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Kelas B

Tugas Mekanika

4.1 Temukan gaya untuk masing" fungsi energi potensial brkt.

a. $V = cxyz + C$

$= \vec{F} = -\vec{\nabla} V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$

$= -C(iyz + \hat{j}xz + \hat{k}xy)$

b. $V = \alpha x^2 + By^2 + \gamma z^2 + C$

$= \vec{F} = -\vec{\nabla} V = -\hat{i} 2\alpha x + \hat{j} 2By + \hat{k} 2\gamma z$

c. $V = Ce^{-(x\alpha + By + \gamma z)}$

$\vec{F} = -\vec{\nabla} V = Ce^{-(\alpha x + By + \gamma z)}(i\alpha + \hat{j}B + \hat{k}\gamma)$

d. $V = Cr^n$

$\vec{F} = -\vec{\nabla} V = -\hat{e}r\frac{\partial V}{\partial r} - \hat{e}\theta\frac{1}{r}\frac{\partial V}{\partial \theta} - \hat{e}\phi\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}$

4.2 Dengan menemukan curl, tentukan mana dan gaya"

a. $F = ix + jy + kz$

$\Rightarrow \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix} = 0$ (konservatif)

b. $F = ix - jy + kz^2$

$\Rightarrow \vec{\nabla} \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & -y & z^2 \end{vmatrix} = \hat{k}(-1-1) \neq 0$ (tidak konservatif)

c. $F = ix + jy + kz^2$

$\Rightarrow \vec{\nabla} \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z^3 \end{vmatrix} = \hat{k}(1-1) = 0$ (konservatif)

d. $F = -kr^{-n}\,er$

$\Rightarrow \vec{\nabla} \times \vec{F} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{e}r & \hat{e}\theta r & \hat{e}\phi r\sin\theta \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ -kr^{-n} & 0 & 0 \end{vmatrix} = 0$ (konservatif)

4.3 Tentukan nilai konstan C sehingga masing" gaya brkt.

a. $F = ixy + jcx^2 + kz^3$

$= \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy & cx^2 & z^3 \end{vmatrix} = k(2cx - x)$

$2cx - x = 0$

$c = \frac{1}{2}$

b. $F = i\,(z/y) + cj\,(xz/y^2) + k\,(x/y)$

$\Rightarrow \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z/y & cxz/y^2 & x/y \end{vmatrix} = \hat{i}\left(-x/y - cx/y^2\right) + \hat{j}\left(1/y - 1/y\right) + k\left(cz/y^2 + z/y^2\right)$

$= \dfrac{x}{y^2} - \dfrac{cx}{y^2} = 0$, $c = -1$

juga $\dfrac{cz}{y^2} + \dfrac{z}{y^2} = 0$, $c = -1$

4. Diket = fungsi $Ep = V(x,y,z) = \alpha x + \beta y^2 + \gamma z^2$

kecepatan V_0 melewati titik $(1,1,1)$

a. E = konstan $= V(x,y,z) + \frac{1}{2}mv^2$

titik asal = $E = 0 + \frac{1}{2}mv^2$

pada $(1,1,1)$ = $E = \alpha + \beta + \gamma + \frac{1}{2}mv^2$

$= \frac{1}{2}mv^2$

b. Jika titik $(1,1,1)$ adalah titik balik gerak $v = 0$ berapa V_0?

$V_0 = -2/m\,(\alpha + \beta + \gamma) = 0$

$= \left[\dfrac{2}{m}(\alpha + \beta + \gamma)\right]^{1/2}$

c. Apa pers. diferensial komponen gerak partikel?

