Nama : Sihfa Zhainita
NPM : 2013022062
Prodi : Pendidikan Fisika (B)

"Mekanika"

4.1 (a). $V = Cxyz + C$

Jawab: $\vec{F} = -\vec{\nabla} V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z} = -C(\hat{i}yz + \hat{j}xz + \hat{k}xy)$

(b). $V = \alpha x^2 + \beta y^2 + \gamma z^2 + C$

Jawab: $\vec{F} = -\vec{\nabla} V = -\hat{i}\,2\alpha x + \hat{j}\,2\beta y + \hat{k}\,2\gamma z$

(c). $V = Ce^{-(\alpha x + \beta y + \gamma z)}$

Jawab. $\vec{F} = -\vec{\nabla} V = Ce^{-(\alpha x + \beta y + \gamma z)}(\hat{i}\alpha + \hat{j}\beta + \hat{k}\gamma)$

(d). $V = cr^n$ dalam koordinat bola

Jawab: $\vec{F} = -\vec{\nabla} V = -\hat{e}_r\frac{\partial V}{\partial r} - \hat{e}_\theta\frac{1}{r}\frac{\partial V}{\partial \theta} - \hat{e}_\phi\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}$

4.2 (a). $F = \hat{i}x + \hat{j}y + \hat{k}z$

Jawab: $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0 \rightarrow$ konservatif

(b). $F = \hat{i}x - \hat{j}y + \hat{k}z^2$

Jawab: $\vec{\nabla} + \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -y & z^2 \end{vmatrix} = \hat{k}(-1-1) \neq 0 \rightarrow$ tidak konservatif

(c). $F = \hat{i}x + \hat{j}y + \hat{k}z^3$

Jawab: $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z^3 \end{vmatrix} = \hat{k}(1-1) = 0 \rightarrow$ konservatif

(d). $F = -kr^{-n}e_r$ dalam koordinat bola

Jawab: $\vec{\nabla} \times \vec{F} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{e}_r & \hat{e}_\theta r & \hat{e}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ -kr & 0 & 0 \end{vmatrix} = 0 \rightarrow$ konservatif

4.3 (a). $F = \hat{i}xy + \hat{j}\,cx^2 + \hat{k}z^3$

Jawab: $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & cx^2 & z^3 \end{vmatrix} = k(2cx - x)$

$2cx - x = 0 \rightarrow C = \frac{1}{2}$

(b). $F = i\,(z/y) + cj\,(xz/y^2) + k\,(x/y)$

Jawab: $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & cx^2 & z^3 \end{vmatrix} = k$

(b). $F = i\,(z/y) + cj\,(xz/y^2) + k(x/y)$

Jawab: $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{z}{y} & \frac{cxz}{y^2} & \frac{x}{y} \end{vmatrix} = \hat{i}\left(\frac{-x}{y} - \frac{cx}{y^2}\right) + \hat{j}\left(\frac{1}{y} - \frac{1}{y}\right) + \hat{k}\left(\frac{cz}{y^2} + \frac{z}{y^2}\right)$

$$-\frac{x}{y^2} - \frac{cx}{y^2} = 0 \longrightarrow c = -1$$

Juga $\frac{cz}{y^2} + \frac{z}{y^3} = 0 \longrightarrow c = -1$

4.4 (a). Jawaban : $E = \text{konstan} = V(x,y,z) + \frac{1}{2} mv^2$

diasalnya. $E = 0 + \frac{1}{2} mv^2$

pada $(1,1,1)$ $E = \alpha + \beta + y + \frac{1}{2} mv^2 = \frac{1}{2} mv^2$

(b). Jawaban : $V_0 - \frac{2}{m}(\alpha + \beta + y) = 0$

$$V_0 = \left[\frac{2}{m}(\alpha + \beta + y)\right]^{\frac{1}{2}}$$

(c). Jawaban : $m\ddot{x} = F_x = -\frac{2V}{2x}$

$$m\ddot{x} = -\alpha$$

$$m\ddot{y} = -\frac{\partial V}{\partial y} = -2\beta y$$

$$m\ddot{z} = -\frac{\partial V}{\partial z} = -3\gamma z^2$$

4.5 Pertimbangkan 2 fungsi gaya

(a). $F = \hat{i}x + \hat{j}y$

Jawab: $\vec{F} = \hat{i}x + \hat{j}y$

di $x = y$; $\quad d\vec{r} = \hat{i}\,dx + \hat{j}\,dy$

$$\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_x\,dx + \int_0^1 F_y\,dy = \int_0^1 x\,dx + \int_0^1 y\,dy = 1$$

