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Ayo Berlatih chapter 4

4.1 Find the force each of the following potential energy functions :

(a) $V = cxyz + c$

(b) $V = \alpha x^2 + \beta y^2 + \gamma z^2 + c$

(c) $V = ce^{-(\alpha x + \beta y + \gamma z)}$

(d) $V = cr^n$ in spherical coordinates

Jawaban :

(a) $\vec{F} = -\vec{\nabla} V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$

$\vec{F} = -c(\hat{i}yz + \hat{j}xz + \hat{k}xy)$

(b) $\vec{F} = -\vec{\nabla} V = -\hat{i}\,2\alpha x - \hat{j}\,2\beta y - \hat{k}\,2\gamma z$

(c) $\vec{F} = -\vec{\nabla} V = ce^{-(\alpha x + \beta y + \gamma z)}(\hat{i}\alpha + \hat{j}\beta + k\gamma)$

(d) $\vec{F} = -\vec{\nabla} V = -\hat{e}_r\frac{\partial V}{\partial r} - \hat{e}_\theta \frac{1}{r}\frac{\partial V}{\partial \theta} - \hat{e}_\phi \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}$

$\vec{F} = -\hat{e}_r cnr^{n-1}$

4.2 By finding the curl, determine which of the following force are conservative

(a) $F = ix + jy + kz$

(b) $F = iy - jx + kz^2$

(c) $F = iy + jx + kz^3$

(d) $F = -kr^{-n}e$, in spherical coordinates

Jawaban :

(a)

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0 \quad \text{(conservative)}$$

(b)

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & z^2 \end{vmatrix} = \hat{k}(-1-1) \neq 0 \quad \text{(non conservative)}$$

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(c)

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x & z^3 \end{vmatrix} = \hat{k}(1-1) = 0 \quad \text{(conservative)}$$

(d)

$$\vec{\nabla} \times \vec{F} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{e}_r & \hat{e}_\theta r & \hat{e}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ -kr^{-n} & 0 & 0 \end{vmatrix} = 0 \quad \text{(conservative)}$$

4.3 Find the value of the constant c such that each of the following force is conservative:

(a) $F = ixy + jcx^2 + kz^3$

(b) $F = i(z/y) + cj(xz/y^2) + k(x/y)$

Jawaban:

(a)

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & cx^2 & z^3 \end{vmatrix} = k(2cx - x)$$

$$2cx - x = 0$$

$$c = \frac{1}{2}$$

(b)

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{z}{y} & \frac{cxz}{y^2} & \frac{x}{y} \end{vmatrix}$$

$$= \hat{i}\left(-\frac{x}{y^2} - \frac{cx}{y^2}\right) + \hat{j}\left(\frac{1}{y} - \frac{1}{y}\right) + \hat{k}\left(\frac{cz}{y^2} + \frac{z}{y^3}\right)$$

$$-\frac{x}{y^2} - \frac{cx}{y^2} = 0$$

$$c = -1$$

also $\frac{cz}{y^2} + \frac{z}{y^2} = 0$ implies that $c = -1$ as it must

4.4 A particle of mass m moving in three dimensions under the potential energy function $V(x,y,z) = \alpha x + \beta y^2 + \gamma z^3$ has speed v_0 when it passes through the origin.

(a) what will its speed be if and when it passed through the point (1,1,1)

(b) if the point (1,1,1) is a turning point in the motion ($v=0$), what v_0?

(c) what are the component differental equations of motion of the particle?

(Note: it is necessary to solve the differential equations of motion in this problem)

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Jawaban :

(a) $E = \text{constant} = V(x,y,z) + \frac{1}{2}mv^2$

at the origin $E = 0 + \frac{1}{2}mv^2$

at $(1,1,1)$ $\quad E = \alpha + \beta + \gamma + \frac{1}{2}mV^2 = \frac{1}{2}mv^2$