$= m\ddot{x} = Fx = -\dfrac{\partial V}{\partial x}$

$m\ddot{x} = -\alpha$

$m\ddot{y} = -\dfrac{\partial V}{\partial y} = -2\beta y$

$m\ddot{z} = -\dfrac{\partial V}{\partial z} = -3\gamma z^2$

4.5. a. $\vec{F} = \hat{i}x + \hat{j}y$

di $x = y$

$d\vec{r} = i\,dx + j\,dy$

$\displaystyle\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 Fx\,dx + \int_0^1 Fy\,dy = \int_0^1 x\,dx + \int_0^1 y\,dy = 1$

Pada lintasan sepanjang sumbu $x = d\vec{r} = \hat{i}\,dx$

$x = 1 = d\vec{r} = j\,dy$

$\displaystyle\int_{(0,0)}^{(1,1)} \vec{F}\, d\vec{r} = \int_0^1 Fx\,dx + \int_0^1 Fy\,dy = 1$

$\vec{F}$ adalah konservatif

6. $\vec{F} = \hat{i}y - \hat{j}x$

di $x = y$

$\displaystyle\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 Fx\,dx + \int_0^1 Fy\,dy = \int_0^1 y\,dx - \int_0^1 x\,dy$

dan $x = y$ $\displaystyle\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 x\,dx - \int_0^1 y\,dy = 0$

pada sumbu x = $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_x \, dx = \int_0^1 y \, dx$

pada $y=0$ dengan sumbu x

pada garis $x=1$ $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_y dy = \int_0^1 x \, dy$

$x=1$ $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 dy = 1$

pada jalur ini $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = 0 + 1 = 1$

$\vec{F}$ adalah tidak konservatif.

4.6 Dari contoh 2.3.2, $V(z) = -mg \dfrac{re^2}{(re+z)}$

$$V(z) = -mgre \left(1 + z/re\right)^{-1}$$

$$(1+x)^{-1} = 1 - x + x^2 + \ldots$$

Maka = $V(z) = -mgre \left(1 - z/re + z^2/re + \ldots\right)$

$$V(z) = -mgre + mgz - mgz^2/r^2 + \ldots$$

dengan $-mgre$ sebuah konstan tambahan

$$V(z) \approx mgz \left(1 - z/re\right)$$

$$\vec{F} = -\vec{\nabla} V = -\hat{k} \, \partial/\partial z \, V(z) : -\hat{k} \, mg \left[1 - \frac{z}{re} + z\left(-\frac{1}{re}\right)\right]$$

$$= -\hat{k} \, mg \left(1 - 2z/re\right)$$

$m\ddot{x} = F_x = 0 \qquad m\ddot{y} = F_y = 0$

$$m\dot{z} \, d\dot{z}/dz = -mg\left(1 - \frac{2z}{re}\right)$$

$$\int_{V_z}^{0} \dot{z} \, d\dot{z} = -g \int_0^h \left(1 - 2z/re\right) dz$$

$$-\frac{1}{2} V_{0z}^2 = -g\left(h - \frac{h^2}{re}\right)$$

$$h^2 - reh + \frac{re V_{0z}^2}{2g} = 0$$

$$h = \frac{re}{2} - \frac{re}{2}\sqrt{\frac{1 - 2V_{0z}^2}{gre}}$$

$$(1+x)^{1/2} = 1 + x/2 + x^2/8 + \ldots$$

$$h = re/2 - re/2 + V_{0z}^2/2g + V_{0z}^4/4gre + \ldots$$

$$h \approx \frac{V_{0z}^2}{2g}\left(1 + \frac{V_{0z}^2}{2gre}\right)$$

dari contoh 2.3.2 $h = \dfrac{V_0^2}{g}\left(1 - \dfrac{V_0^2}{2gre}\right)^{-1}$

dan $(1-x)^{-1} \approx 1 + x$, $h \approx \dfrac{V_0^2}{2g}\left(1 + \dfrac{V_0^2}{2gre}\right)$

4.7 Tunjukkan bahwa ketinggian terbesar di tanah

$$b + \frac{V_0^2}{2g} + \frac{gb^2}{2V_0}$$

Pada titik mana roda lumpur bergulir?