pada lintasan sepanjang sumbu x : $d\vec{r} = \hat{i}\,dx$

dan pada garis $x = 1$ $\quad d\vec{r} = \hat{j}\,dy$

$\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_x\,dx + \int_0^1 F_y\,dy = 1$, $\vec{F}$ = konservatif (b). $\vec{F} = \hat{i}y - \hat{j}x$

di $x = y$: $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_x dx + \int_0^1 F_y dy = \int_0^1 y\,dx - \int_0^1 x\,dy$

dan, dengan $x = y$ $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 x\,dx - \int_0^1 y\,dy = 0$

pada sumbu x : $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_x dx = \int_0^1 y\,dx$

dan, dengan $y = 0$ pada sumbu x $\int_{(0,0)}^{(1,0)} \vec{F} \cdot d\vec{r} = 0$

Pada garis $x = 1$ $\int_{(1,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 F_y dy = \int_0^1 x\,dy$

dan, dengan $x = 1$ $\int_{(1,0)}^{(1,1)} \vec{F}\, d\vec{r} = \int_0^1 dy = 1$

Pada jalur ini $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = 0 + 1 = 1$, $\vec{F}$ = Tidak konservatif

4.6 Jawaban :

Dari contoh 2.3.2, $V(z) = -mg \dfrac{r_e^2}{(r_e + z)}$

$$V(z) = -mg\,r_e \left(1 + \frac{z}{r_e}\right)^{-1}$$

$$V(z) = -mg\,r_e \left(1 - \frac{z}{r_e} + \frac{z^2}{r_e} + \dots\right)$$

$$V(z) = -mg\,r_e + mgz - \frac{mgz^2}{r_e} + \dots$$

dengan $-mg\,r_e$ sebuah konstan

$$V(z) \approx mgz \left(1 - \frac{z}{r_e}\right)$$

$$\vec{F} = -\vec{\nabla} V = -\hat{k} \frac{\partial}{\partial z} V(z)$$

$$= -\hat{k}mg \left[1 - \frac{z}{r_e} + z\left(-\frac{1}{r_e}\right)\right]$$

$$\vec{F} = -\hat{k}mg \left(1 - \frac{2z}{r_e}\right)$$

$$m\ddot{x} = F_x = 0$$

$$m\dot{z}\frac{d\dot{z}}{dz} = -mg \left(1 - \frac{2z}{r_e}\right)$$

$$\int_{v_z}^{0} \dot{z}\,d\dot{z} = -g \int_0^h \left(1 - \frac{2z}{r_e}\right) dz$$

$$-\frac{1}{2} v_{0z}^2 = -g \left(h - \frac{h^2}{r_e}\right)$$

$$h^2 - r_e h + \frac{r_e v_{0z}^2}{2g} = 0$$

$$h - r_e h + \frac{r_e v_{0z}^2}{2g} = 0$$

$$h = \frac{r_e}{2} - \frac{1}{2}\sqrt{r_e^2 - \frac{2 r_e v_{0z}^2}{g r_e}} \qquad (h, z << r_e)$$

$$h = \frac{r_e}{2} - \frac{r_e}{2}\sqrt{1 - \frac{2v_{0z}^2}{g r_e}}$$

$(1+x)^{1/2} = 1 + \frac{x}{2} - \frac{x^2}{8} + \dots$

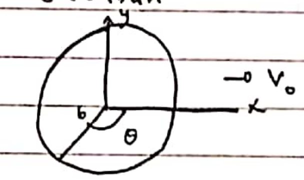
$h = \frac{r_e}{2} - \frac{r_e}{2} + \frac{V_{oz}^2}{2g} + \frac{V_{oz}^4}{4g r_e} + \dots$

