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Ayo berlatih, (General Motion of a particle in three dimension)

2.1 a)  $V = Cxyz + c$

$$\Rightarrow \vec{F} = -\vec{\nabla}V = -\hat{i} \frac{\partial V}{\partial x} - \hat{j} \frac{\partial V}{\partial y} - \hat{k} \frac{\partial V}{\partial z} = -c(\hat{i}yz + \hat{j}xz + \hat{k}xy)$$

b)  $V = \alpha x^2 + \beta y^2 + \gamma z^2 + c$

$$\Rightarrow \vec{F} = -\vec{\nabla}V = -\hat{i}2\alpha x - \hat{j}2\beta y - \hat{k}2\gamma z$$

c)  $V = Ce^{-(2x + 4y + 8z)}$

$$\Rightarrow \vec{F} = -\vec{\nabla}V = Ce^{-(2x + 4y + 8z)}(-2\hat{i} - 4\hat{j} - 8\hat{k})$$

d)  $V = Cr^n$  dalam koordinat bola

$$\Rightarrow \vec{F} = -\vec{\nabla}V = -\hat{e}_r \frac{\partial V}{\partial r} - \hat{e}_\theta \frac{1}{r} \frac{\partial V}{\partial \theta} - \hat{e}_\phi \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi}$$

4.2 a)  $\vec{F} = \hat{i}x + \hat{j}y + \hat{k}z$

Jawab:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0 \rightarrow \text{konservatif}$$

b)  $\vec{F} = \hat{i}x - \hat{j}y + \hat{k}z$

Jawab:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -y & z \end{vmatrix} = \hat{k}(-1-1) \neq 0$$

↳ tidak konservatif

c)  $F = i x + j y + k z^2$

Jawab:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z^2 \end{vmatrix} = \hat{k} (1-1) = 0 \rightarrow \text{konservatif}$$

d)  $F = -kr^n \hat{e}_r$  dalam koordinat bola

Jawab:

$$\vec{\nabla} \times \vec{F} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \hat{e}_r & \hat{e}_\theta & \hat{e}_\phi \sin \theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ -kr & 0 & 0 \end{vmatrix} = 0 \quad (\text{konservatif})$$

4.2 a)  $F = i x y + j c x^2 + k z^2$

Jawab:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & cx^2 & z^2 \end{vmatrix} = k (z(x-y))$$

$$2cx - x = 0 \rightarrow c = \frac{1}{2}$$

c)  $F = i (z/y) + c j (xz/y^2) + k (x/y)$

Jawab:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{z}{y} & \frac{cxz}{y^2} & \frac{x}{y} \end{vmatrix} = \hat{i} \left( -\frac{x}{y} - \frac{cx}{y^2} \right) + \hat{j} \left( \frac{1}{y} - \frac{1}{y} \right) + \hat{k} \left( \frac{cz}{y^2} + \frac{z}{y^2} \right)$$

$$-\frac{x}{y} - \frac{cx}{y^2} = 0 \rightarrow c = -1$$

$$\text{Juga } \frac{cz}{y^2} + \frac{z}{y^2} = 0 \rightarrow c = -1$$

$$d) F = i(x/y) + cj(xz/y^2) + k(x/y)$$

Jawab:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & cx^2 & z^2 \end{vmatrix} = k$$

4.4 a) Jawab:

$$E = \text{konstan} = V(x, y, z) + \frac{1}{2} mv^2$$

$$\text{diasumsinya, } E = 0 + \frac{1}{2} mv^2$$

$$\text{Pada } (1, 1, 1) \quad E = \alpha + \beta + \gamma + \frac{1}{2} mv^2 = \frac{1}{2} mv^2$$

b) Jawaban:

$$V_0 = \frac{2}{m} (\alpha + \beta + \gamma) = 0$$

$$V_0 = \left[ \frac{2}{m} (\alpha + \beta + \gamma) \right]^{\frac{1}{2}}$$

c) Jawab:

$$m\ddot{x} = F_x = -\frac{2V}{2x}$$

$$m\ddot{x} = -\alpha$$

$$m\ddot{y} = -\frac{2V}{2y} = -2\beta y$$

$$m\ddot{z} = -\frac{2V}{2z} = 3\gamma z^2$$

4.5 Pertimbangan z fungsi gaya

$$a) F = i x + j y$$

Jawab:

$$\vec{F} = \hat{i} x + \hat{j} y$$

$$\text{di } x = y \quad d\vec{r} = \hat{i} dx + \hat{j} dy$$

$$\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_x dx + \int_0^1 F_y dy = \int_0^1 x dx + \int_0^1 y dy = -1$$



