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Mekanika.

(4.1) a) $V = Cxyz + C$

$\hookrightarrow \vec{F} = -\bar{\nabla} V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z} = -C(\hat{i}yz + \hat{j}xz + \hat{k}xy)$

b) $V = \alpha x^2 + \beta y^2 + \gamma z^2 + C$

$\hookrightarrow \vec{F} = -\bar{\nabla} V = -\hat{i}\,2\alpha x + \hat{j}\,2\beta y + \hat{k}\,2\gamma z$

c) $V = Ce^{-(\alpha x + \beta y + \gamma z)}$

$\hookrightarrow \vec{F} = -\bar{\nabla} V = Ce^{-(\alpha x + \beta y + \gamma z)}\,(\hat{i}\alpha + \hat{j}\beta + \hat{k}\gamma)$

d) $V = Cr^n$ dalam koordinat bola

$\hookrightarrow \vec{F} = -\bar{\nabla} V = -\hat{e}_r\frac{\partial V}{\partial r} - \hat{e}_\theta\frac{1}{r}\frac{\partial V}{\partial \theta} - \hat{e}_\phi\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}$

(4.2) a) $F = \hat{i}x + \hat{j}y + \hat{k}z$

$\hookrightarrow \vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0 \Rightarrow$ konservatif

b) $F = \hat{i}x - \hat{j}y + kz^2$

$\hookrightarrow \vec{\nabla} + \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -y & z^2 \end{vmatrix} = \hat{k}(-1-1) \neq 0 \Rightarrow$ tidak konservatif.

c) $F = \hat{i}x + \hat{j}y + kz^3$

$\hookrightarrow \vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z^3 \end{vmatrix} = \hat{k}\,(1-1) = 0 \Rightarrow$ konservatif

d) $F = -kr^{-n}\,e_r$ dalam koordinat bola

$\hookrightarrow \vec{\nabla}\times\vec{F} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{e}_r & \hat{e}_{\theta r} & \hat{e}_{\phi}r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ -kr & 0 & 0 \end{vmatrix} = 0 \Rightarrow$ konservatif

(4.3) a) $F = \hat{i}xy + \hat{j}(x^2 + kz^3)$

$\hookrightarrow \vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & Cx^2 & z^3 \end{vmatrix} = k\,(2Cx - x)$

$2Cx - x = 0 \longrightarrow C = \frac{1}{2}$

b) $F = \hat{i}\,(z/y) + c\hat{j}\,(xz/y^2) + k\,(x/y)$

$\rightarrow \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & Cx^2 & z^3 \end{vmatrix} = k$

c) $F = \hat{i}\,(z/y) + c\hat{j}\,(xz/y^2) + k\,(x/y)$

$\rightarrow \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{z}{y} & \frac{Cxz}{y^2} & \frac{x}{y} \end{vmatrix} = \hat{i}\left(\frac{-x}{y} - \frac{Cx}{y^2}\right) + \hat{j}\left(\frac{1}{y} - \frac{1}{y}\right) + \hat{k}\left(\frac{Cz}{y^2} + \frac{z}{y^2}\right)$

$= -\frac{x}{y^2} - \frac{Cx}{y^2} = 0 \longrightarrow C = -1$

Juga $\frac{Cz}{y^2} + \frac{z}{y^3} = 0 \longrightarrow C = -1$

(4.5) a) $F = \hat{i}x + \hat{j}y$

$\rightarrow \vec{F} = \hat{i}x + \hat{j}y$

di $x = y$; $\quad \vec{dr} = \hat{i}\,dx + \hat{j}\,dy$

$\int_{(0,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = \int_0^1 F_x\,dx + \int_0^1 F_y\,dy = \int_0^1 x\,dx + \int_0^1 y\,dy = -1$

Pada lintaran sepanjang rumbu x $= \vec{dr} = \hat{i}\,dx$ dan pada garis $x = 1$

$= \vec{dr} = \hat{j}\,dy$

$\int_{(0,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = \int_0^1 F_x dx + \int_0^1 F_y\,dy = 1$, $\vec{F}$ = konservatif

b) $\vec{F} = \hat{i}y + \hat{j}x$

$\rightarrow$ di $x = y$; $\int_{(0,0)}^{(1,1)} \vec{F} \cdot \vec{dr} \cdot \int_0^1 F_x\,dx + \int_0^1 F_y\,dy = \int_0^1 y\,dx - \int_0^1 x\,dy$

dan, dengan $x = y$ $\int_{(0,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = \int_0^1 x\,dx - \int_0^1 y\,dy = 0$

pada rumbu x $\int_{(0,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = \int_0^1 F_x\,dx = \int_0^1 y\,dx$

dengan $y = 0$, pada rumbu x $\int_{(0,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = 0$

pada garis $x = 1$ $\int_{(1,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = \int_0^1 F_y\,dx = \int_0^1 x\,dy$

pada $x = 1$ $\int_{(1,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = \int_0^1 dy = 1$

pada jalur ini $\int_{(0,0)}^{(1,1)} \vec{F} \cdot \vec{dr} = 0 + 1 = 1$, $\vec{F}$ = tidak konservatif.

