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4.1 find the force each of the following potensial ~~energi~~ energi funcitions

a. $V = Cxyz + c$

b. $V = \alpha x^2 + \beta y^2 + \gamma z^2 + c$

c. $V = Ce^{-(\alpha x + \beta y + \gamma z^2)}$

d. $V = Cr^n$ in spherical coordinates

Answer :

a. $\vec{F} = -\vec{\nabla} V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$

$\vec{F} = -C \langle \hat{i}yz + \hat{j}xz + \hat{k}xy \rangle$

b. $\vec{F} = -C\vec{\nabla} V = -\hat{i}\,2\alpha x - \hat{j}\,2\beta y - \hat{k}\,2\gamma z$

c. $\vec{F} = -\vec{\nabla} V = Ce^{-(\alpha x + \beta y + \gamma z)}(\hat{i}\alpha + \hat{j}\beta + \hat{k}\gamma)$

d. $\vec{F} = -\vec{\nabla} V = -\hat{e} \cdot \frac{\partial V}{\partial r} - \hat{e}_\theta \frac{1}{r}\frac{\partial V}{\partial \theta} - \hat{e}_\theta \frac{1}{r\sin\theta}\frac{\partial V}{\partial \theta}$

$\vec{F} = -\hat{e} \cdot Cnr^{n-1}$

4.2 By finding the curl, determine which of the following forces are conservative :

a. $F = \hat{i}x + \hat{j}y + kz$

b. $F = iy - jx + kz^2$

c. $F = iy + jx + kz^2$

d. $F = -kr^{-n} e_r$ in spherical coodinates

Answer :

a)

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0$$

d.)

$$\vec{\nabla} \times \vec{F} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{e}_r & \hat{e}_{\theta r} & \hat{e}_{\theta r}\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \theta} \\ -kr^{-n} & 0 & 0 \end{vmatrix} = 0$$

b

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & j & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x & z^2 \end{vmatrix} = \hat{k}(1-1) \neq 0$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} i & j & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x & z^2 \end{vmatrix} = \hat{k}(1-1) = 0$$

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4.3. find the value of the constant c such that each of the following forces is the conservative:

a. $f = ixy + jcx^2 + kz^2$

b. $f = i(z/y) + cj(xz/y^2) + k(x/y)$

Answer:

a.
$$\vec{\nabla}\times\vec{f} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & cx^2 & z^3 \end{vmatrix} = k(2cx - x)$$

$= 2cx - x = 0$

$c = \frac{1}{2}$

b.
$$\vec{\nabla}\times\vec{f} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{z}{y} & cx^2 & \frac{x}{y} \end{vmatrix}$$

$$= i\left[-\frac{x}{y^2} - \frac{cx}{y^2}\right] + j\left[\frac{1}{y} - \frac{1}{y}\right] + \hat{k}\left[\frac{cz}{y^2} + \frac{z}{y^2}\right] - \frac{x}{y^2} - \frac{cx}{y^2} = 0 \quad c = 1$$

also $\frac{cz}{y^2} + \frac{z}{y^2} = 0$ implies that $c = -1$ at it mast

4.4. A particle of massa m moving in the three dimensions under the potential energy function $V(x,y,z) = \alpha x + \beta yz^2 + \gamma z^3$ has speed if and when it passes through the origin

a. What whill its speed V_0 when it passes though the point(1,1,1)?

b. If the point (1,1,1) is a turning point in the motion (V=0) what is V_0 = ?

c. What are the component differontial equations of montion of the parcticel) ?

(Note : It is not necessay to save the differential equation in this problem).

Answer:

a. $E = \text{Constant} = V(x,y,z) + \frac{1}{2}mV^2$

at the origin $E = 0 + \frac{1}{2}mV^2$

at $(1,1,1)$ $E = \alpha + \beta + \gamma + \frac{1}{2}mV^2 = \frac{1}{2}mV^2$

$V = [V^2 - \frac{2}{m}(\alpha + \beta + \gamma)$

$V = [V^2 - \frac{2}{m}(\alpha + \beta + \gamma)^{\frac{1}{2}}$

b. $V^2 - \frac{2}{m}(\alpha + \beta + \gamma) = 0$

$V = [\frac{2}{m}(\alpha + \beta + \gamma)]^{\frac{1}{2}}$

c. $m\ddot{x} = -\alpha$

$m\ddot{y} = -\frac{\partial V}{\partial y} = -2\beta y$

$m\ddot{z} = -\frac{\partial V}{\partial z} = -3\gamma z^2$

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4.5. Consider the two functions