Pada lintasan sepanjang  $dx$  :  $\vec{dr} = \hat{i} dx$

dan pada garis  $x=1$

$$\vec{dr} = \hat{j} dy$$

$$\int_{(0,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = \int_0^1 F_x dx + \int_0^1 F_y dy = 1, \vec{F} = \text{konservatif}$$

$$b) \vec{F} = \hat{i}y - \hat{j}x$$

$$\text{di } x=y : \int_{(0,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = \int_0^1 F_x dx + \int_0^1 F_y dy = \int_0^1 y dx - \int_0^1 x dy$$

$$\text{dan di } x=y : \int_{(0,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = \int_0^1 x dx - \int_0^1 y dy = 0$$

$$\text{Pada sumbu } x : \int_{(0,0)}^{(1,0)} \vec{F} \cdot \vec{dr} = \int_0^1 F_x dx = \int_0^1 y dx$$

$$\text{dan, di } y=0 \text{ pd sb } x : \int_{(0,0)}^{(1,0)} \vec{F} \cdot \vec{dr} = 0$$

$$\text{Pada garis } x=1 : \int_{(1,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = \int_0^1 F_y dy = \int_0^1 x dy$$

$$\text{dan di } x=1 : \int_{(1,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = \int_0^1 dy = 1$$

$$\text{Pada jalur ini } \int_{(0,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = 0 + 1 = 1, \vec{F} = \text{tidak konservatif}$$

4.6 Jawab:

$$\text{Dari contoh 2.3.2, } V(z) = -mg \frac{r_e^2}{(r_e + z)}$$

$$V(z) = -mg r_e \left(1 + \frac{z}{r_e}\right)^{-1}$$

$$V(z) = -mg r_e \left(1 - \frac{z}{r_e} + \frac{z^2}{r_e^2} + \dots\right)$$

$$V(z) = -mg r_e + mg z - \frac{mg z^2}{r_e} + \dots$$

di  $-mg r_e$  sebuah konstan

$$V(z) \approx mg z \left(1 - \frac{z}{r_e}\right)$$

$$\vec{F} = -\vec{\nabla} V = -\hat{k} \frac{\partial}{\partial z} V(z)$$

$$= -\hat{r} mg \left[ 1 - \frac{z}{r_e} + z \left( -\frac{1}{r_e} \right) \right]$$

$$\vec{F} = -\hat{r} mg \left( 1 - \frac{2z}{r_e} \right)$$

$$m\ddot{x} = F_x = 0$$

$$m\ddot{z} \frac{dz}{dz} = -mg \left( 1 - \frac{2z}{r_e} \right)$$

$$\int_{V_z}^0 \dot{z} dz = -g \cdot \int_0^m \left( 1 - \frac{2z}{r_e} \right) dz$$

$$-\frac{1}{2} V_0^2 z = -g \left( h - \frac{h^2}{r_e} \right)$$

$$h^2 - r_e h + \frac{r_e V_0^2 z}{2g} = 0$$

$$h - r_e h + \frac{r_e V_0^2 z}{2g} = 0$$

$$h = \frac{r_e}{2} - \frac{1}{2} \sqrt{r_e^2 - \frac{2r_e V_0^2 z}{g r_e}} \quad (h \ll r_e)$$

$$h = \frac{r_e}{2} - \frac{r_e}{2} \sqrt{1 - \frac{2V_0^2 z}{g r_e}}$$

$$(1+x)^{\frac{1}{2}} = 1 + \frac{x}{2} - \frac{x^2}{2} + \dots$$

$$h = \frac{r_e}{2} - \frac{r_e}{2} + \frac{V_0^2 z}{2g} + \frac{V_0^4 z^2}{4g r_e} + \dots$$

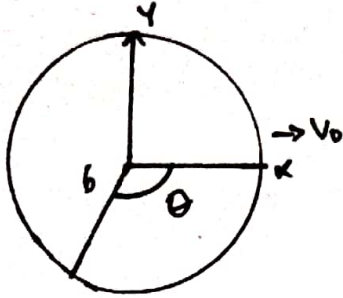
$$h \approx \frac{V_0^2 z}{2g} \left( 1 + \frac{V_0^2 z}{2g r_e} \right)$$

$$\text{dari contoh 2.3.2 } h = \frac{V_0^2}{2g} \left( 1 - \frac{V_0^2}{2g r_e} \right)^{-1}$$

$$\text{dan dg } (1-x)^{-1} \approx 1 + x, h \approx \frac{V_0^2}{2g} \left( 1 + \frac{V_0^2}{2g r_e} \right)$$