$h \approx \frac{V_{oz}^2}{2g}\left(1 + \frac{V_{oz}^2}{2g r_e}\right)$

dari contoh 2.3.2 $h = \frac{V_0^2}{2g}\left(1 - \frac{V_0^2}{2g r_e}\right)^{-1}$

dan dengan $(1-x)^{-1} \approx 1 + x$, $h \approx \frac{V_0^2}{2g}\left(1 + \frac{V_0^2}{2g r_e}\right)$

4.7. Jawaban:



$\vec{r} = \hat{i}\, b\cos\theta - \hat{j}\, b\sin\theta$

$\theta = \omega t = \frac{V_0 t}{b}$ → jadi, $\dot{\vec{r}} = -\hat{i} V_0 \sin\theta - \hat{j} V_0 \cos\theta$

relatif ke tanah, $\vec{v} = \hat{i} V_0 (1 - \sin\theta) - \hat{j} V_0 \cos\theta$

partikel keluar tepi : $y_0 = -b\sin\theta$ dan $V_{oy} = -V_0\cos\theta$

Jadi, $V_y = V_{oy} - gt = -V_0\cos\theta - gt$ dan $y = -b\sin\theta - V_0 t\cos\theta - \frac{1}{2}gt^2$

tinggi maksimum $V_y = 0$

$t = -\frac{V_0\cos\theta}{g}$

$h = -b\sin\theta - V_0\left(-\frac{V_0\cos\theta}{g}\right)\cos\theta - \frac{1}{2}g\left(-\frac{V_0\cos\theta}{g}\right)^2$

$h = -b\sin\theta = \frac{V_0^2\cos^2\theta}{2g}$

h maksimum terjadi untuk $\frac{dh}{d\theta} = 0 = -b\cos\theta - \frac{2V_0^2\cos\theta\sin\theta}{2g}$

$\sin\theta = -\frac{gb}{V_0^2}$

$\cos^2\theta = 1 - \sin^2\theta = \frac{V_0^4 - g^2b^2}{V_0^4}$

h max $= \frac{gb^2}{V_0^2} + \frac{V_0^4 \; g^2b^2}{2gV_0^2} = \frac{gb^2}{2V_0^2} + \frac{V_0^2}{2g}$

lumpur meninggalkan roda di $\theta = \sin^{-1}\left(-\frac{gb}{V_0^2}\right)$

4.8 Jawaban :

$x = R\cos\phi$ dan $x = V_{ox}t = (V_0\cos\alpha)t$

Jadi, $t = \frac{R\cos\phi}{V\cos\alpha}$

$y = R\sin\phi$ dan $y = V_{y}t = -\frac{1}{2}gt^2 = (V_0\sin\alpha)t - \frac{1}{2}gt^2$

$$R \sin \phi = (V_0 \sin \alpha) \frac{R \cos \phi}{V_0 \cos \alpha} - \frac{1}{2} g \left(\frac{R \cos \phi}{V_0 \cos \alpha} \right)^2$$

$$\sin \phi = \tan \alpha \cos \phi - \frac{g R \cos^2 \phi}{}$$

$$R = \frac{2 V_0^2 \cos^2 \alpha}{g \cos \phi} \left(\tan \alpha \cos \phi \, V_0^2 \cos^2 \alpha - \sin \phi \right) = \frac{2 V_0^2 \cos \alpha}{g \cos^2 \phi} (\sin \alpha \cos \phi - \cos \alpha \sin \phi)$$

$$\sin (\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi$$

$$R = \frac{2 V_0^2 \cos \alpha}{g \cos^2 \phi} \sin (\alpha - \phi)$$

R maks . untuk $\frac{dR}{d\alpha} = 0 = \frac{2 V_0^2}{g \cos^2 \phi} \left[- \sin \alpha \sin (\alpha - \phi) + \cos \alpha \cos (\alpha - \phi) \right]$

$$\cos (\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi$$

jadi $\cos (2\alpha - \phi) = 0$

$$2\alpha - \phi = \frac{\pi}{2} \qquad \alpha = \frac{\pi}{4} + \frac{\phi}{2}$$