$\vec{r} = \hat{i} b\cos\theta - \hat{j} b\sin\theta$

$\theta = \omega t = \frac{V_0 t}{b}$, jadi $\vec{r} = \hat{i}\, V_0 \sin\theta - \hat{j}\, V_0 \cos\theta$

$\vec{V} = \hat{i} V_0 (1 - \sin\theta) - \hat{j} V_0 \cos\theta$

$y_0 = -b\sin\theta$ dan $V_{0y} = -V_0\cos\theta$

jadi

$V_y = V_{0y} - gt = -V_0 \cos\theta - gt$

dan

$y = -b\sin\theta - V_0 t\cos\theta - \frac{1}{2} g t^2$

tinggi maksimum $V_y = 0$

$t = \frac{-V_0 \cos\theta}{g}$

$$h = -b\sin\theta - V_0\left(\frac{-V_0\cos\theta}{g}\right)\cos\theta - \frac{1}{2} g\left(\frac{-V_0\cos\theta}{g}\right)^2$$

$$h = -b\sin\theta + \frac{V_0^2\cos^2\theta}{2g}$$

h maksimum terjadi untuk $\frac{dh}{d\theta} = 0 = -b\cos\theta - \frac{2V_0^2\cos\theta\sin\theta}{2g}$

$\sin\theta = \frac{-gb}{V_0^2}$

$\cos^2\theta = 1 - \sin^2\theta = \frac{V_0^4 - g^2 b^2}{V_0}$

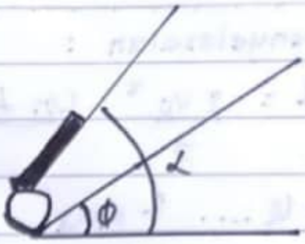
$h_{max} = \frac{gb^2}{V_0^2} + \frac{V_0^4 - g^2b^2}{2gV_0^2} = \frac{gb^2}{2V_0^2} + \frac{V_0^2}{2g}$

diukur dari atas tanah $h_{max} = b + \frac{gb^2}{2V_0^2} + \frac{V_0^2}{2g}$

lumpur meninggalkan roda di $\theta = \sin^{-1}\left(\frac{-gb}{V_0^2}\right)$

1.8 Penyelesaian

$x = R\cos\phi$ dan $x = V_{0x}\ t = (V_0\cos\alpha)t$



Jadi $t = \dfrac{R\cos\phi}{v\cos\gamma}$

$y = R\sin\phi$ dan $y = V - yt - \frac{1}{2}gt^2 = (V_0\sin\alpha)t - \frac{1}{2}gt^2$

$$R\sin\phi = (V_0\sin\alpha)\frac{R\cos\phi}{V_0\cos\alpha} - \frac{1}{2}g\left(\frac{R\cos\phi}{V_0\cos\alpha}\right)^2$$

$$\sin\phi = \tan\alpha.\cos\phi - \frac{gr\cos^2\phi}{2V_0^2\cos^2\alpha}$$

$$R = \frac{2V_0^2\cos^2\alpha}{g\cos\phi}(\tan\alpha\cos\phi - \sin\phi) = \frac{2V_0^2\cos\alpha}{g\cos^2\phi}(\sin\alpha\cos\phi - \cos\alpha\sin\phi)$$

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

$$R = \frac{2V_0^2\cos\alpha\sin(\alpha-\phi)}{g\cos^2\phi}$$

R max untuk $\dfrac{dr}{d\alpha} = 0 = \dfrac{2V_0^2}{g\cos^2\phi}\left[-\sin\alpha\sin(\alpha-\phi) + \cos\alpha\cos(\alpha-\phi)\right]$

$\cos(\theta+\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$ jadi, $\cos(2\alpha-\phi) = 0$