a. $f = ix + jy$

b. $f = iy - jx$

verify that (a) is conservative and that (b) is noncorservative by showing that the integral $\int r \cdot dr$ is indefendent of the path of integration for (a) but not for (b) by talking two paths in wich the starting points is the origin (0,0) and the and point is (1,1) for one path take the line $x = y$. for the other path take x-axis out to the points (1,0) and the lime $x = 1$ up to the point (1,1)

Answer:

a.) $\bar{f} = \hat{i}x + jy$

on the path $x = y$ $\qquad d\vec{r} = \hat{i}dx + \hat{j}dy$

$$\int_{(0,0)}^{(1,1)} \bar{f} \cdot d\bar{r} = \int_0^1 fx\,dx + \int_0^1 fy\,dy = \int_0^1 x\,dx + \int_0^1 y\,dy = 1$$

on the path along the x-axis: $d\vec{r} = i\,dx$

and on the line $x = 1$ $\quad$: $d\vec{r} = \hat{j}\,dy$

$$\int_{(0,0)}^{(1,1)} \bar{f} \cdot d\bar{r} = \int_1^1 fx\,dx + \int_0^1 y\,dy = 1$$

$\bar{f}$ is conservative

b). $f = iy - jx$

on the path $x = y$.

$$\int_{(0,0)}^{(1,1)} f\,d\bar{r} = \int_0^1 fx\,dx + \int_0^1 fy\,dy = \int_0^1 y\,dx - \int_0^1 x\,dy$$

and with $x = y$ $\int_{0,0}^{1,1} \bar{f} \cdot d\bar{r} = \int_0^1 x\,dx - \int_0^1 y\,dy = 0$

on the x-azis $\int_{0,0}^{1,0} f \cdot dr = \int_0^1 fx\,dx = \int_0^1 y\,dx$

and, with $y = 0$ on the x-axis $\int_{0,0}^{1,0} \bar{f} \cdot d\bar{r} = 0$

one the line $x = 1$ $\int_{0,0}^{1,1} \bar{f} \cdot d\bar{r} = \int_0^1 fy\,dy = \int_0^1 x\,dy$

and with $x = 1$ $\int_{0,0}^{1,1} f\,d\bar{r} = \int dy = 1$

On this path $= \int_{0,0}^{1,1} f \cdot dr = 0 + 1 = 1$

f is not conservative.

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4.6 Show that the variation of grafity with beight can be accuanted for approximaty by the following potensial energy function.

$$V = mgz\left(1 - \frac{z}{r_e}\right)$$

In which r_e is the radius of the earth. find the force given by the above potensial function. from this find the component differensial equations of montion, of a projective under such a force. if the vertical komponent of the initial velocity is V_{0z}, how high does the projectile go (compare with example 2.3.2)

Answer:

from exampel 2.3.2 $V(z) = -mg\frac{r_e^2}{(r_e+z)}$

$$V(z) = -mg\, r_e\, (1+z)^{-1}$$

from Appendix D $(1+x)^{-1} = 1 - x + x^2 + r_e$

$$V(z) = -mgr \cdot \left[1 - \frac{z}{r_e} + \frac{z^2}{r^2 e} + \cdots\right]$$

$$V(z) = -mgr_e + mgz - \frac{mgz^2}{r_z} + \cdots$$

whith $-mgr_e$ an anaditive constant

$$V(z) = mgz\left(1 - \frac{z}{r_e}\right)$$

$$\bar{f} = -\bar{\nabla} V = -\hat{k}\frac{d}{dz}V(z)$$

$$= -\hat{k}mg\left[1 - \frac{z}{r_e} + z\left(-\frac{1}{r_e}\right)\right]$$

$$\vec{f} = -\hat{k}mg\left(1 - \frac{2z}{r_e}\right)$$

$m\ddot{x} = f_x = 0 \quad m\ddot{y} = f_y = 0 \quad m\ddot{z} = -mg\left(1 - \frac{z^2}{r_e}\right)$

