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Prodi : Pendidikan Fisika
Kelas : B

Ayo Berlatih (General Motion of a Particle in Three Dimensions)

(4.1) a.) $V = Cxyz + C$

Jawab:

$$\vec{F} = -\vec{\nabla}V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z} = -C(\hat{i}yz + \hat{j}xz + \hat{k}xy)$$

b.) $V = \alpha x^2 + \beta y^2 + \gamma z^2 + C$

Jawab:

$$\vec{F} = -\vec{\nabla}V = -\hat{i}2\alpha x - \hat{j}2\beta y - \hat{k}2\gamma z$$

c.) $V = Ce^{-(\alpha x + \beta y + \gamma z)}$

Jawab:

$$\vec{F} = -\vec{\nabla}V = Ce^{-(\alpha x + \beta y + \gamma z)}(-\hat{i}\alpha - \hat{j}\beta - \hat{k}\gamma)$$

d.) $V = cr^n$ dalam koordinat bola

Jawab:

$$\vec{F} = -\vec{\nabla}V = -\hat{e}_r \frac{\partial V}{\partial r} - \hat{e}_\theta \frac{1}{r} \frac{\partial V}{\partial \theta} - \hat{e}_\phi \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi}$$

(4.2) a.) $F = \hat{i}x + \hat{j}y + \hat{k}z$

Jawab:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0 \rightarrow \text{konservatif.}$$

b.) $F = \hat{i}x - \hat{j}y + \hat{k}z^2$

Jawab:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -y & z^2 \end{vmatrix} = \hat{k}(-1-1) \neq 0 \rightarrow \text{tidak konservatif}$$

c.) $F = \hat{i}x + \hat{j}y + \hat{k}z^3$

Jawab:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z^3 \end{vmatrix} = \hat{k}(1-1) = 0 \rightarrow \text{konservatif}$$

d.) $\vec{F} = -kr^{-n} \hat{e}_r$ dalam koordinat bola

Jawab:

$$\vec{\nabla} \times \vec{F} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \hat{e}_r & \hat{e}_\theta & \hat{e}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ -kr & 0 & 0 \end{vmatrix} = 0 \rightarrow \text{konservatif}$$

4.3 a.) $F = \hat{i}xy + \hat{j}cx^2 + \hat{k}z^3$

Jawab:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & cx^2 & z^3 \end{vmatrix} = k(2cx - x)$$

$$2cx - x = 0 \rightarrow c = \frac{1}{2}$$

b.) $F = \hat{i}(z/y) + c\hat{j}(xz/y^2) + \hat{k}(x/y)$

Jawab:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & cx^2 & z^3 \end{vmatrix} = k$$

b.) $F = \hat{i}(z/y) + c\hat{j}(xz/y^2) + \hat{k}(x/y)$

Jawab:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{z}{y} & \frac{cxz}{y^2} & \frac{x}{y} \end{vmatrix} = \hat{i} \left(-\frac{x}{y} - \frac{cx}{y^2} \right) + \hat{j} \left(\frac{1}{y} - \frac{1}{y} \right) + \hat{k} \left(\frac{cz}{y^2} + \frac{z}{y^2} \right)$$

$$-\frac{x}{y} - \frac{cx}{y^2} = 0 \rightarrow c = -1$$

$$\text{juga } \frac{cz}{y^2} + \frac{z}{y^2} = 0 \rightarrow c = -1$$

4.4 a.) Jawaban: $E = \text{konstan} = V(x, y, z) + \frac{1}{2}mv^2$

$$\text{diasalnya, } E = 0 + \frac{1}{2}mv^2$$

$$\text{Pada } (1, 1, 1) \quad E = \alpha + \beta + \gamma + \frac{1}{2}mv^2 = \frac{1}{2}mv^2$$

b.) Jawaban : $V_0 = \frac{2}{m} (\alpha + \beta + \gamma) = 0$

$$V_0 = \left[\frac{2}{m} (\alpha + \beta + \gamma) \right]^{\frac{1}{2}}$$

c.) Jawaban : $m\ddot{x} = F_x = -\frac{2V}{2x}$

$$m\ddot{x} = -\alpha$$

$$m\ddot{y} = -\frac{2V}{2y} = -2\beta y$$

$$m\ddot{z} = -\frac{2V}{2z} = -3\gamma z^2$$

4.5 Pertimbangkan 2 fungsi gaya

a.) $\vec{F} = \hat{i}x + \hat{j}y$

Jawab :

$$\vec{F} = \hat{i}x + \hat{j}y$$

di $x = y$; $d\vec{r} = \hat{i} dx + \hat{j} dy$

$$\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_x dx + \int_0^1 F_y dy = \int_0^1 x dx + \int_0^1 y dy = -1$$