$2\alpha - \phi = \dfrac{\pi}{2}$ $\qquad \alpha = \dfrac{\pi}{4} + \dfrac{\phi}{2}$

$$R\ max = \frac{2V_0^2}{g\cos\phi}\cos\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\sin\left(\frac{\pi}{4}-\frac{\phi}{2}\right)$$

$$\sin\left(\frac{\pi}{4}-\frac{\phi}{2}\right) = \cos\left[\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{\phi}{2}\right)\right] = \cos\left(\frac{\pi}{4}+\frac{\phi}{2}\right)$$

$$R\ max = \frac{2V_0^2}{g\cos^2\phi}\cos^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)$$

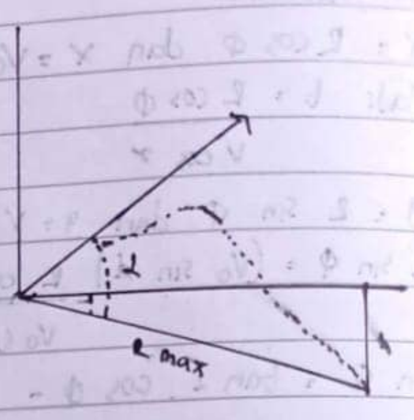
$$R\ max = \frac{2V_0^2}{g\cos^2\phi}\left[\frac{1}{2}\cos\left(\frac{\pi}{2}+\phi\right)+\frac{1}{2}\right] = \frac{V_0^2}{g\cos^2\phi}\left[\cos\left(\frac{\pi}{2}+\phi\right)+1\right]$$

[redacted]unakan $\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$$R\ max = \frac{V_0^2}{g(1-\sin^2\phi)}(1-\sin\phi)$$

$$R\ max = \frac{V_0^2}{g(1+\sin\phi)}$$

4.9 Penyelesaian :



$$R = \frac{2 V_0^2}{g} \cdot \frac{\cos\alpha \sin(\alpha + \varphi)}{\cos^2\phi}$$

$- \varphi \ldots\ R_{max} = V_0^2 (1 + \sin\varphi)$ dan ... $2\alpha = \frac{\pi}{2} - \varphi$

$$R_{max} = \frac{h}{\sin\varphi} = \frac{V_0^2}{g} \frac{(1 + \sin\varphi)}{\cos^2\varphi} = \frac{V_0^2}{g(1 - \sin\varphi)}$$

untuk $\sin\varphi = \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2}\right)$

$$2\sin^2\alpha = \frac{2}{\csc^2\alpha} = 1 - \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2}\right) = \frac{1}{1 + \frac{gh}{V_0^2}}$$

sehingga didapatkan $\csc^2\alpha = 2\left(1 + \frac{gh}{V_0^2}\right)$

b. Berapakan jarak maksimum R dari malam itu?

Penyelesaian : $R_{max} = \frac{h}{\sin\varphi} = \frac{h}{1 - 2\sin^2\alpha} = \frac{h}{1 - 2/\csc^2\alpha}$

$$R_{max} = \frac{V_0^2}{g}\left(1 + \frac{gh}{V_0^2}\right)$$

4.12 Penyelesaian :

Posisi x dia z bola dan waktu adalah

$x = V_0 + \cos\frac{1}{2}\theta \qquad z = V_0 + \cos\frac{1}{2}\theta, \sin\theta - \frac{1}{2} g r^2$

$V = V_0 \cos\frac{1}{2}\theta$

Jangkauan mendatar adalah $R = \frac{V_0^2}{g} \cos^2\frac{1}{2}\theta, \sin 2\theta$

Jangkauan max $\frac{dr}{d\theta} = 0$

a. $\frac{dr}{d\theta} = \frac{V_0^2}{g}\left(2\cos^2\frac{1}{2}\theta, \cos 2\theta, \cos\frac{1}{2}\theta, \sin\frac{1}{2}\theta, \sin 2\theta\right) = 0$

$2\cos^2\frac{1}{2}, \cos 2\theta = \cos\frac{1}{2}\theta, \sin\frac{1}{2}\theta, \sin 2\theta$

dengan $2\cos^2\theta = 1 + \cos 2\theta$ dan $\sin 2\theta = 2\sin\theta - \cos\theta$