$$m\dot{z}\frac{d\dot{z}}{dz} = mg\left(1 - \frac{2z}{r_e}\right)$$

$$\int_v^0 \dot{z}\,dz = -g\int_0^1 \left(1 - \frac{2z}{r_e}\right)de$$

$$-\frac{1}{2}V_z^2 = -g\left(h - \frac{h^2}{r_e}\right)$$

$$h^2 = r_e h + \frac{r_e v_z^2}{2g} = 0$$

$$h = \frac{r_e}{2} - \frac{1}{2}\sqrt{r_e^2 - 2r_e\frac{V_z^2}{g}} \; - (h \cdot z(r_e))$$

$$h = \frac{r_e}{2} - \frac{r_e}{2}\sqrt{1 - \frac{2V_e^2}{gr_e}}$$

from appendix D $(1+x)^{\frac{1}{2}} = 1 + \frac{x}{2} - \frac{x^2}{8} + \cdots$

$$h = \frac{r_e}{2} - \frac{r_e}{2} + \frac{V_{0z}^2}{2g} + \frac{V_{0z}^4}{4g^2 r_e} + \cdots$$

$$h = \frac{V_{0z}^2}{2g}\left(1 + \frac{V_{0z}^2}{2gr_e}\right)$$

from example 2.3.2 $h = \frac{V_{0z}^2}{2g}\left(1 - \frac{V_z^2}{2gr}\right)^{-1}$

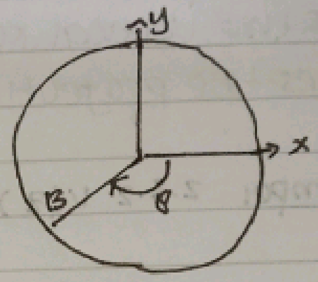
and witch $(1-x)^{-1} = 1 + x$, $h = \frac{V_z^2}{2g}\left(1 + \frac{V_z^2}{2gr}\right)$

4.7 particles of mud are throw from the rim of a rolling wheel, If the greatest height above the ground that the mud can go:

$$b + \frac{v^2}{2g} + \frac{gb^2}{2v_0^2}$$

at what point on the rolling wheel does this mud leave? (note: It is necessary to assume that $v_0^2 \geq bg$)

Answer:



for a point rim meansurea from the Center of the wheel.

$\vec{r} = \hat{i}b\cos\theta - \hat{j}b\sin\theta$

$\theta = \omega t = \frac{vt}{b}$ so $\bar{r} = -\hat{i}v\sin\theta - \hat{j}v\cos\theta$

relative to the ground

$$\vec{v} = \hat{i}v(1-\sin\theta) - \hat{j}v\cos\theta$$

for a particle of mud leaving the rim:

$y = -b\sin\theta$ and, $v_y = -v_1\cos\theta$

$v_y = v_{0y} - gt = -v_1\cos\theta - gt$

$y = -b\sin\theta - v_0 t\cos\theta - \frac{1}{2}gt^2$

maximum height $v_y = 0$

$$t = -\frac{v_0\cos\theta}{g}$$

$$h = -b\sin\theta - v_0\left(\frac{-v_0\cos\theta}{g}\right)\cos\theta \frac{1}{2}g.\left(\frac{-v_0\cos\theta}{g}\right)$$

$$h = -b\sin\theta + \frac{v_1^2\cos^2\theta}{2g}$$

$$\sin\theta = \frac{-gb}{v^2}$$

$$\cos^2\theta = 1 - \sin^2\theta = \frac{v^2 g^2 b^2}{v^4}$$

$$h_{max} = \frac{gb^2}{v^2} + \frac{v^4 - g^2b^2}{2gv^2} = \frac{gb^2}{2v^2} + \frac{v^2}{2g}$$

meansured from the ground

$$h_{max} = b + \frac{gb^2}{2v^2} + \frac{v^2}{2g}$$

the mud leaves the wheel at $\theta = \sin^{-1}\left(-\frac{gb}{v^2}\right)$

4.8. A gun is located at the bottom of a hill of constant slope $\emptyset$. Show that the range of the gun meansured up the slope the hiils.

$$\frac{2v_0^2\cos\alpha\sin(\alpha-\emptyset)}{g\cos^2\emptyset}$$

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where α is the engle elevation of the gun and that the maximum value of the slope range is

$$\frac{V_0^2}{g(1+\sin\theta)}$$

Answer :

