theta_0 + z + \frac{gr^2}{2V_0^2} = 0$

(r, z) titik koordinat

$\tan \theta_0 = 1/gr\ [V_0^4 \pm (V_0^4 - gz V_0^2 - g^2 r^2)^{1/3}]$

$V_0^4 - 2gz V_0^2 - g^2 r^2 \geq 0$

Permukaan kritis oleh sebab itu $V_0^4 - 2gz - gt^2 = 0$

4.17). $m\ddot{x} = F_x = -\frac{\partial V}{\partial x} = -kx = -\pi^2 mx$

$\dot{x} = A = \cos \left(\sqrt{\frac{k}{m}}\, t + \alpha\right) \pm A \cos (\pi t + \alpha)$

$m\ddot{y} = -\frac{\partial V}{\partial y} = 4\pi^2 mU$

$y = B \cos (2\pi t + \beta)$

$m\ddot{z} = -\frac{\partial V}{\partial z} = -9\pi^2 mz$

$z = \cos(3\pi t - \gamma)$

Setelah $x = y = z = 0$ pada $t = 0$ dan $\beta = \gamma = \pi/2$

$x = A\cos(\pi t - \pi/2) = A\sin\pi t$

$\dot{x}_0 = A\pi\cos\pi t$

$v_0^2 = \dot{x}_0 + \dot{y}_0^2 + \dot{z}_0^2$ dan $\dot{x}_0 = \dot{y}_0 = \dot{z}_0$

$\dot{x}_0 = \frac{v_0}{\sqrt{3}} = A\pi$, $A = \frac{v_0}{\pi\sqrt{3}}$, $x = \frac{v_0}{\pi\sqrt{3}}\sin\pi t$

$y = B\sin 2\pi t$, $\dot{y} = 2B\pi\cos 2\pi t$, $\dot{z}_0 = v_0/\sqrt{3} = 3\pi t$; $\dot{z} = 3c\pi\cos 3\pi t$

$\dot{y}_0 = \frac{v_0}{\sqrt{3}} = 2\pi$, $y = \frac{v_0}{2\pi\sqrt{3}}\sin 2\pi t$, $\dot{z}_0 = v_0/\sqrt{3} = 3C\pi$

$B = \frac{v_0}{2\pi\sqrt{3}}$, $z = C\sin 3\pi t$, $C = v_0/3\pi\sqrt{3}$, $z = v_0/3\pi\sqrt{3}\,\sin 3\pi t$

Maka $t_{min} = \frac{2\pi n_1}{\omega_x} = \frac{2\pi n_2}{\omega_y}$; $\frac{2\pi n_3}{\omega_z}$

$t_{min} = \frac{2\pi}{2} = 2$