, maka diperoleh

$(1 + \cos\theta)(2\cos^2\theta - 1) = \sin\theta_0 \cos\theta_0 : (1 - \cos^2\theta_0)\cos\theta_0$

atau $(1 + \cos\theta_0)(3\cos^2\theta_0 - \cos\theta_0 - 1) = 0$

Dengan demikian $\cos\theta_0 = -1 \quad \cos\theta_0 = \frac{1}{6}(1 \pm \sqrt{13})$

Hanya akar positif yang berlaku untuk θ, dengan jangkauan $0 \le \theta_1 \le \frac{\pi}{2}$

$\cos\theta_1 = \frac{1}{6}(1 + \sqrt{13}) = 0{,}7676$

b. Untuk $V_0 = 25\ ms^{-1}$ $R_{max} = 55{,}4m$ $\theta_0 = 39°51'$

ketinggian maksimum terjadi pada $dz/dt = 0$

$V_0 \cos\frac{1}{2}, \sin\theta = gt$ atau $T = \frac{V_0 \cos\frac{1}{2}\theta, \sin\theta}{g}$ atau

$H = \frac{V_0^2}{2g} \cos^2\frac{1}{2}\theta, \sin^2\theta$

maksimum pada ketinggian tetap : $\frac{dH}{d\alpha} = 0$

maka, $\frac{dH}{d\alpha} = \frac{V_0^2}{2g}\left(2\cos^2\frac{1}{2}\theta_0 \sin\theta_0 - \cos\frac{1}{2}\theta, \sin\frac{1}{2}\theta_0 \sin^2\theta\right) = 0$

$(1+\cos\theta_0)\sin\theta_0\cos\theta_0 = \frac{1}{2}\sin\theta_0\sin^2\theta_0 = \frac{1}{2}\sin\theta_0(1-\cos^2\theta_0)$ atau

$\sin\theta_0\ (1+\cos\theta_0)(3\cos\theta_0 - 1) = 0$

$\sin\theta_0 = 0$ $\cos\theta_0 = -1$ $\cos\theta_0 = \frac{1}{3}$

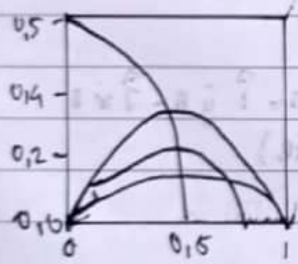
Jadi, $H_{max} = 10{,}9\ m$ dan $\theta_0 = \cos^{-1}\frac{1}{2} = 70{,}32$

1.13 Penyelesaian :

r menggantikan x

$z = \frac{z_0}{r_0} r - \frac{g}{2r^2} r^2$ dimana $r_0 = V_0\cos\theta_0$ $z_0 = V_0\sin\theta_0$

$z = r\tan\theta_0 - \frac{gr^2}{2V_0^2}\sec^2\theta$ sejak $\sec^2\theta_0 = 1 + \tan^2\theta$



$\frac{gr^2}{2V_0^2}\tan^2\theta_0 - r\tan\theta_0 + z + \frac{gr^2}{2V_0^2} = 0$

(r, z) titik kordinat

$\tan\theta_0 = \frac{1}{gr}\left[V_0^2 \pm (V_0^4 - gzV_0^2 - g^2r^2)^{1/2}\right]$