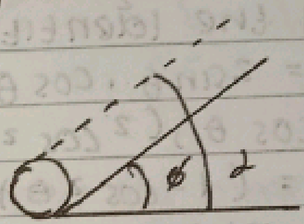
$x = R\cos\theta$ dan $x = V.t.(V_0\cos\alpha)t$

Jadi $t = \dfrac{R\cos\theta}{V\cos\alpha}$

$y = R\sin\theta$ dan $y = V.t - \frac{1}{2}gt^2 = (V.\sin\alpha)t - \frac{1}{2}gt^2$

$R\sin\theta = (V_0\sin\alpha)\dfrac{R\cos\theta}{V_0\cos\alpha} - \frac{1}{2}g\left(\dfrac{R\cos\theta}{V_0\cos\alpha}\right)^2$

$\sin\theta = \tan\alpha\cos\theta - \dfrac{gR\cos^2\theta}{2V_0^2\cos^2\alpha}$



$R = \dfrac{2V_2^{\,2}\cos^2\alpha}{g\cos^2\theta}(\tan\alpha\cos\theta - \sin\theta)$

$= \dfrac{2V_0^2\cos\alpha}{g\cos^2\theta}(\sin\alpha\cos\theta - \cos\alpha\sin\theta)$

Dari Lampiran B. $\sin(\theta+\theta) = 0$

Implies that $\cos a\cos(a-\theta) = \sin\alpha\sin(\alpha-\theta) = 0$

from appendix B. $\cos(0+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$

So $\cos(2\alpha-\theta) = 0$

$2\alpha - \theta = \frac{\pi}{2}$ $\quad \alpha = \frac{\pi}{4} + \frac{\theta}{2}$

again using Appendix B, $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$R_{max} = \dfrac{2V^2}{g\cos^2\theta}\left(\frac{1}{2}\cos\left(\frac{\pi}{2}+\theta\right)\right] + \frac{1}{2}\right) = \dfrac{V_0^2}{g\cos^2\theta}\left(\cos\left(\frac{\pi}{2}+\theta\right)+1\right)$

using $\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$R_{max} = \dfrac{V^2}{g(1-\sin^2\theta)}(1-\sin\theta)$

$R_{max} = \dfrac{V^2}{g(1+\sin\theta)}$

4.12 A Baseball pitcher can throw a ball more easily horizontally than vertically, assume that the pitcher's throwing speed varies with elevation angle approximately as $V_0\cos\frac{1}{2}\theta_0$ m/s, where θ_0 is the initial when the bal is thrown horizontally. fin the angle θ_0 at which the ball must be thrown to achieve maximum (a) hight and (b). range fina the values of the maximum (c) height and (d) range, assume no air resistano and let $V_0 = 25$ m/s

Answer :

The x and z positions of the ball vs time are

$x = V_1 t \cos\frac{1}{2}\theta$ $\qquad z = V_1 t \cos\frac{1}{2}\theta \sin\theta \ \frac{1}{2}gr^2$

since $V_2 = V_0 \cos\frac{1}{2}\theta$

The horizontal range is $R = \frac{V^2}{g}\cos^2\frac{1}{2}\theta \, . \sin 2\theta$

The maximum range $\quad a : \frac{dR}{d\theta} = 0$

$$\frac{dr}{d\theta} = \frac{V^2}{g}\left(2\cos^2\frac{1}{2}\theta \, . \cos 2\theta - \frac{k}{2}\cos\frac{1}{2}\theta \, . \sin\frac{1}{2}\theta \, . \sin 2\theta\right) = 0$$

thus $= 2\cos^2\frac{1}{2}\cos 2\theta = \cos\frac{1}{2}\theta \sin\frac{1}{2}\theta \, , \sin 2\theta$