(4.4) a) E = kostan = $V_{(x,y,z)} + \frac{1}{2} mv^2$

diawalnya $\Rightarrow E = 0 + \frac{1}{2} mv^2$

pada(1,1,1) $E = \alpha + \beta + \gamma + \frac{1}{2} mv^2 \neq \frac{1}{2} mv^2$

b) $V_0 - \frac{2}{m}(\alpha + \beta + \gamma) = 0$

$V_0 = \left[\frac{2}{m}(\alpha + \beta + \gamma)\right]^{\frac{1}{2}}$

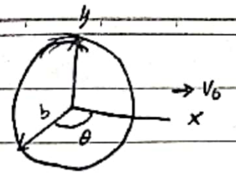
c) $m\ddot{x} = F_x = -\frac{\partial V}{\partial x}$

$m\ddot{x} = -\alpha$

$m\ddot{y} = -\frac{\partial V}{\partial y} = -2\beta y$

$m\ddot{z} = -\frac{\partial V}{\partial z} = -3\gamma z^2$

(4.7)



$\bar{r} = \hat{i}\, b \cos\theta - \hat{j}\, b \sin\theta$

$\theta = \omega t = \dfrac{V_0 t}{b}$ = Jadi, $\vec{r} = -\hat{i}\, V_0 \sin\theta - \hat{j}\, V_0 \cos\theta$

= relatif ke tanah, $\vec{V} = \hat{i}\, V_0 (1 - \sin\theta) - \hat{j}\, V_0 \cos\theta$

Partikel keluar tepi = $y_0 = -b\sin\theta$ dan $V_{0y} = -V_0 \cos\theta$

Jadi, $V_y = V_{0y} - gt = -V_0 \cos\theta - gt$ dan $y = -b\sin\theta - V_0 t \cos\theta - \frac{1}{2} g t^2$

tinggi maks $V_y = 0$

$t = -\dfrac{V_0 \cos\theta}{g}$

$h = -b\sin\theta - V_0 \left(-\dfrac{V_0 \cos\theta}{g}\right)\cos\theta - \frac{1}{2} g \left(-\dfrac{V_0 \cos\theta}{g}\right)^2$

$h = -b\sin\theta = \dfrac{V_0^2 \cos^2\theta}{2g}$

h maks terjadi untuk $\dfrac{dh}{d\theta} = 0 = -b\cos\theta - \dfrac{2V_0^2 \cos\theta \sin\theta}{2g}$

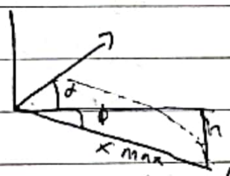
$\sin\theta = -\dfrac{gb}{V_0^2}$

$\cos^2\theta = 1 - \sin^2\theta = \dfrac{V_0^4 - g^2 b^2}{V_0^4}$

$h\ max = gb^2/V_0^2 + \dfrac{V_0^4\ g^2 b^2}{2g V_0^2} = \dfrac{gb^2}{2V_0^2} + \dfrac{V_0^2}{2g}$

lumpur meninggalkan noda di $\theta = \sin^{-1}\left(-\dfrac{gb}{V_0^2}\right)$

(4.9) a)



$R = \dfrac{2V_0^2}{g} \dfrac{\cos\alpha \sin(\alpha + \phi)}{\cos^2\phi}$

$-b = \cdots$ R maks $= V_0^2 (1 + \sin\phi$ dan $2\alpha = \frac{\pi}{2} = \phi$

$R\,max = \dfrac{h}{\sin\phi} = \dfrac{V_0^2}{g} \dfrac{1 + \sin\phi}{\cos^2\phi} = \dfrac{V_0^2}{g(\sin\phi)}$

$\sin\phi = \dfrac{gh}{V_0^2} \Big/ \left(1 + \dfrac{gh}{V_0^2}\right)$

$\sin\phi = \sin\left(\frac{\pi}{2} - 2\alpha\right) = \cos 2\alpha = 1 - 2\sin^2\alpha$

$1 - 2\sin^2\alpha = \dfrac{gh}{V_0^2} \Big/ \left(1 + \dfrac{gh}{V_0^2}\right)$

$2\sin^2\alpha = \dfrac{2}{\csc^2\alpha} = 1 - \dfrac{gh}{V_0^2} \Big/ \left(1 + \dfrac{gh}{V_0^2}\right) = \dfrac{1}{1 + \frac{gh}{V_0^2}}$

akhirnya $= \csc^2\alpha = 2\left(1 + \dfrac{gh}{V_0^2}\right)$

b) R maks $= \dfrac{h}{\sin\phi} = \dfrac{h}{1 - 2\sin^2\alpha} = \dfrac{h}{1 - 2/\csc^2\alpha}$