Pada lintasan sepanjang sumbu x : $d\vec{r} = \hat{i} dx$

dan pada garis $x = 1$ $d\vec{r} = \hat{j} dy$

$$\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_x dx + \int_0^1 F_y dy = 1, \vec{F} = \text{Konservatif}$$

b.) $\vec{F} = \hat{i}y - \hat{j}x$

di $x = y$: $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_x dx + \int_0^1 F_y dy = \int_0^1 y dx - \int_0^1 x dy$

dan, dengan $x = y$ $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 x dx - \int_0^1 y dy = 0$

pada sumbu x = $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_x dx = \int_0^1 y dx$

dan, dengan $y = 0$ pada sumbu x $\int_{(0,0)}^{(1,0)} \vec{F} \cdot d\vec{r} = 0$

Pada garis $x = 1$ $\int_{(1,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_y dy = \int_0^1 x dy$

dan, dengan $x = 1$ $\int_{(1,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 dy = 1$

pada jalur ini $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = 0 + 1 = 1$, $\vec{F}$ = tidak konservatif.

4.6. Jawaban :

Dari contoh 2.3.2, $V(z) = -mg \frac{r_c^2}{(r_c + z)}$

$$V(z) = -mg r_c \left(1 + \frac{z}{r_c}\right)^{-1}$$

$$V(z) = -mg r_c \left(1 - \frac{z}{r_c} + \frac{z^2}{r_c^2} + \dots\right)$$

$$V(z) = -mg r_c + mg z - \frac{mg z^2}{r_c} + \dots$$

dengan $-mg r_c$ sebuah konstanta

$$V(z) \approx mg z \left(1 - \frac{z}{r_c}\right)$$

$$\vec{F} = -\vec{\nabla} V = -\hat{k} \frac{\partial}{\partial z} V(z)$$

$$= -\hat{k} mg \left[1 - \frac{z}{r_c} + z \left(-\frac{1}{r_c}\right)\right]$$

$$\vec{F} = -\hat{k} mg \left(1 - \frac{2z}{r_c}\right)$$

$$m\ddot{x} = F_x = 0$$

$$m\dot{z} \frac{dz}{dz} = -mg \left(1 - \frac{2z}{r_c}\right)$$

$$\int_{V_z}^0 \dot{z} dz = -g \int_0^M \left(1 - \frac{2z}{r_c}\right) dz$$

$$-\frac{1}{2} V_{0z}^2 = -g \left(h - \frac{h^2}{r_c}\right)$$

$$h^2 - r_c h + \frac{r_c V_{0z}^2}{2g} = 0$$

$$h - r_c h + \frac{r_c V_{0z}^2}{2g} = 0$$

$$h = \frac{r_c}{2} - \frac{1}{2} \sqrt{r_c^2 - \frac{2r_c V_{0z}^2}{g r_c}} \quad (h, z \ll r_c)$$

$$h = \frac{r_c}{2} - \frac{r_c}{2} \sqrt{1 - \frac{2V_{0z}^2}{g r_c}}$$

$$(1+x)^{\frac{1}{2}} = 1 + \frac{x}{2} - \frac{x^2}{2} + \dots$$

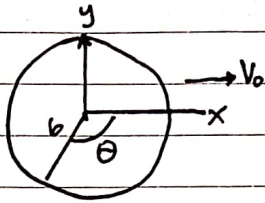
$$h = \frac{r_c}{2} - \frac{r_c}{2} + \frac{V_{0z}^2}{2g} + \frac{V_{0z}^4}{4g r_c} + \dots$$

$$h \approx \frac{V_{0z}^2}{2g} \left(1 + \frac{V_{0z}^2}{2g r_c}\right)$$

dari contoh 2.3.2 $h = \frac{V_{0z}^2}{2g} \left(1 - \frac{V_{0z}^2}{2g r_c}\right)^{-1}$

dan dengan $(1-x)^{-1} \approx 1+x$, $h \approx \frac{V_{0z}^2}{2g} \left(1 + \frac{V_{0z}^2}{2g r_c}\right)$