$$V^2 = V_0^2 - \frac{2}{m}(\alpha + \beta + \gamma)$$

$$V = \sqrt{V_0^2 - \frac{2}{m}(\alpha + \beta + \gamma)}$$

(b) $V_0^2 - \frac{2}{m}(\alpha + \beta + \gamma) = 0$

$$V_0 = \sqrt{\frac{2}{m}(\alpha + \beta + \gamma}$$

(c) $m\ddot{x} = F_x = -\frac{\partial V}{\partial x}$

$m\ddot{x} = -\alpha$

$m\ddot{y} = -\frac{\partial V}{\partial y} = -2\beta y$

$m\ddot{z} = -\frac{\partial V}{\partial z} = -3\gamma z^2$

4.5 Consider the two force functions :

(a) $F = ix + jy$

(b) $F = iy - jx$

Verify that (a) is conservative and that (b) is nonconservative by showing that the integral $\int F \cdot dx$ its independent of the path of integration for (a) but not for (b), by talking two paths in which the strating point is the origin (0,0), and the end point is (1,1). For one path take the line $x = y$. For the other path take the x-axis out the point (1,0) and then the line $x=1$ up the poth (1,1)

Jawaban :

$\vec{F} = iy - jx$

On the path $x = y$;

$$\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_x dx + \int_0^1 F_y dy$$

$$= \int_0^1 y\, dx - \int_0^1 x\, dy$$

and, with $x = y$ $\quad \int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 x dx - \int_0^1 y dy = 0$

On the x-axis : $\int_{(0,0)}^{(1,0)} \vec{F} \cdot d\vec{r} = \int_0^1 F_x\, dx = \int_0^1 y\, dx$

and, with $y = 0$ on the x-axis $\int_{(0,0)}^{(1,0)} \vec{F} \cdot d\vec{r} = 0$

On the line $x = 1$ $\quad \int_{(1,0)}^{(1,0)} \vec{F} \cdot d\vec{r} = \int_0^1 F_y\, dy = \int_0^1 x\, dx$

and, with $x = 1$ $\quad \int_{(1,0)}^{(1,0)} \vec{F} \cdot d\vec{r} = \int_0^1 dy = 1$

On this path $\int_{(0,0)}^{(1,1)} \vec{F} \cdot dr = 0 + 1 = 1$

$\vec{F}$ is not conservative

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4.6 Show that the variation of gravity with height can be accounted for approximately by the following potential energi function:

$$V = mgz\left(1-\frac{z}{r_e}\right)$$

in which r_e is the radius of the earth. Find the force given by the above potential function. From this find the component differential equations of motion of a projectile under such a force. If the vertical component of the initial velocity is v_{0z} How high does the projectile go? (compare with Example 2.3.2)

Jawaban:

From Example 2.3.2 $V(z) = -mg\dfrac{r_e^2}{r_e+z}$

$$V(z) = -mgr_e\left(1+\frac{z}{r_e}\right)^{-1}$$

From Appendix D, $(1+x)^{-1} = 1-x+x^2+\ldots$

$V(z) = -mgr_e\left(1-\frac{z}{r_e}+\frac{z^2}{r_e^2}+\ldots\right)$

$V(z) = -mgr_e + mgz - \frac{mgz^2}{r_e}+\ldots$

with $-mgr_e$ an additive constant

$V(z) \approx mgz\left(1-\frac{z}{r_e}\right)$

$\vec{F} = -\nabla V = -\hat{k}\frac{d}{dz}V(z)$

$= -\hat{k}mg\left[1-\frac{z}{r_e}+z\left(-\frac{1}{r_e}\right)\right]$

$\vec{F} = -\hat{k}mg\left(1-\frac{2z}{r_e}\right)$

$m\ddot{x} = F_x = 0 \qquad m\ddot{y} = F_y = 0 \qquad m\ddot{z} = -mg\left(1-\frac{2z}{r_e}\right)$

$m\dot{z}\frac{d\dot{z}}{dz} = -mg\left(1-\frac{2z}{r_e}\right)$

$\int_{v_z}^{0}\dot{z}\,d\dot{z} = -g\int_0^h\left(1-\frac{2z}{r_e}\right)dz$

$-\frac{1}{2}v_{0z}^2 = -g\left(h-\frac{h^2}{r_e}\right)$

$h^2 - r_eh + \frac{r_ev_{0z}^2}{2g} = 0$

$h = \frac{r_e}{2} - \frac{1}{2}\sqrt{r_e^2 - \frac{2r_ev_{0z}^2}{g}} \qquad (h, z \ll r_e)$