$R\,mak = \dfrac{V_0^2}{g}\left(1 + \dfrac{gh}{V_0^2}\right)$

(4.12) x dan z adalah posisi bola

$x = V_0 t \cos\frac{1}{2}\theta_0$, $z = V_0 t \cos\frac{1}{2}\theta_0 \sin\theta_0 - \frac{1}{2} g t^2$

sejak $V_x = V_0 \cos\frac{1}{2}\theta$,

$R = V_0^2/g \cos^2 \frac{1}{2}\theta^2{}_0 \sin 2\theta_0$

maks $\dfrac{dR}{d\theta} = 0$

$\dfrac{dR}{d\theta} : \dfrac{V_0^2}{g}\left(2\cos^2\frac{1}{2}\theta, \cos 2\theta_0 - \cos\frac{1}{2}\theta_0 \sin\frac{1}{2}\theta \sin 2\theta_0\right)^2 = 0$

$2\cos^2\frac{1}{2}\theta_0 \cos\theta_0 = \cos\frac{1}{2}\theta_0 \sin\frac{1}{2}\theta_0 \sin 2\theta_0$

$2\cos^2\theta_0 = 1 + \cos 2\theta_0$ dan $\sin\theta = 2\sin\theta_0 \sin 2\theta_0$

didapat : $(1 + \cos\theta_0)(2\cos^2\theta_0 - 1) = \sin\theta_0 \sin\theta_0 \cos\theta_0 = (1 - \cos^2\theta \cos\theta_0)$

atau $(1 + \cos\theta_0)(3\cos^2\theta_0 - \cos\theta_0 - 1) = 0$

hanya akar positif yang berlaku adalah θ, $0 \le \theta \le \frac{\pi}{2}$

$\cos\theta_0 = \frac{1}{6}(1 + \sqrt{13}) = 0{,}7676$ $\theta_0 = 39°51°$

b) untuk $V_0 = 25$ m/s $R_{max} = 55,4$ m $\theta_0 = 39^\circ 51^\circ$, maks $\frac{dz}{dt} = 0$

$V_0 \cos \frac{1}{2}\theta_0 \sin \theta_0 = gT$ atau $T = \frac{V_0 \cos 1/2 \, \theta_0 \sin \theta_0}{g}$

atau $H = \frac{V_0^2}{2g} \cos^2 \frac{1}{2}\theta_0 \sin^2 \theta$, maks. tetap di θ.

Maks pada ketinggian tetap terjadi $\frac{dH}{d\theta} = 0$

$\frac{dH}{d\theta} = \frac{V_0^2}{2g}\left(2\cos^2 \frac{1}{2}\theta_0 \sin \theta_0 \cos \theta_0 - \cos \frac{1}{2}\theta_0 \sin \frac{1}{2}\theta_0 \sin^2 \theta\right) = 0$

$(1 + \cos\theta_0)\sin\theta_0 \cos\theta_0 = \frac{1}{2}\sin\theta_0 \sin^2\theta_0 = \frac{1}{2}\sin\theta_0 (1 - \cos^2\theta_0)$ /

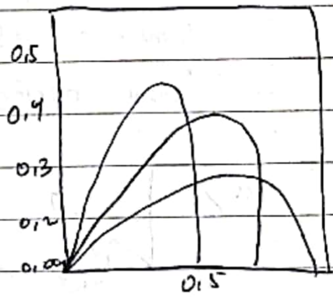
$\sin\theta_0 (1 + \cos\theta_0)(3\cos\theta_0 - 1) = 0$

$\sin\theta_0 = 0$ $\cos\theta_0 = -1$

Jadi H maks : 18,9 m $\theta_0 = \cos^{-1}\frac{1}{3} = 70,32^\circ$

(4.13) r menggantikan x

$z = \frac{\dot{z}_0}{\dot{r}_0} r - \frac{g}{2\dot{r}_0^2} r^2$ dimana $\dot{r}_0 = V_0 \cos\theta_0$ $\dot{z} = V_0 \sin\theta_0$



$z = r\tan\theta_0 - \frac{gr^2}{2V_0^2}\sec^2\theta$ sejak $\sec^2\theta_0 = 1 + \tan^2\theta$

$\frac{gr^2}{2V_0^2}\tan^2\theta_0 - r\tan\theta_0 + z + \frac{gr^2}{2V_0^2} = 0$

(r, z) titik koordinat

$\tan\theta_0 = \frac{1}{gr}\left[V_0^2 \pm (V_0^4 - 2zV_0^2 - g^2r^2)^{1/2}\right]$