using the identities $2\cos^2\theta = 1 \mp \cos 2\theta$ and $\sin 2\theta$

$= 2\sin\theta \, . \cos\theta$, we get

$(1+\cos\theta)(2\cos^2\theta - 1) = \sin\theta \, . \sin\theta \, . \cos\theta$

$= (1-\cos^2\theta)\cos\theta$. or

$(1+\cos\theta)(3\cos^2\theta - \cos\theta - 1) = 0$

This $\cos\theta_1 = -1 \quad \cos\theta = \frac{1}{6}(1 \pm \sqrt{3})$

only the positive root applies for the θ range $= 0 \leq 0, \frac{\pi}{2}$

$\cos\theta_1 = \frac{1}{6}(1+\sqrt{13}) = 0.7676 \ \theta_0 = 39'51$

Thus (b) for $V_6 = 25\ ms^{-2}$ $R_{max} = 55.4m \qquad \theta_0 = 39'51$

(a) The maximum height occurs at $\frac{dz}{dt} = 0$

$V_0 \cos\frac{1}{2}\theta = gT \qquad$ or at $T = \frac{V\cos t\ \theta, \sin\theta}{g}$

or $H = \frac{V_0^2}{2g}\cos^2\frac{1}{2}\theta \, , \sin^2\theta$ maximum at fixed θ

The maximum possible height occurs $\frac{dH}{da} = 0$

$\frac{dH}{da} = \frac{V^2}{\ }$

4.13 A gun can fire an artellery shell with a sped V_0 in any direction. Show that a shell can strike any target within the surface given by.

$$g^2 r^2 = V_0^4 - 2gV_0^2 z$$

Where z is the height of the target and r is it's horizontal distance from the gun. assume no air resistane

Answer :

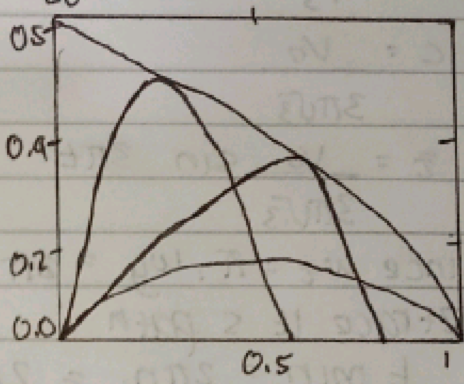
The trajectory of the shell is given by Eq 4.3.11 with r replacing x

$z = \frac{z}{r} r - \frac{g}{2r^2} r^2$ who $r_0 = V_0 \cos\theta_0 \quad z = V_0 \sin\theta$

Thus, $z = r\tan\theta - \frac{gr^2}{2v^2}\sec^2\theta$

since $\sec^2\theta = 1 + \tan^2\theta$

we have : $\frac{gr^2}{2v^2}\tan^2\theta - r^2\tan\theta + z + \frac{gr^2}{2v^2} = 0$



(r, z) are target coordinates, the above equation yields two possible roots:

$\tan\theta = \frac{1}{gr}\left[v^2 \pm (v^4 \cdot 2gv^2 - g^2r^2)^{\frac{1}{2}}\right]$

the roots are only real if

$v^4, - 2gzv^2 - g^2r^2 \geq 0$

the critical surface is therefore :

$v^4 - 2gzv^2 - g^2r^2 = 0$

4.19 A small lead ball of mass m is suspended by means of six light springs as shown in figure 4.4.1 The stiffness constants are in the ration 1:4:9, so that the potensial energy function can be expressed as

$$V = \frac{V}{2}(x^2 + 4y^2 + 9z^2)$$

a time $t = 0$ the ball receives a pust in the $(1,1,1)$ Direction that imparts to it a speed V_0 at the origin.

Answer.

$m\ddot{x} = f_1 = -\frac{dV}{dx} = -kx = -\pi^2 mx$

$\dot{x} = A\cos\left(\sqrt{\frac{k}{m}}\,t + \alpha\right) = A\cos(\pi t + \alpha)$