$h = \frac{r_e}{2} - \frac{r_e}{2}\sqrt{1-\frac{2v_{0z}^2}{gr_e}}$

From Appendix D $(1+x)^{\frac{1}{2}} = 1+\frac{x}{2}-\frac{x^2}{8}+\ldots$

$h \approx \frac{r_e}{2} - \frac{r_e}{2} + \frac{v_{0z}^2}{2g} + \frac{v_{0z}^4}{4g^2r_e}+\ldots$

$h \approx \frac{v_{0z}^2}{2g}\left(1+\frac{v_{0z}^2}{2gr_e}\right)$

From Example 2.3.2 $h = \frac{v_0^2}{2g}\left(1-\frac{v_0^2}{2gr_e}\right)^{-1}$

And with $(1-x)^{-1} \approx 1+x$, $h = \frac{v_0^2}{2g}\left(1+\frac{v_0^2}{2gr_e}\right)$

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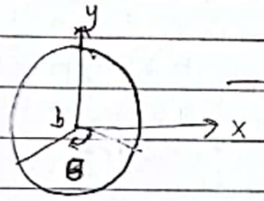
4.7 Particles of mud are thrown from the rim of a rolling wheel. If the forward speed of the wheel is V_0 dan the radius of the wheel is b, show that the greatest height above the ground that the mud can go is:

$$b + \frac{V_0^2}{2g} + \frac{gb^2}{2V_0^2}$$

At what point on the rolling wheel does this mud leave? (Note: It is necessary to assume that $V_0^2 \geq bg$)

Jawaban :



For a point on the rim measured from the center of the wheel

$\vec{r} = \hat{i}\, b\cos\theta - \hat{j}\, b\sin\theta$

$\theta = \omega t = \frac{V_0 t}{b}$ so, $\dot{\vec{r}} = -\hat{i}V_0 \sin\theta - \hat{j}V_0\cos\theta$

Relative to the ground $\vec{v} = \hat{i}V_0(1-\sin\theta) - \hat{j}V_0\cos\theta$

For a particle of mud leaving the rim:

$y_0 = -b\sin\theta$ and $V_{0y} = -V_0\cos\theta$

so $V_y = V_{0y} - gt = -V_0\cos\theta - gt$

and $y = -b\sin\theta - V_0 t\cos\theta - \frac{1}{2}gt^2$

At maximum height $V_y = 0$

$$t = -\frac{V_0\cos\theta}{g}$$

$$h = -b\sin\theta - V_0\left(-\frac{V_0\cos\theta}{g}\right)\cos\theta - \frac{1}{2}g\left(-\frac{V_0\cos\theta}{g}\right)^2$$

$$h = -b\sin\theta + \frac{V_0^2\cos^2\theta}{2g}$$

maximum h occurs for $\frac{dh}{d\theta} = 0 = -b\cos\theta - \frac{2V^2\cos\theta\sin\theta}{2g}$

$$\sin\theta = -\frac{gb}{V_0^2}$$

$$\cos^2\theta = 1 - \sin^2\theta = \frac{V_0^4 - g^2b^2}{V_0^4}$$

$$h_{max} = \frac{gb^2}{V_0^2} + \frac{V_0^4 - g^2b^2}{2gV_0^2} = \frac{gb^2}{2V_0^2} + \frac{V_0^2}{2g}$$

Measured from the ground

$$h'_{max} = b + \frac{gb^2}{2V_0^2} + \frac{V_0^2}{2g}$$

The mud leaves the wheel at $\theta = \sin^{-1}\left(-\frac{gb}{V_0^2}\right)$

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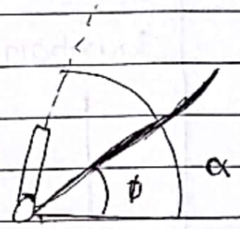
4.8 A gun is located at the bottom of a hill of constant slope ϕ. Show that the range of the gun measured up the slope of the hill is

$$\frac{2V_0^2 \cos\alpha \sin(\alpha-\phi)}{g\cos^2\phi}$$

where α is the angle of elevation of the gun, and that the maximum value of the slope range is $\frac{V_0^2}{g(1+\sin\phi)}$

Jawaban :



$x = R\cos\phi$ and $x = V_{0x} t = (V_0\cos\alpha)t$

so $t = \frac{R\cos\phi}{V_0\cos\alpha}$

$y = R\sin\phi$ and $y = V_y t - \frac{1}{2}gt^2 = (V_0\sin\alpha)t - \frac{1}{2}gt^2$