$V_0^4 - 2gzV_0^2 - g^2r^2 \geq 0$

Permukaan kritis oleh karena itu $V_0^4 - 2gz - g^2r^2 = 0$

(4.17) $m\ddot{x} = F_x = -\frac{\partial U}{\partial x} = -kx = -\pi^2 mx$

$\ddot{x} = A\cos\left(\sqrt{\frac{k}{m}} + t\alpha\right) = A\cos(\pi\alpha + \alpha)$

$m\ddot{y} = \frac{\partial V}{\partial y} = 4\pi^2 m\alpha$

$y = B\cos(2\pi t + \beta)$

$m\ddot{z} = -\frac{\partial U}{\partial z} = -9\pi^2 mz$

$z = \cos(3\pi t + \gamma)$

sejak $x = y = z = 0$ pada $t = 0$ $\alpha = \beta = \gamma = -\frac{\pi}{2}$

$x = A\cos(\pi t - \frac{\pi}{2}) = A\sin\pi t$, $\dot{x}_0 = A\pi\cos\pi t$

$V_0^2 = \dot{x}_0^2 + \dot{y}_0^2 + \dot{z}_0^2$ dan $\dot{x}_0 = \dot{y}_0 = \dot{z}_0$

$\dot{x}_0 = \frac{V_0}{\sqrt{3}} = A\pi$

$A = \frac{V_0}{\pi\sqrt{3}}$

$x = \frac{V_0}{\pi\sqrt{3}}\sin\pi t$

$y = B\sin 2\pi t$, $\dot{y} = 2B\pi\cos 2\pi t$

$\dot{y}_0 = \frac{V_0}{\sqrt{3}} = 2\pi$

$B = \frac{V_0}{2\pi\sqrt{3}}$

$y = \frac{V_0}{4\pi\sqrt{3}}\sin 2\pi t$

$z = C\sin 3\pi t$

$\dot{z}_0 = \frac{V_0}{\sqrt{3}} = 3\pi t$ $\dot{z} = 3C\pi\cos 3\pi t$

$$\dot{z}_0 = \frac{v_0}{\sqrt{3}} = 3C\pi$$

$$C = \frac{v_0}{3\pi\sqrt{3}}$$

$$z = \frac{v_0}{3\pi\sqrt{3}} \sin \pi t$$

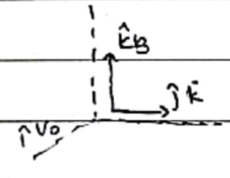
sejak $\omega_x = \pi$. $\omega_y = 2\pi$ dan $\omega_z = 3\pi$,

maka $T\ min = \frac{2\pi n_1}{\omega_x} = \frac{2\pi n_2}{\omega_y} = \frac{2\pi n_3}{\omega_z}$

waktu maks pada $n_1 = 1$, $n_2 = 2$, dan $n_3 = 3$.

$$t_{min} = \frac{2\pi}{\pi} = 2$$

(4.20)



$$\bar{F} = q(\hat{E} + \vec{v} + \vec{B})$$

$$\vec{V} \times \vec{B} = (\hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z}) \times kB = \hat{i}\,\dot{y}B - \hat{j}\dot{x}B$$

$$\bar{F} = \hat{i}\,q\dot{y}B + \hat{j}\,q(E - \dot{x}B)$$

$$m\ddot{x} = F_x = q\dot{y}B$$

$$\dot{x} - \dot{x}_0 = \frac{qB}{m} y$$

$$m\ddot{y} = F_y = qE - q\dot{x}B = qE - qB\left(\dot{x}_0 + \frac{qB}{m} y\right)$$

$$\ddot{y} = qE/m - \frac{qB\dot{x}}{m} - \left(\frac{qB}{m}\right)^2 y = \frac{eE}{m} + \frac{EB\dot{x}}{m} - \left(\frac{eB}{m}\right)^2 y$$

$$\ddot{y} + \omega^2 y = -\frac{eE}{m} + \omega\dot{x}_0$$

$$y = \frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega x\right) + A\cos(\omega t + \theta_0)$$

$$\dot{y} = -A\,\omega \sin(\omega t + \theta)$$

$$\dot{y}_0 = 0, \text{ Jadi } \theta_0 = 0,\ A = -\frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}_0\right)$$

$$y = a(1 - \cos\omega t),\ a = \frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}_0\right)$$

$$\dot{x} = \dot{x}_0 = \frac{qB}{m} y = \dot{x} - \omega y = \dot{x}_0 - \omega a(1 - \cos\omega t)$$

$$x = (\dot{x}_0 - \omega a)t + a\sin\omega t$$

$$x = a\sin\omega t + bt \qquad b = \dot{x}_0 - \omega a$$

$$m\ddot{z} = F_z = 0, \quad z = \dot{z}_0 t + z_0 = \underline{\underline{0}}$$