4.2) Jawab :



$$\vec{r} = i b \cos \theta - j b \sin \theta$$

$$\theta = \omega t = \frac{V_0 t}{b} \text{, jadi, } \vec{r} = -i V_0 \sin \theta - j V_0 \cos \theta$$

selanjut ke kanan,  $\vec{v} = i V_0 (1 - \sin \theta) - j V_0 \cos \theta$

Paralel keluar tepi :  $y_0 = -b \sin \theta$  dan  $V_{0y} = -V_0 \cos \theta$

Jadi,  $V_y : V_{0y} - gt = -V_0 \cos \theta - gt$  dan  $y = -b \sin \theta - V_0 t \cos \theta - \frac{1}{2} gt^2$

tinggi maks  $V_y = 0$

$$t = -\frac{V_0 \cos \theta}{g}$$

$$h = -b \sin \theta - V_0 \left( -\frac{V_0 \cos \theta}{g} \right) \cos \theta - \frac{1}{2} g \left( -\frac{V_0 \cos \theta}{g} \right)^2$$

$$h = -b \sin \theta = \frac{V_0^2 \cos^2 \theta}{2g}$$

h maks terjadi untuk  $\frac{dh}{d\theta} = 0 = -b \cos \theta - \frac{2 V_0^2 \cos \theta \sin \theta}{2g}$

$$\sin \theta = -\frac{gb}{V_0^2}$$

$$\cos^2 \theta = 1 - \sin^2 \theta = \frac{V_0^4 - g^2 b^2}{V_0^4}$$

$$h_{\max} = \frac{gb^2}{V_0^2} + \frac{V_0^4 g^2 b^2}{2g V_0^4} = \frac{gb^2}{2V_0^2} + \frac{V_0^2}{2g}$$

lumpur meninggalkan roda di  $\theta = \sin^{-1} \left( -\frac{gb}{V_0^2} \right)$

4.8) Jawab:

$$x = R \cos \phi \text{ dan } x = v_0 \cos \alpha \cdot t \cdot (v_0 \cos \alpha) t$$

$$\text{Jadi, } t = \frac{R \cos \phi}{v_0 \cos \alpha}$$

$$y = R \sin \phi \text{ dan } y = v_0 \sin \alpha \cdot t - \frac{1}{2} g t^2 = (v_0 \sin \alpha) t - \frac{1}{2} g t^2$$

$$R \sin \phi = (v_0 \sin \alpha) \frac{R \cos \phi}{v_0 \cos \alpha} - \frac{1}{2} g \left( \frac{R \cos \phi}{v_0 \cos \alpha} \right)^2$$

$$\sin \phi = \tan \alpha \cos \phi - \frac{g R \cos^2 \phi}{2 v_0^2 \cos^2 \alpha}$$

$$R = \frac{2 v_0^2 \cos^2 \alpha}{g \cos \phi} (\tan \alpha \cos \phi - \sin \phi) = \frac{2 v_0^2 \cos^2 \alpha}{g \cos^2 \phi} (\sin \alpha \cos \phi - \cos \alpha \sin \phi)$$

$$\sin (\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi$$

$$R = \frac{2 v_0^2 \cos^2 \alpha}{g \cos^2 \phi} \sin (\alpha - \phi)$$

$$R_{\max} \text{ untuk } \frac{dR}{d\alpha} = 0 = \frac{2 v_0^2}{g \cos^2 \phi} \left[ -\sin \alpha \sin (\alpha - \phi) + \cos \alpha \cos (\alpha - \phi) \right]$$

$$\cos (\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi \text{ jadi } \cos (2\alpha - \phi) = 0$$

$$2\alpha - \phi = \frac{\pi}{2}$$

$$R_{\max} = \frac{2 v_0^2}{g \cos^2 \phi} \cos \left( \frac{\pi}{4} + \frac{\phi}{2} \right) \sin \left( \frac{\pi}{4} - \frac{\phi}{2} \right)$$

$$\sin \left( \frac{\pi}{4} - \frac{\phi}{2} \right) = \cos \left[ \frac{\pi}{2} - \left( \frac{\pi}{4} - \frac{\phi}{2} \right) \right] = \cos \left( \frac{\pi}{4} + \frac{\phi}{2} \right)$$