$$R_{max} = \frac{2 V_0^2}{g \cos \phi} \cos \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \sin \left(\frac{\pi}{4} - \frac{\phi}{2} \right)$$

$$\sin \left(\frac{\pi}{4} - \frac{\phi}{2} \right) = \cos \left[\frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{\phi}{2} \right) \right] = \cos \left(\frac{\pi}{4} + \frac{\phi}{2} \right)$$

$$R_{max} = \frac{2 V_0^2}{g \cos^2 \phi} \cos^2 \left(\frac{\pi}{4} + \frac{\phi}{2} \right)$$

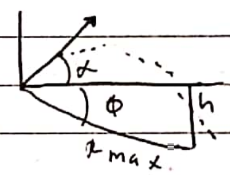
$$R_{max} = \frac{2 V_0^2}{g \cos^2 \phi} \left[\frac{1}{2} \cos \left(\frac{\pi}{2} + \phi \right) + \frac{1}{2} \right] = \frac{V_0^2}{g \cos^2 \phi} \left[\cos \left(\frac{\pi}{2} + \phi \right) + 1 \right]$$

menggunakan $\cos \left(\frac{\pi}{2} + \theta \right) = - \sin \theta$

$$R_{max} = \frac{V_0^2}{g (1 - \sin \phi)} (1 - \sin \phi)$$

$$= \frac{V_0^2}{g (1 + \sin \phi)}$$

4.9 (a) : Jawaban :



$$R = \frac{2 V_0^2}{g} \frac{\cos \alpha \sin (\alpha + \phi)}{\cos^2 \phi}$$

$-\varphi = \ldots$ $R_{max} = V_0^2 (1 + \sin \varphi)$ dan $\ldots 2\alpha = \frac{\pi}{2} - \varphi$

$$R_{max} = \frac{h}{\sin \varphi} = \frac{V_0^2}{g} \frac{(1 + \sin \varphi)}{\cos^2 \varphi} = \frac{V_0^2}{g (- \sin \varphi)}$$

$$\sin \varphi = \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2} \right)$$

$$\sin \varphi = \sin \left(\frac{\pi}{2} - 2\alpha \right) = \cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$1 - 2 \sin^2 \alpha = \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2} \right)$$

$$2 \sin^2 \alpha = \frac{2}{\csc^2 \alpha} = 1 - \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2} \right) = \frac{1}{1 + \frac{gh}{V_0^2}}$$

akhirnya $= \csc^2 \alpha = 2 \left(1 + \frac{gh}{V_0^2} \right)$

(b). Jawaban : $R_{max} = \frac{h}{\sin\varphi} = \frac{h}{1-2\sin^2\alpha} = \frac{h}{1-2/\csc^2\alpha}$

$$R_{max} = \frac{V_0^2}{g}\left(1+\frac{gh}{V_0^2}\right)$$

4.12 .(a) Jawaban :

x dan z adalah posisi bola

$x = V_0 t \cos\frac{1}{2}\theta_0$, $z = V_0 t \cos\frac{1}{2}\theta_0 \sin\theta_0 - \frac{1}{2}gt^2$

Sejak $V_x = V_0 \cos\frac{1}{2}\theta_0$

$R = \frac{V_0^2}{g}\cos^2\frac{1}{2}\theta_0 \sin 2\theta_0$

maks $\frac{dR}{d\theta} = 0$

$\frac{dR}{d\theta} = \frac{V_0^2}{g}\left(2\cos^2\frac{1}{2}\theta_0 \cos 2\theta_0 - \cos\frac{1}{2}\theta_0 \sin\frac{1}{2}\theta \sin 2\theta_0\right) = 0$

$2\cos^2\frac{1}{2}\theta_0 \cos 2\theta_0 = \cos\frac{1}{2}\theta_0 \sin\frac{1}{2}\theta_0 \sin 2\theta_0$