4.7) Jawaban :



$$\vec{r} = \hat{i} b \cos \theta - \hat{j} b \sin \theta$$

$$\theta = \omega t = \frac{V_0 t}{b} \quad \text{Jadi, } \vec{r} = -\hat{i} V_0 \sin \theta - \hat{j} V_0 \cos \theta$$

$$\text{relatif ke tanah, } \vec{v} = \hat{i} V_0 (1 - \sin \theta) - \hat{j} V_0 \cos \theta$$

partikel keluar tepi : $y_0 = -b \sin \theta$ dan $V_{0y} = -V_0 \cos \theta$

$$\text{Jadi, } V_y = V_{0y} - gt = -V_0 \cos \theta - gt \quad \text{dan } y = -b \sin \theta - V_0 t \cos \theta - \frac{1}{2} gt^2$$

tinggi maksimum $V_y = 0$

$$t = -\frac{V_0 \cos \theta}{g}$$

$$h = -b \sin \theta - V_0 \left(-\frac{V_0 \cos \theta}{g} \right) \cos \theta - \frac{1}{2} g \left(-\frac{V_0 \cos \theta}{g} \right)^2$$

$$h = -b \sin \theta = \frac{V_0^2 \cos^2 \theta}{2g}$$

$$h \text{ maksimum terjadi untuk } \frac{dh}{d\theta} = 0 = -b \cos \theta - \frac{2 V_0^2 \cos \theta \sin \theta}{2g}$$

$$\sin \theta = -\frac{gb}{V_0^2}$$

$$\cos^2 \theta = 1 - \sin^2 \theta = \frac{V_0^4 - g^2 b^2}{V_0^4}$$

$$h_{\max} = \frac{gb^2}{V_0^2} + \frac{V_0^4 g^2 b^2}{2g V_0^2} = \frac{gb^2}{2V_0^2} + \frac{V_0^2}{2g}$$

$$\text{Lumpur Meninggalkan roda di } \theta = \sin^{-1} \left(-\frac{gb}{V_0^2} \right)$$

4.8) Jawaban :

$$x = R \cos \phi \quad \text{dan} \quad x = V_0 t \cos \phi = (V_0 \cos \phi) t$$

$$\text{Jadi, } t = \frac{R \cos \phi}{V \cos \phi}$$

$$y = R \sin \phi \quad \text{dan} \quad y = V_y t = -\frac{1}{2} gt^2 = (V_0 \sin \phi) t - \frac{1}{2} gt^2$$

$$R \sin \theta = (V_0 \sin \phi) \frac{R \cos \phi}{V_0 \cos \phi} - \frac{1}{2} g \left(\frac{R \cos \phi}{V_0 \cos \phi} \right)^2$$

$$\sin \phi = \tan \phi \cos \phi - \frac{g R \cos^2 \phi}{2 V_0^2 \cos^2 \phi}$$

$$R = \frac{2V_0^2 \cos^2 \theta}{g \cos \phi} (\tan \theta \cos \phi - \sin \phi) = \frac{2V_0^2 \cos \theta}{g \cos^2 \phi} (\sin \theta \cos \phi - \cos \theta \sin \phi)$$

$$\sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi$$

$$R = \frac{2V_0^2 \cos \theta}{g \cos^2 \phi} \sin(\theta - \phi)$$

$$R_{\max} \text{ Untuk } \frac{dR}{d\theta} = 0 = \frac{2V_0^2}{g \cos^2 \phi} [-\sin \theta \sin(\theta - \phi) + \cos \theta \cos(\theta - \phi)]$$

$$\cos(\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi \text{ Jadi } \cos(2\theta - \phi) = 0$$

$$2\theta - \phi = \frac{\pi}{2} \quad \theta = \frac{\pi}{4} + \frac{\phi}{2}$$

$$R_{\max} = \frac{2V_0^2}{g \cos \phi} \cos\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \sin\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$

$$\sin\left(\frac{\pi}{4} - \frac{\phi}{2}\right) = \cos\left[\frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{\phi}{2}\right)\right] = \cos\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