$$R_{\max} = \frac{2 v_0^2}{g \cos^2 \phi} \cos^2 \left( \frac{\pi}{4} + \frac{\phi}{2} \right)$$

$$R_{\max} = \frac{2 v_0^2}{g \cos^2 \phi} \left[ \frac{1}{2} \cos \left( \frac{\pi}{2} + \phi \right) + \frac{1}{2} \right] = \frac{v_0^2}{g \cos^2 \phi} \left[ \cos \left( \frac{\pi}{2} + \phi \right) + 1 \right]$$

$$\text{menggunakan } \cos \left( \frac{\pi}{2} + \phi \right) = -\sin \phi$$



$$R_{\max} = \frac{V_0^2}{g(1 - \sin \phi)}$$

$$= \frac{V_0^2}{g(1 + \sin \phi)}$$

4.9) Jawab:

a)



$$R = \frac{2V_0^2}{g} \frac{\cos \alpha \sin(\alpha + \phi)}{\cos^2 \phi}$$

$$-p = \dots R_{\max} : V_0^2(1 + \sin \phi) \text{ dan } \dots 2\alpha = \frac{\pi}{2} = p$$

$$R_{\max}, \frac{h}{\sin \phi} = \frac{V_0^2}{g} \frac{(1 + \sin \phi)}{\cos^2 \phi} = \frac{V_0^2}{g(\sin \phi)}$$

$$\sin \phi = \frac{gh}{V_0^2} \left( 1 + \frac{gh}{V_0^2} \right)$$

$$\sin \phi = \sin \left( \frac{\pi}{2} - 2\alpha \right) = \cos 2\alpha = 1 - 2\sin^2 \alpha$$

$$1 - 2\sin^2 \alpha = \frac{gh}{V_0^2} \left( 1 + \frac{gh}{V_0^2} \right)$$

$$2\sin^2 \alpha = \frac{2}{\csc^2 \alpha} \cdot 1 - \frac{gh}{V_0^2} \left( 1 + \frac{gh}{V_0^2} \right) = \frac{1}{1 + \frac{gh}{V_0^2}}$$

$$\text{akhirnya} = \csc^2 \alpha = 2 \left( 1 + \frac{gh}{V_0^2} \right)$$

b) Jawab:

$$R_{\max} = \frac{h}{\sin \phi} = \frac{h}{1 - 2\sin^2 \alpha} = \frac{h}{1 - 2/\csc^2 \alpha}$$

$$R_{\max} = \frac{V_0^2}{g} \left( 1 + \frac{gh}{V_0^2} \right)$$



Jawab:  
 (4.12) a).  $x$  dan  $z$  adalah posisi bola

$$x = V_0 t \cos \frac{1}{2} \theta_0, \quad z = V_0 t \cos \frac{1}{2} \theta_0 \sin \theta_0 - \frac{1}{2} g t^2$$

$$\text{Sejak } v_x = V_0 \cos \frac{1}{2} \theta_0$$

$$R = \frac{V_0^2}{g} \cos^2 \frac{1}{2} \theta_0 \sin 2\theta_0$$

$$\max \frac{dR}{d\theta} = 0$$

$$\frac{dR}{d\theta} = \frac{V_0^2}{g} (2 \cos^2 \frac{1}{2} \theta_0 \cos 2\theta_0 - \cos \frac{1}{2} \theta_0 \sin \frac{1}{2} \theta_0 \sin 2\theta_0) = 0$$

$$2 \cos^2 \frac{1}{2} \theta_0 \cos 2\theta_0 = \cos \frac{1}{2} \theta_0 \sin \frac{1}{2} \theta_0 \sin 2\theta_0$$

$$2 \cos^2 \theta_0 = 1 + \cos 2\theta_0 \quad \text{dan} \quad \sin 2\theta_0 = 2 \sin \theta_0 \cos \theta_0$$

$$\text{didapat: } (1 + \cos \theta_0) (2 \cos^2 \theta_0 - 1) = \sin \theta_0 \sin \theta_0 \cos \theta_0 \cdot (1 - \cos^2 \theta_0)$$

$$\text{atau } (1 + \cos \theta_0) (3 \cos^2 \theta_0 - \cos \theta_0 - 1) = 0$$

hanya akar positif yang berlaku adalah  $\theta_0, 0 \leq \theta_0 \leq \frac{\pi}{2}$