$2\cos^2\theta_0 = 1+\cos 2\theta_0$ dan $\sin 2\theta = 2\sin\theta_0 \sin 2\theta_0$

didapat : $(1+\cos\theta_0)(2\cos^2\theta_0 - 1) = \sin\theta_0 \sin\theta_0 \cos\theta_0 = (1-\cos^2\theta_0)\cos\theta_0$

atau $(1+\cos\theta_0)(3\cos^2\theta_0 - \cos\theta_0 - 1) = 0$

hanya akar positif yang berlaku adalah θ_0, $0 \le \theta \le \frac{\pi}{2}$

$\cos\theta_0 = \frac{1}{6}(1+\sqrt{13}) = 0,7676$ $\theta_0 = 39°51'$

(b). untuk $V_0 = 25$ m/s $R_{max} = 55,4$ m $\theta_0 = 39°51'$

maks $\frac{dz}{dt} = 0$

$V_0 \cos\frac{1}{2}\theta_0 \sin\theta_0 = gT$ atau $T = \frac{V_0 \cos 1/2\,\theta_0 \sin\theta_0}{g}$

atau $H = \frac{V_0^2}{2g}\cos^2\frac{1}{2}\theta_0 \sin^2\theta_0$ maks. tetap di θ_0

maks. pada ketinggian tetap terjadi $\frac{dH}{d\alpha} = 0$

$\frac{dH}{d\alpha} = \frac{V_0^2}{2g}\left(2\cos^2\frac{1}{2}\theta_0 \sin\theta_0 \cos\theta_0 - \cos\frac{1}{2}\theta_0 \sin\frac{1}{2}\theta_0 \sin^2\theta\right) = 0$

$(1+\cos\theta_0)\sin\theta_0\cos\theta_0 = \frac{1}{2}\sin\theta_0\ \cancel{\cos}\ \sin^2\theta_0 = \frac{1}{2}\sin\theta_0(1-\cos^2\theta_0)/$

$\sin\theta_0 (1+\cos\theta_0)(3\cos\theta_0 - 1) = 0$

$\sin\theta_0 = 0$ $\cos\theta_0 = -1$

Jadi $H_{max} = 18,9$ m $\theta_0 = \cos^{-1}\frac{1}{3} = 70,32°$

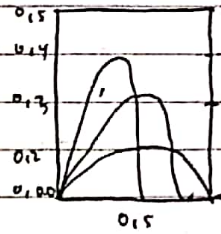
4.13 Jawaban :

r menggantikan x

$$z = \frac{\dot z_0}{\dot r_0} r - \frac{g}{2\dot r_0^2} r^2 \quad \text{dimana } \dot r_0 = V_0 \cos \theta_0 \quad \dot z_0 = V_0 \sin \theta_0$$

$$z = r \tan \theta_0 - \frac{gr^2}{2V_0^2} \sec^2 \theta \quad \text{sejak } \sec^2 \theta_0 = 1 + \tan^2 \theta$$

$$\frac{gr^2}{2V_0^2} \tan^2 \theta_0 - r \tan \theta_0 + z + \frac{gr^2}{2V_0^2} = 0$$



(r, z) titik koordinat

$$\tan \theta_0 = \frac{1}{gr} \left[V_0^2 \pm (V_0^4 - gzV_0^2 - g^2 r^2)^{1/2} \right]$$

$$V_0^4 - 2gzV_0^2 - g^2 r^2 \geq 0$$

Permukaan kritis oleh karena itu $V_0^4 - 2gz - g^2 r^2 = 0$

4.17 Jawaban :

$$m\ddot x = F_x = -\frac{\partial V}{\partial x} = -kx = -\pi^2 m x$$

$$x = A \cos \left(\sqrt{\frac{k}{m}}\, t + \alpha \right) = A \cos (\pi t + \alpha)$$

$$m\ddot y = -\frac{\partial V}{\partial y} = 4\pi^2 m x$$

$$y = B \cos (2\pi t + \beta)$$

$$m\ddot z = -\frac{\partial V}{\partial z} = -9\pi^2 m z$$

$$z = C \cos (3\pi t + \gamma)$$

Sejak $x = y = z = 0$ pada $t = 0$ $\quad \alpha = \beta = \gamma = -\frac{\pi}{2}$