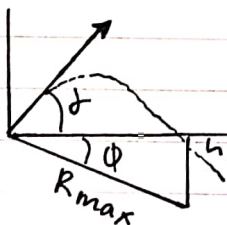
$$R_{\max} = \frac{2V_0^2}{g \cos^2 \phi} \cos^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

$$R_{\max} = \frac{2V_0^2}{g \cos^2 \phi} \left[\frac{1}{2} \cos\left(\frac{\pi}{2} + \phi\right) + \frac{1}{2} \right] = \frac{V_0^2}{g \cos^2 \phi} \left[\cos\left(\frac{\pi}{2} + \phi\right) + 1 \right]$$

$$\text{Menggunakan } \cos\left(\frac{\pi}{2} + \phi\right) = -\sin \phi$$

$$R_{\max} = \frac{V_0^2}{g(1 - \sin \phi)} \\ = \frac{V_0^2}{g(1 + \sin \phi)}$$

4.9) a.) Jawaban:



$$R = \frac{2V_0^2}{g} \frac{\cos \theta \sin(\theta + \phi)}{\cos^2 \phi}$$

$$-\phi = \dots R_{\max} = V_0^2(1 + \sin \phi) \text{ dan } \dots 2\theta = \frac{\pi}{2} = \phi$$

$$R_{\max} = \frac{h}{\sin \phi} = \frac{V_0^2}{g} \frac{(1 + \sin \phi)}{\cos^2 \phi} = \frac{V_0^2}{g(1 - \sin \phi)}$$

$$\sin \phi = \frac{gh}{V_0^2} \left/ \left(1 + \frac{gh}{V_0^2} \right) \right.$$

$$\sin \phi = \sin\left(\frac{\pi}{2} - 2\theta\right) = \cos 2\theta = 1 - 2\sin^2 \theta$$

$$1 - 2\sin^2 \theta = \frac{gh}{V_0^2} \left/ \left(1 + \frac{gh}{V_0^2} \right) \right.$$

$$2\sin^2 \theta = \frac{2}{\cos^2 \theta} = 1 - \frac{gh}{V_0^2} \left/ \left(1 + \frac{gh}{V_0^2} \right) \right. = \frac{1}{1 + \frac{gh}{V_0^2}}$$

$$\text{akhirnya} = \csc^2 \alpha = 2 \left(1 + \frac{gh}{V_0^2} \right)$$

b.) Jawaban :

$$R_{\max} = \frac{h}{\sin \varphi} = \frac{h}{1 - 2 \sin^2 \alpha} = \frac{h}{1 - 2 / \csc^2 \alpha}$$

$$R_{\max} = \frac{V_0^2}{g} \left(1 + \frac{gh}{V_0^2} \right)$$

4.12 a.) Jawaban

x dan z adalah posisi bola

$$x = V_0 t \cos \frac{1}{2} \theta_0, \quad z = V_0 t \cos \frac{1}{2} \theta_0 \sin \theta_0 - \frac{1}{2} g t^2$$

$$\text{Sejak } V_x = V_0 \cos \frac{1}{2} \theta,$$

$$R = \frac{V_0^2}{g} \cos^2 \frac{1}{2} \theta_0 \sin 2 \theta_0$$

$$\text{maka } \frac{dR}{d\theta} = 0$$

$$\frac{dR}{d\theta} = \frac{V_0^2}{g} (2 \cos^2 \frac{1}{2} \theta_0 \cos 2 \theta_0 - \cos \frac{1}{2} \theta_0 \sin \frac{1}{2} \theta \sin 2 \theta_0) = 0$$

$$2 \cos^2 \frac{1}{2} \theta_0 \cos 2 \theta_0 = \cos \frac{1}{2} \theta_0 \sin \frac{1}{2} \theta_0 \sin 2 \theta_0$$

$$2 \cos^2 \theta_0 = 1 + \cos 2 \theta_0 \quad \text{dan} \quad \sin 2 \theta = 2 \sin \theta_0 \cos \theta_0$$

didapat: $(1 + \cos \theta_0)(2 \cos^2 \theta_0 - 1) = \sin \theta_0 \sin \theta_0 \cos \theta_0 = (1 - \cos^2 \theta \cos \theta)$