$$\cos \theta_0 = \frac{1}{6} (1 + \sqrt{13}) = 0,7676 \quad \theta_0 = 39^\circ \text{ si}$$

b) untuk  $V_0 = 25 \text{ m/s}$  &  $\max. H = 4 \text{ m}$   $\theta_0 = 39^\circ \text{ si}$

$$\max \frac{dz}{dt} = 0$$

$$V_0 \cos \frac{1}{2} \theta_0 \sin \theta_0 = g t \quad \text{atau} \quad T = \frac{V_0 \cos \frac{1}{2} \theta_0 \sin \theta_0}{g}$$

$$\text{atau } H = \frac{V_0^2}{2g} \cos^2 \frac{1}{2} \theta_0 \sin^2 \theta_0, \quad x \text{ tetap di } \theta, \max \text{ pada}$$

$$\text{kelingasan tetap terjadi } \frac{dH}{d\theta} = 0$$

$$\frac{dH}{d\theta} = \frac{V_0^2}{2g} (2 \cos^2 \frac{1}{2} \theta \sin \theta \cos \theta - \cos \frac{1}{2} \theta \sin \frac{1}{2} \theta \sin^2 \theta) = 0$$

$$(1 + \cos \theta_0) \sin \theta_0 \cos \theta_0 = \frac{1}{2} \sin \theta_0 \sin^2 \theta_0 + \frac{1}{2} \sin \theta_0 (1 - \cos^2 \theta_0)$$

$$\sin \theta_0 = 0$$

$$\cos \theta_0 = -1$$

$$\text{Jadi, } H_{\max} = 10,9 \text{ m}$$

$$\theta_0 = \cos^{-1} \frac{1}{3} = 70,32^\circ$$

4.1) Jawab:

r menggantikan x

$$z = \frac{\dot{z}_0}{\dot{r}_0} r + \frac{g}{2\dot{r}_0^2} r^2 \text{ dimana } \dot{r}_0 = V_0 \cos \theta_0 \quad \dot{z}_0 = V_0 \sin \theta_0$$

$$z = r \tan \theta_0 - \frac{gr^2}{2V_0^2} \sec^2 \theta \text{ sejak } \sec^2 \theta_0 = 1 + \tan^2 \theta$$

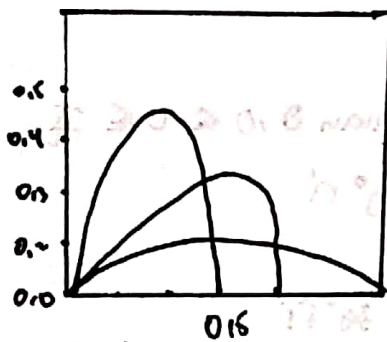
$$\frac{gr^2}{2V_0^2} \tan^2 \theta_0 - r \tan \theta_0 + z + \frac{gr^2}{2V_0^2} = 0$$

(r, z) titik koordinat

$$\tan \theta_0 = \frac{1}{gr} \left[ V_0^4 \pm (V_0^4 - 2gzV_0^2 - g^2r^2)^{1/2} \right]$$

$$V_0^4 - 2gzV_0^2 - g^2r^2 \geq 0$$

$$\text{Permukaan kritis oleh kamarnya } V_0^4 - 2gzV_0^2 - g^2r^2 = 0$$



4.12) Jawab:

$$m\ddot{x} = F_x = -\frac{\partial V}{\partial x} = -kx = -\pi^2 mx$$

$$\dot{x} = A \cos \left( \sqrt{\frac{k}{m}} t + \alpha \right) = A \cos (\pi t + \alpha)$$

$$m\ddot{y} = -\frac{\partial V}{\partial y} = 4\pi^2 my$$

$$y = B \cos (2\pi t + \beta)$$

$$m\ddot{z} = -\frac{\partial V}{\partial z} = -g\pi^2 mz$$

$$z = C \cos (3\pi t + \gamma)$$

Sejak  $x = y = z = 0$  pada  $t = 0$  dan  $\beta = \gamma = \frac{\pi}{2}$

$$x = A \cos \left( \pi t - \frac{\pi}{2} \right) = A \sin \pi t$$



$$\dot{x}_0 = A \pi \cos \pi t$$

$$V_0^2 = \dot{x}_0^2 + \dot{y}_0^2 + \dot{z}_0^2 \text{ dan } \dot{x}_0 = \dot{y}_0 = \dot{z}_0$$