$$x = A \cos (\pi t - \tfrac{\pi}{2}) = A \sin \pi t$$

$$\dot x_0 = A\pi \cos \pi t$$

$$V_0^2 = \dot x_0^2 + \dot y_0^2 + \dot z_0^2 \quad \text{dan } \dot x_0 = \dot y_0 = \dot z_0$$

$$\dot x_0 = \frac{V_0}{\sqrt 3} = A\pi$$

$$A = \frac{V_0}{\pi \sqrt 3}$$

$$x = \frac{V_0}{\pi \sqrt 3} \sin \pi t$$

$$y = B \sin 2\pi t, \quad \dot y = 2B\pi \cos 2\pi t$$

$$\dot y_0 = \frac{V_0}{\sqrt 3} = 2\pi$$

$$B = \frac{V_0}{2\pi \sqrt 3}$$

$$y = \frac{V_0}{2\pi\sqrt{3}} \sin 2\pi t$$

$$z = C \sin 3\pi t$$

$$\dot{z}_0 = \frac{V_0}{\sqrt{3}} = 3\pi t \qquad \dot{z} = 3C\pi \cos 3\pi t$$

$$\dot{z}_0 = \frac{V_0}{\sqrt{3}} = 3C\pi$$

$$C = \frac{V_0}{3\pi\sqrt{3}}$$

$$z = \frac{V_0}{3\pi\sqrt{3}} \sin 3\pi t$$

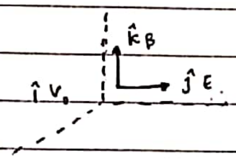
Sejak $\omega_x = \pi \cdot \omega_y = 2\pi$ dan $\omega_z = 3\pi$, maka

$$T_{min} = \frac{2\pi n_1}{\omega_x} = \frac{2\pi n_2}{\omega_y} = \frac{2\pi n_3}{\omega_z}$$

waktu maks pada $n_1 = 1$, $n_2 = 2$, $n_3 = 3$

$$t_{min} = \frac{2\pi}{\pi} = 2$$

2.20 Jawaban :



$$\vec{F} = q(\hat{E} + \vec{v} + \vec{B})$$

$$\vec{v} \times \vec{B} = (\hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z}) \times kB = \hat{i}\dot{y}B - \hat{j}\dot{x}B$$

$$\vec{F} = \hat{i}\, q\dot{y}B + \hat{j}q\,(E - \dot{x}B)$$

$$m\ddot{x} = F_x = q\dot{y}B$$

$$\dot{x} - \dot{x}_0 = \frac{qB}{m} y$$

$$m\ddot{y} = F_y = qE - q\dot{x}B = qE - qB\left(\dot{x}_0 + \frac{qB}{m} y\right)$$

$$\ddot{y} = \frac{qE}{m} - \frac{qB_x}{m} - \left(\frac{qB}{m}\right)^2 y = \frac{eE}{m} + \frac{EB\dot{x}}{m} - \left(\frac{eB}{m}\right)^2 y$$

$$\ddot{y} + \omega^2 y = -eE + \omega\dot{x}_0$$

$$y = \frac{1}{\omega^2}\left(-\frac{eE}{m}^m + \omega x\right) + A\cos(\omega t + \theta_0)$$

$$\dot{y} = -A\,\omega \sin(\omega t + \theta)$$

$y_0 = 0$, jadi $\theta_0 = 0$, $A = -\frac{1}{\omega^2}\left(\frac{eE}{m} + \omega\dot{x}_0\right)$

$$y = a(1 - \cos\omega t), \quad a = \frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}_0\right)$$

$$\dot{x} = \dot{x}_0 = \frac{qB}{m} y = \dot{x} - \omega y = \dot{x}_0 - \omega a\,(1 - \cos\omega t)$$

$$x = (\dot{x}_0 - \omega a)\,t + a \sin \omega t$$

$$x = a\sin\omega t + bt \qquad b = \dot{x}_0 - \omega a$$

$$m\ddot{z} = F_z = 0 \qquad z = \dot{z}_0 t + z_0 = 0$$