$$\text{atau } (1 + \cos \theta_0)(3 \cos^2 \theta_0 - \cos \theta_0 - 1) = 0$$

hanya akar positif yang berlaku adalah $\theta, 0 \leq \theta \leq \frac{\pi}{2}$

$$\cos \theta_0 = \frac{1}{6} (1 + \sqrt{13}) = 0,7676 \quad \theta_0 = 39^\circ 51'$$

b.) untuk $V_0 = 25 \text{ m/s}$ $R_{\max} = 55,4 \text{ m}$ $\theta_0 = 39^\circ 51'$

$$\text{maka } \frac{dz}{dt} = 0$$

$$V_0 \cos \frac{1}{2} \theta_0 \sin \theta_0 = g t \quad \text{atau} \quad t = \frac{V_0 \cos \frac{1}{2} \theta_0 \sin \theta_0}{g}$$

$$\text{atau } H = \frac{V_0^2}{2g} \cos^2 \frac{1}{2} \theta_0 \sin^2 \theta, \quad \text{maks. tetap di } \theta,$$

$$\text{maks. pada ketinggian tetap terjadi } \frac{dH}{d\theta} = 0$$

$$\frac{dH}{dt} = \frac{V_0^2}{2g} \left(2 \cos^2 \frac{1}{2} \theta_0 \sin \theta_0 \cos \theta_0 - \cos \frac{1}{2} \theta_0 \sin \frac{1}{2} \theta_0 \sin^2 \theta_0 \right) = 0$$

$$(1 + \cos \theta_0) \sin \theta_0 \cos \theta_0 = \frac{1}{2} \sin \theta_0 \sin^2 \theta_0 = \frac{1}{2} \sin \theta_0 (1 - \cos^2 \theta_0) / \sin \theta_0 (1 + \cos \theta_0) (3 \cos \theta_0 - 1) = 0$$

$$\sin \theta_0 = 0 \quad \cos \theta_0 = -1$$

$$\text{Jadi, } H_{\max} = 18,9 \text{ m} \quad \theta_0 = \cos^{-1} \frac{1}{3} = 70,32^\circ$$

4.13) Jawaban :

r menggantikan x

$$z = \frac{\dot{z}_0}{\dot{r}_0} r - \frac{g}{2\dot{r}_0^2} r^2 \text{ dimana } \dot{r}_0 = V_0 \cos \theta_0 \quad \dot{z}_0 = V_0 \sin \theta_0$$

$$z = r \tan \theta_0 - \frac{gr^2}{2V_0^2} \sec^2 \theta \text{ sebab } \sec^2 \theta_0 = 1 + \tan^2 \theta$$

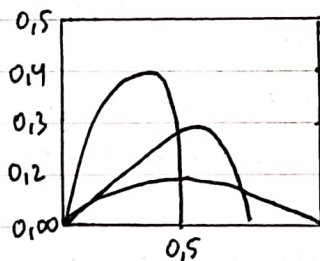
$$\frac{gr^2}{2V_0^2} \tan^2 \theta_0 - r \tan \theta_0 + z + \frac{gr^2}{2V_0^2} = 0$$

(r, z) titik koordinat

$$\tan \theta_0 = \frac{1}{gr} \left[V_0^2 \pm (V_0^4 - 2gzV_0^2 - g^2 r^2)^{\frac{1}{2}} \right]$$

$$V_0^4 - 2gzV_0^2 - g^2 r^2 \geq 0$$

$$\text{Permukaan kritis oleh karena itu } V_0^4 - 2gzV_0^2 - g^2 r^2 = 0$$



4.17) Jawaban :

$$m\ddot{x} = F_x = -\frac{\partial V}{\partial x} = -kx = -\pi^2 mx$$

$$\dot{x} = A \cos \left(\sqrt{\frac{k}{m}} t + \phi \right) = A \cos (\pi t + \phi)$$

$$m\ddot{y} = -\frac{\partial V}{\partial y} = 4\pi^2 mx$$

$$y = B \cos (2\pi t + \beta)$$

$$m\ddot{z} = -\frac{\partial V}{\partial z} = -g\pi^2 m z$$

$$z = \cos(3\pi t + \gamma)$$

Sejak $x = y = z = 0$ pada $t = 0$ $\alpha = \beta = \gamma = -\frac{\pi}{2}$

$$x = A \cos(\pi t - \frac{\pi}{2}) = A \sin \pi t$$

$$\dot{x}_0 = A\pi \cos \pi t$$

$$V_0^2 = \dot{x}_0^2 + \dot{y}_0^2 + \dot{z}_0^2 \quad \text{dan} \quad \ddot{x}_0 = \ddot{y}_0 = \ddot{z}_0$$