$$\dot{x}_0 = \frac{V_0}{\sqrt{3}} = A \pi$$

$$A = \frac{V_0}{\pi \sqrt{3}}$$

$$x = \frac{V_0}{\pi \sqrt{3}} \sin \pi t$$

$$y = B \sin 2\pi t, \dot{y} = 2B\pi \cos 2\pi t$$

$$\dot{y}_0 = \frac{V_0}{\sqrt{3}} = 2\pi$$

$$B = \frac{V_0}{2\pi \sqrt{3}}$$

$$y = \frac{V_0}{2\pi \sqrt{3}} \sin 2\pi t$$

$$z = C \sin 3\pi t$$

$$\dot{z}_0 = \frac{V_0}{\sqrt{3}} = 3\pi \quad \dot{z} = 3C\pi \cos 3\pi t$$

$$\dot{z}_0 = \frac{V_0}{\sqrt{3}} = 3\pi$$

$$C = \frac{V_0}{3\pi \sqrt{3}}$$

$$z = \frac{V_0}{3\pi \sqrt{3}} \sin 3\pi t$$

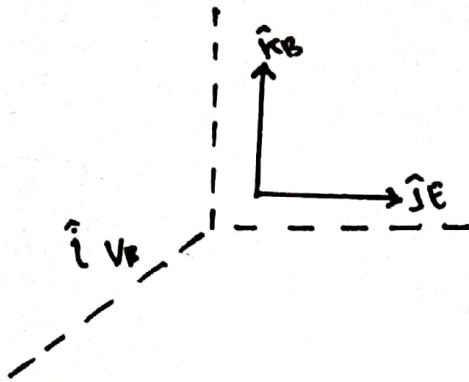
Sejajar  $\omega_x = \pi$ ,  $\omega_y = 2\pi$ , dan  $\omega_z = 3\pi$ , maka

$$T_{\min} = \frac{2\pi n_1}{\omega_x} = \frac{2\pi n_2}{\omega_y} = \frac{2\pi n_3}{\omega_z}$$

Waktu max pada  $n_1 = 1$ ,  $n_2 = 2$ ,  $n_3 = 3$

$$t_{\min} = \frac{2\pi}{\pi} = 2$$

(4.20) Jawab:



$$\vec{F} = q(\vec{E} \times \vec{v} + \vec{v} \times \vec{B})$$

$$\vec{v} \times \vec{B} = (\hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z}) \times B\hat{k} = \hat{i}\dot{y}B - \hat{j}\dot{x}B$$

$$\vec{F} = \hat{i}q\dot{y}B + \hat{j}q(E - \dot{x}B)$$

$$m\dot{x} = F_x = q\dot{y}B$$

$$\dot{x} - \dot{x}_0 = \frac{q_0}{m} = y$$

$$m\ddot{y} = F_y = qE - q\dot{x}B = qE - qB(\dot{x}_0 + \frac{q_0}{m}y)$$

$$\ddot{y} = \frac{qE}{m} - \frac{qB\dot{x}}{m} - \left(\frac{qB}{m}\right)^2 y = \frac{eE}{m} + \frac{EB\dot{x}}{m} - \left(\frac{eB}{m}\right)^2 y$$

$$\ddot{y} + \omega^2 y = -\frac{eE}{m} + \omega \dot{x}_0$$

$$y = \frac{1}{\omega^2} \left( -\frac{eE}{m} + \omega \dot{x}_0 \right) + A \cos(\omega t + \theta_0)$$

$$\dot{y} = -A\omega \sin(\omega t + \theta_0)$$

$$\dot{y}_0 = 0, \text{ jadi } \theta_0 = 0, A = -\frac{1}{\omega^2} \left( -\frac{eE}{m} + \omega \dot{x}_0 \right)$$

$$y = a(1 - \cos \omega t), a = \frac{1}{\omega^2} \left( -\frac{eE}{m} + \omega \dot{x}_0 \right)$$

$$\dot{x} = \dot{x}_0 = \frac{qB}{m} y = \dot{x} - \omega y = \dot{x}_0 - \omega a(1 - \cos \omega t)$$

$$x = (\dot{x}_0 - \omega a)t + a \sin \omega t$$

$$x = a \sin \omega t + b t \quad b = \dot{x}_0 - \omega a$$

$$m\ddot{z} = F_z = 0 \quad z = \dot{z}_0 t + z_0 = 0$$