$R\sin\phi = (V_0\sin\alpha)\frac{R\cos\phi}{V_0\cos\alpha} - \frac{1}{2}g\left(\frac{R\cos\phi}{V_0\cos\alpha}\right)^2$

$\sin\phi = \tan\alpha\cos\phi - \frac{gR\cos^2\phi}{2V^2\cos^2\alpha}$

$R = \frac{2V_0^2\cos^2\alpha}{g\cos^2\phi}(\tan\alpha\cos\phi - \sin\phi) = \frac{2V_0^2\cos\alpha}{g\cos^2\phi}(\sin\alpha\cos\phi - \cos\alpha\sin\phi)$

From Appendix B, $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$R = \frac{2V_0^2\cos\alpha}{g\cos^2\phi}\sin(\alpha-\phi)$

R is a maximum for $\frac{dR}{d\alpha} = 0 = \frac{2V_0^2}{g\cos^2\phi}\left[-\sin\alpha\sin(\alpha-\phi) + \cos\alpha\cos(\alpha-\phi)\right]$

Implies that $\cos\alpha(\cos\alpha-\phi) - \sin\alpha\sin(\alpha-\phi) = 0$

From appendix B, $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

So $\cos(2\alpha-\phi) = 0$

$2\alpha - \phi = \frac{\pi}{2}$, $\alpha = \frac{\pi}{4} + \frac{\phi}{2}$

$$R_{max} = \frac{2V^2}{g\cos^2\phi}\cos\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\sin\left(\frac{\pi}{4}-\frac{\phi}{2}\right)$$

Now $\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right) = \cos\left[\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{\phi}{2}\right)\right] = \cos\left(\frac{\pi}{4}+\frac{\phi}{2}\right)$

$R_{max} = \frac{2V^2}{g\cos^2\phi}\cos^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)$

Again using Apendix B, $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$R_{max} = \frac{2V_0^2}{g\cos^2\theta}\left[\frac{1}{2}\cos\left(\frac{\pi}{2}+\phi\right)+\frac{1}{2}\right] = \frac{V_0^2}{g\cos^2\phi}\left[\cos\left(\frac{\pi}{2}+\phi\right)+1\right]$

using $\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$$R_{max} = \frac{V_0^2}{g(1-\sin^2\theta)}(1-\sin\theta)$$

$$R_{max} = \frac{V_0^2}{g(1+\sin\theta)}$$

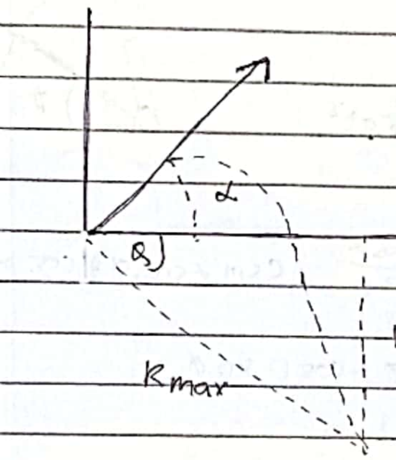
4.9 A cannon that is capable of firing a shell V_0 is mounted on a vertical tower of height h that overlooks a level plain below

(a) Show that the elevation angle α at which the cannon must be set to achieve maximum range is given by expression

$$\text{cosec}^2 \alpha = 2\left(1 + \frac{gh}{V_0^2}\right)$$

(b) What is the minimum range R of the cannon?

Jawaban:



(a) Here we note that projectile is launched "downhill" towards the target, which is located a distance h below the cannon along a line at an angle φ below the horizon. α is the angle of projection that yields maximum range Rmax, we can use the result from problem 4.8 for this problem we result with the angle −φ, to account for the downhill slop, thus we get for the downhill range

$$R = \frac{2V_0^2}{g} \frac{\cos\alpha \sin(\alpha + \theta)}{\cos^2\varphi}$$

The maximum range and the angle is α are obtained from the problem above again by replacing φ with angle −φ

$$R_{max} = \frac{V_0^2}{g} \frac{(1+\sin\varphi)}{\cos^2\varphi} \quad \text{and} \quad 2\alpha = \frac{\pi}{2} - \varphi$$