$V_0^4 - 2gzV_0^2 - g^2r^2 \geq 0$

Permukaan kritis oleh karena itu $V_0^4 - 2gz - g^2r^2 = 0$

1.14 Penyelesaian :

$m\ddot{x}, F_x = -\frac{\partial V}{\partial x} = -kx = -\pi^2 mx$

$x = A\cos\left[\sqrt{\frac{k}{m}}\ t + \alpha\right] = A\cos(\pi t + \alpha)$

$m\ddot{y} = -\frac{\partial V}{\partial y} = -4\pi^2 mx$

$y = B\cos(2\pi t + \beta)$

$m\ddot{z} = -\frac{\partial V}{\partial z} = -9\pi^2 mz$

$z = C(\cos(3\pi t + \gamma)$

sejak $x = y = z = 0$ atau $t = 0$, $\alpha = \beta = \gamma = -\frac{\pi}{2}$

$$x = A \cos\left(\pi t_1 - \frac{\pi}{2}\right) = A \sin \pi t \quad x = A\pi \cos \pi t$$

maka $v_0^2 = x_0^2 + y_0^2 + z_0^2$ dan $x_1 = y_1 = z_1$

$$x_0 = \frac{v_0}{\sqrt{3}} = A\pi, \quad A = \frac{v_0}{\pi\sqrt{3}}, \quad x = \frac{v_0}{\pi\sqrt{3}} \sin \pi t, \quad y = B \sin 2\pi t$$

$$y = 2B\pi \cos 2\pi t \quad y_1 = \frac{v_0}{\sqrt{3}} = 2\pi B, \quad B = \frac{v_0}{2\pi\sqrt{3}}, \quad y = \frac{v_0}{2\pi\sqrt{3}} \sin 2\pi t$$

$$z = \dot{c} \sin 2\pi t. \quad z = 3c\pi \cos 3\pi t, \quad z = \frac{v_0}{\sqrt{3}} = 3c\pi$$

$$c = \frac{v_0}{3\pi\sqrt{3}} \text{ dan } z = \frac{v_0}{3\pi\sqrt{3}} \sin 3\pi t$$

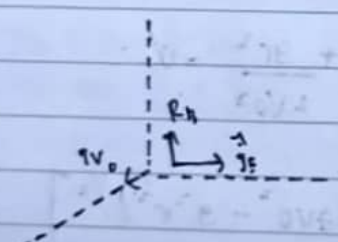
sejak $w_x = \pi$. $w_y = 2\pi$ dan $w_z = 3\pi$, maka

$$T_{min} = \frac{2\pi . n_1}{w_x} = \frac{2\pi . n_2}{w_y} = \frac{2\pi\, n_3}{w_z}$$

waktu minimum pada $n_1 = 1$, $n_2 = 2$, $n_3 = 3$

$$t_{min} = \frac{2\pi}{\pi} = 2$$

7.20 Penyelesaian :



$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{v} \times \vec{B} = (\hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z}) \times kB = \hat{i}\,\dot{y}B - \hat{j}\,\dot{x}B$$

$$\vec{F} = \hat{i}\, q\dot{y}B + \hat{j}\, q(E - \dot{x}B)$$

$$m\ddot{x} = F_x = q\dot{y}B$$

$$\dot{x} - \dot{x}_0 = \frac{qB}{m} y$$

maka, $m\ddot{y} = F_y = qE - q\dot{x}B = qE - qB\left(\dot{x}_0 + \frac{qB}{m} y\right)$

$$\ddot{y} = \frac{qE}{m} - \frac{qB\dot{x}}{m} - \left(\frac{qB}{m}\right)^2 y = \frac{-eE}{m} + \frac{eB\dot{x}}{m} - \left(\frac{eB}{m}\right)^2 y \ldots\ldots$$

$$\ddot{y} + w^2 y = -\frac{eE}{m} + w\dot{x}_0, \quad w = \frac{eB}{m}$$

$$y = \frac{1}{w^2}\left(-\frac{eE}{m} + w\dot{x}\right) + A\cos(wt + \theta_0)$$

$$\dot{y} = -Aw \sin(wt + \theta)$$

$\dot{y}_0 = 0$, Jadi, $\theta_0 = 0$

$y_0 = 0$, Jadi $A = -\frac{1}{w^2}\left(\frac{-eE}{m} + w\dot{x}_0\right)$

$$y = a(1 - \cos wt), \quad a = \frac{1}{w^2}\left(-\frac{eE}{m} + w\dot{x}_0\right)$$