$$\dot{x}_0 = \frac{V_0}{\sqrt{3}} = A\pi$$

$$A = \frac{V_0}{\pi\sqrt{3}}$$

$$x = \frac{V_0}{\pi\sqrt{3}} \sin \pi t$$

$$y = B \sin 2\pi t, \quad \dot{y} = 2B\pi \cos 2\pi t$$

$$\dot{y}_0 = \frac{V_0}{\sqrt{3}} = 2\pi$$

$$B = \frac{V_0}{2\pi\sqrt{3}}$$

$$y = \frac{V_0}{2\pi\sqrt{3}} \sin 2\pi t$$

$$z = C \sin 3\pi t$$

$$\dot{z}_0 = \frac{V_0}{\sqrt{3}} = 3\pi \quad \ddot{z} = 3C\pi \cos 3\pi t$$

$$\dot{z}_0 = \frac{V_0}{\sqrt{3}} = 3C\pi$$

$$C = \frac{V_0}{3\pi\sqrt{3}}$$

$$z = \frac{V_0}{3\pi\sqrt{3}} \sin 3\pi t$$

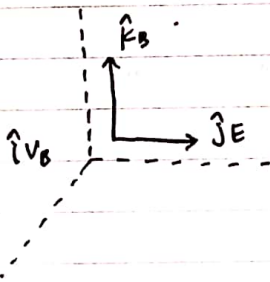
Sejak $\omega_x = \pi$, $\omega_y = 2\pi$ dan $\omega_z = 3\pi$, maka

$$T_{\min} = \frac{2\pi n_1}{\omega_x} = \frac{2\pi n_2}{\omega_y} = \frac{2\pi n_3}{\omega_z}$$

waktu maks pada $n_1 = 1$, $n_2 = 2$, $n_3 = 3$

$$t_{\min} = \frac{2\pi}{\pi} = 2$$

4.20 Jawaban :



$$\vec{F} = q (\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{v} \times \vec{B} = (\hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z}) \times B\hat{k} = \hat{i}\dot{y}B - \hat{j}\dot{x}B$$

$$\vec{F} = \hat{i}q\dot{y}B + \hat{j}q(E - \dot{x}B)$$

$$m\ddot{x} = F_x = q\dot{y}B$$

$$\ddot{x} - \dot{x}_0 = \frac{qB}{m} y$$

$$m\ddot{y} = F_y = qE - q\dot{x}B = qE - qB(\dot{x}_0 + \frac{qB}{m}y)$$

$$\ddot{y} = \frac{qE}{m} - \frac{qB\dot{x}_0}{m} - \left(\frac{qB}{m}\right)^2 y = \frac{eE}{m} + \frac{EB\dot{x}_0}{m} - \left(\frac{eB}{m}\right)^2 y$$

$$\ddot{y} + \omega^2 y = -\frac{eE}{m} + \omega \dot{x}_0$$

$$y = \frac{1}{\omega^2} \left(-\frac{eE}{m} + \omega \dot{x}_0 \right) + A \cos(\omega t + \theta_0)$$

$$\dot{y} = -A\omega \sin(\omega t + \theta)$$

$$\dot{y}_0 = 0, \text{ jadi } \theta_0 = 0, A = -\frac{1}{\omega^2} \left(-\frac{eE}{m} + \omega \dot{x}_0 \right)$$

$$y = a(1 - \cos \omega t), a = \frac{1}{\omega^2} \left(-\frac{eE}{m} + \omega \dot{x}_0 \right)$$

$$\ddot{x} = \ddot{x}_0 = \frac{qB}{m} y = \dot{x} - \omega y = \dot{x}_0 - \omega a(1 - \cos \omega t)$$

$$x = (\dot{x}_0 - \omega a)t + a \sin \omega t$$

$$x = a \sin \omega t + bt \quad b = \dot{x}_0 - \omega a$$

$$m\ddot{z} = F_z = 0 \quad z = \dot{z}_0 t + z_0 = 0$$