We can now calculate α ... $R_{max} = \frac{h}{\sin\varphi} = \frac{V_0^2}{g}\frac{(1+\sin\varphi)}{\cos^2\varphi} = \frac{V_0^2}{g(1-\sin\varphi)}$

Solving for $\sin\theta$ ... $\sin\varphi = \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2}\right)$

But, from the above ... $\sin\varphi = \sin\left(\frac{\pi}{2} - 2\alpha\right) = \cos 2\alpha = 1 - 2\sin^2\alpha$

Thus ... $1 - 2\sin^2\alpha = \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2}\right)$

$$2\sin^2\alpha = \frac{2}{\csc^2\alpha} = 1 - \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2}\right) = \frac{1}{1 + \frac{gh}{V_0^2}}$$

Finally ... $\csc^2\alpha = 2\left(1 + \frac{gh}{V_0^2}\right)$

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(b) Solving for R_{max} ...

$$R_{max} = \frac{h}{\sin\phi} = \frac{h}{1-2\sin^2\alpha} = \frac{h}{1-2/\csc^2\alpha}$$

Substituting for $\csc^2\alpha$ and solving

$$R_{max} = \frac{V_0^2}{g}\left(1+\frac{gh}{V_0^2}\right)$$

4.12 A baseball pitcher can throw a ball more easily horizontally than vertically. Assume that the pitcher's throwing speed varies with elevation angle aproximately as $V_0 \cos \frac{1}{2}\theta_0$ m/s, where θ_0 is the initial elevation angle and V_0 is the initial velocity when the ball is thrown horizontally. Find the angle θ_0 at which the ball must be thrown to achieve maximum

(a) height and

(b) range

Find the valeus of the maximum (c) height and (d) range. Assume no air resistance and let $V_0 = 25$ m/s

Jawaban :

The x and z positions of the ball vs. time are

$x = V_0 t \cos\frac{1}{2}\theta$ $\quad\quad$ $z = V_0 t \cos\frac{1}{2}\theta_0 \sin\theta_0 - \frac{1}{2}gt^2$

Since $V_x = V_0 \cos\frac{1}{2}\theta$

The Horizontal range is $R = \frac{V_0^2}{g}\cos^2\frac{1}{2}\theta_0 \sin 2\theta$

The maximum range occurs @ $\frac{dR}{d\theta} = 0$

$$\frac{dR}{d\theta} = \frac{V_0^2}{g}\left(2\cos^2\tfrac{1}{2}\theta_0 \cos 2\theta_0 - \cos\tfrac{1}{2}\theta_0 \sin\tfrac{1}{2}\theta_0 \sin 2\theta_0\right) = 0$$

Thus, $2\cos^2\frac{1}{2}\theta_0 \cos 2\theta_0 = \cos\frac{1}{2}\theta_0 \sin\frac{1}{2}\theta_0 \sin 2\theta_0$

Using the Identities : $2\cos^2\theta = 1+\cos 2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$

We get :

$(1+\cos\theta_0)(2\cos^2\theta_0 - 1) = \sin\theta \sin\theta \cos\theta = (1-\cos^2\theta)\cos\theta$

or $(1+\cos\theta_0)(3\cos^2\theta_0 - \cos\theta - 1) = 0$

Thus $\cos\theta_0 = -1$ $\quad$ $\cos\theta = \frac{1}{6}(1\pm\sqrt{13})$

Only the positive root applies for the θ -range :

$\cos\theta = \frac{1}{6}(1+\sqrt{13}) = 0.7676$ $\quad\quad$ $\theta = 39°51'$

Thus (c) for $V_0 = 25$ m/s $\quad$ $R_{max} = 55.4$ m $\quad$ $\theta = 39°51'$

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(a) The maximum height accours at $\frac{dz}{dt} = 0$

$V_0 \cos\frac{1}{2}\theta \sin\theta = gt$ or at $r = \frac{V_0 \cos\frac{1}{2}\theta \sin\theta}{g}$

or $H = \frac{V_0^2}{2g} \cos^2\frac{1}{2}\theta . \sin^2\theta$ maximum at fixed θ