$m\ddot{y} = -\frac{\partial U}{\partial y} = -9\pi^2 mz$

$z = C\cos(3\pi t + y)$

Since $x = y = z = 0$ at $t = 0$ $\quad \alpha = \beta = y = -\frac{\pi}{2}$

$x = A\cos(\pi t - \frac{\pi}{2}) = A\sin\pi t$

$x = A\pi\cos\pi t$

$V_0^2 = x^2 + y^2 + z^2$ an $\dot{x} = \dot{y} = \dot{z}$

$\dot{x} = \frac{V_0}{\sqrt{2}} = A\pi$

$A = \frac{V_0}{A\sqrt{3}}$

$x = \frac{V_0}{\pi\sqrt{3}} \cdot \sin\pi t$

$y = \beta\sin 2\pi t \quad \dot{y} = 2\beta\pi\cos^2\pi t$

$y_0 = \frac{V_0}{2\pi\sqrt{3}}$

$$B = \frac{V_0}{2\pi\sqrt{3}} \sin 2\pi t$$

$$z = C \sin 3\pi t \qquad \dot{z} = 3C\pi \cos 3\pi t$$

$$\dot{z} : \frac{V_0}{\sqrt{3}} = 3C\pi$$

$$C = \frac{V_0}{3\pi\sqrt{3}}$$

$$z = \frac{V_0}{3\pi\sqrt{3}} \sin 3\pi t$$

Since $\omega_x = \pi$, $\omega_y = 2\pi$ and $\omega_z = 3\pi$ the ball does retrace it s path

$$t_{min} = \frac{2\pi n_1}{\omega_x} = \frac{2\pi n_2}{\omega_y} = \frac{2\pi n_3}{\omega_z}$$

the minimum time occurs at $n_1 = 1$ $n_2 = 2$ $n_3 = 3$

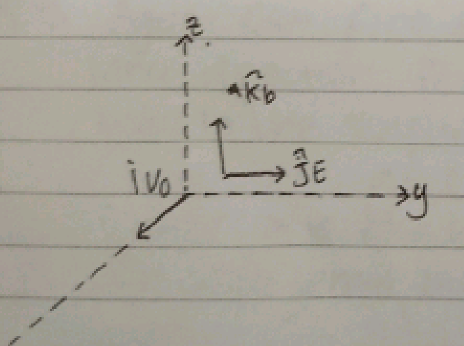
$$t_{min} = \frac{2\pi}{\pi} = 2$$

4.20. an electron maves in a force field due to a uniform Electric field E and a uniform magnetc flield B that is at right angles to E. let $E = \hat{j}E$ and $B = \hat{k}B$ take Initial position of the electron at the origin whith initial velocity $v_0 = \hat{i}v_0$ in the x direction. find the resulting motion of the particel. Show that the path of motion is a cyctoid:

$$x = a \sin \omega t + bt$$
$$y = a(1 - \cos \omega t)$$
$$z = 0$$

Cycloidal motion of electrons is used in an electronic tube called a magnetron to produce the microwave in a microwave

Answer:



$$\vec{f} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{v} \times \vec{B} = (\hat{i}\dot{x} + \hat{j}\dot{x} + \hat{k}\dot{z}) \times \hat{k}B = \hat{i}\dot{y}B - \hat{j}\dot{x}B$$

$$f = \hat{i}q\dot{y}B + \hat{j}q(E - \dot{x}B)$$

$$m\ddot{x} = f_x = q\dot{y}B$$

$$\dot{x} - \dot{x} = \frac{qB}{m} y$$

$$m\ddot{y} = f_y = qE - q\dot{x}B = qE - qB\left(\dot{x}_1 + \frac{qB}{m}y\right)$$

$$\ddot{y} = \frac{qE}{m} - \frac{qB\dot{x}}{m} - \left(\frac{qB}{m}\right)^2 y = \frac{-eE}{m} + \frac{eB\dot{x}}{m} - \left[\frac{EB}{m}\right]y^2$$

$$\ddot{y} + \omega^2 y = --\frac{eE}{m} + \omega\dot{x} \qquad \omega = \frac{eB}{m}$$

$$y = \frac{1}{\omega^2}\left(\frac{-eE}{m} + \omega x\right) + A\cos(\omega t - \theta)$$

$\dot{y} = A\omega \sin(\omega t + \theta)$
$\dot{y} = 0$, Jadi $\theta_0 = 0$
$y_0 = 0$, Jadi $A = -\frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}\right)$

$$\dot{x} = \dot{x}_1 + \frac{qB}{m}y = \dot{x}_1 - \omega y = \dot{x}_1 - \omega x(1 - \cos\omega t)$$

$\dot{x} = (\dot{x}_0 - \omega x) + \omega x \cos \omega t$
$x = (\dot{x}_0 - \omega x)t + d \sin \omega t$
$x = d \sin \omega t + bt$. $b = \dot{x} - \omega d$
$m\ddot{z} = f_z = 0$
$z = \dot{z} \cdot t + z_0 \neq 0$