The maximum possible height occours $\frac{dH}{d\alpha} = 0$

$$\frac{dH}{d\alpha} = \frac{V_0^2}{2g}\left(2\cos^2\tfrac{1}{2}\theta \sin\theta \cos\theta - \cos\tfrac{1}{2}\theta \sin\tfrac{1}{2}\theta . \sin^2\theta\right) = 0$$

Using the above trigonometric identities, we get

$(1+\cos\theta) \sin\theta \cos\theta = \frac{1}{2}\sin\theta \quad \sin^2\theta = \frac{1}{2}\sin\theta\,(1-\cos^2\theta)$

or $\sin\theta . (1+\cos\theta)(3\cos\theta - 1) = 0$

There are 3 roots. $\sin\theta = 0$, $\cos\theta = -1$ $\cos\theta = \frac{1}{3}$

The first to roots give minimum heights: the last gives the maximum. Thus $H_{max} = 18.9$ m $\theta = \cos^{-1}\frac{1}{3} = 70°32'$

4.13 A gun can fire an artillery shell with a speed V_0 in any direction. Show that a shell can strike any target within the surface given by

$$g^2 r^2 = V_0^4 - 2gV_0^2 z$$

Where z is the height of the target and r is its horizontal distance from the gun. Assume no air resistance

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The trajectory of the shell is given by Eq. 4.3.11 with replacing x

$z = \frac{\dot{z}_0}{\dot{r}_0} r - \frac{g}{2\dot{r}_0^2} r^2$ wher $\dot{r}_0 = V_0\cos\theta$. $\dot{z}_0 = V_0 \sin\theta$.

Thes, $z = r\tan\theta - \frac{gr^2}{2V_0^2}\sec^2\theta$

Since $\sec^2\theta = 1 + \tan^2\theta$

We have :

$$\frac{gr^2}{2V_0^2}\tan^2\theta - r\tan\theta . + z + \frac{gr^2}{2V_0^2} = 0$$

(r, z) are target coordinates

The above equation yields two posible roots :

$$\tan\theta_0 = \frac{1}{gr}\left[V_0^2 \pm \left(V_0^4 - 2gzV_0^2 - g^2r^2\right)^{\frac{1}{2}}\right]$$

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The roots are only real if

$v_0^2 - 2gzv_0^2 - g^2r^2 \geq 0$

The critical surface is therefore

$v_0^4 - 2gzv_0^2 - g^2r^2 = 0$

4.17 A small lead ball of mass m is suspended by means of six light springs as shown in Figure 4.4.1. The stiffness constants are in the ratio 1 : 4 : 9, so that the potential energy function can be expressed as $V = \frac{k}{2}(x^2 + 4y^2 + 9z^2)$
At time $t = 0$ the ball receives a push in the (1,1,1) direction that imparts to it a speed v_0 at the origin. If $k = \pi^2 m$, numerically find x, y, and z as functions of the time t. Does the ball ever retrace its path? If so, for what value of t does it first return to the origin with the same velocity that it had at $t=0$
.

Jawab :

$m\ddot{x} = F_x = -\frac{\partial V}{\partial x} = -kx = -\pi^2 m x$

$\ddot{x} = A \cos\left(\sqrt{\frac{k}{m}}t + \alpha\right) = A \cos(\pi t + \alpha)$

$m\ddot{y} = -\frac{\partial V}{\partial y} = -4\pi^2 m x$

$y = B \cos(2\pi t + \beta)$

$m\ddot{z} = -\frac{\partial V}{\partial z} = -9\pi^2 m z$

$z = C \cos(3\pi t + \gamma)$

Since $x = y = z = 0$ at $t = 0$ $\quad \alpha = \beta = \gamma = -\frac{\pi}{2}$

$x = A \cos\left(\pi t - \frac{\pi}{2}\right) = A \sin \pi t$

$\dot{x} = A \pi \cos \pi t$

Since $v_0^2 = \dot{x}_0^2 + \dot{y}_0^2 + \dot{z}_0^2$ and $\dot{x}_0 = \dot{y}_0 = \dot{z}_0$

$\dot{x}_0 = \frac{v_0}{\sqrt{3}} = A\pi$

$A = \frac{v_0}{\pi\sqrt{3}}$

$x = \frac{v_0}{\pi\sqrt{3}} \sin \pi t$

$y = B \sin 2\pi t \quad , \quad \dot{y} = 2B\pi \cos 2\pi t$

$\dot{y}_0 = \frac{v_0}{\sqrt{3}} = 2\pi B$

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$b = \frac{v_0}{2\pi\sqrt{3}}$

$y = \frac{v_0}{2\pi\sqrt{3}} \sin 2\pi t$

$z = c \sin 3\pi t \qquad \dot{z} = 3c\pi \cos 3\pi t$

$\dot{z}_0 = \frac{v_0}{\sqrt{3}} = 3c\pi$

$c = \frac{v_0}{3\pi\sqrt{3}}$

$z = \frac{v_0}{3\pi\sqrt{3}} \sin 3\pi t$

Since $\omega_x = \pi . \omega_y = 2\pi$ and $\omega_z = 3\pi$ the ball does retrace its path

$t_{min} = \frac{2\pi n_1}{\omega_x} = \frac{2\pi n_2}{\omega_y} = \frac{2\pi n_3}{\omega_z}$

The minimum time occours at $n_1 = 1 \quad n_2 = 2 \quad n_3 = 3$

$t_{min} = \frac{2\pi}{\pi} = 2$

4.20 An electron moves in a force field due to a Uniform Electric field E and a uniform magnetic electron at the origin with initial velocity $V_0 = iv_0$ in the x direction. Find the resulting motion of the particle. Show that the path of motion is a cycloid

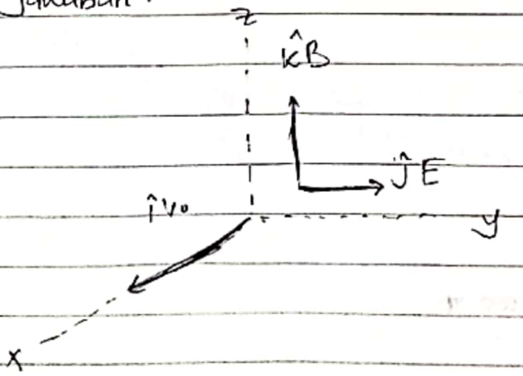
$x = a \sin \omega t + bt$

$y = a(1 - \cos \omega t)$

$z = 0$

Cycloidal motion of electrons is used in an electronic tube called a magnetron to produce the microwaves in a mirco wave oven.

Jawaban :



$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

$\vec{v} \times \vec{B} = (i\dot{x} + j\dot{y} + k\dot{z}) \times kB$

$= i\dot{y}B - j\dot{x}B$

$\vec{F} = iq\dot{y}B + jq(E - \dot{x}B)$

$m\ddot{x} = F_x = q\dot{y}B$

$\dot{x} - \dot{x}_0 = \frac{qB}{m} y$

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$m\ddot{y} = F_y = qE - q\dot{x}B = qE - qB\left(\dot{x}_0 + \frac{qB}{m}y\right)$

$\ddot{y} = \frac{qE}{m} - \frac{qB\dot{x}}{m} - \left(\frac{qB}{m}\right)^2 \quad y = -\frac{eE}{m} + \frac{eB\dot{x}}{m} - \left(\frac{eB}{m}\right)^2 y$

$\ddot{y} + w^2 y = -\frac{eE}{m} + w\dot{x}_0. \qquad w = \frac{eB}{m}$

$y = \frac{1}{w^2}\left(-\frac{eE}{m} + w\dot{x}_0\right) + A\cos(wt + \theta_0)$

$\dot{y} = -Aw\sin(wt+\theta)$

$\dot{y}_0 = 0$. so $\theta = 0$

$y = 0$. so $A = -\frac{1}{w^2}\left(-\frac{eE}{m} + w\dot{x}\right)$

$y = a(1 - \cos wt) \quad a = \frac{1}{w^2}\left(-\frac{eE}{m} + w\dot{x}_0\right)$

$\dot{x} = \dot{x}_0 + \frac{qB}{m}y = \dot{x} - wy = \dot{x} - wa(1 - \cos wt)$

$\dot{x} = (\dot{x}_0 - wa) + wa\cos wt$

$x = (\dot{x}_0 - wa)t + a\sin wt$

$x = a\sin wt + bt, \qquad b = \dot{x}_0 - wa$

$m\ddot{z} = F_z = 0$

$z = \dot{z}_0 t + z_0 = 0$