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4.1 Find the Force each of the following Potential energy functions :

a. $V = cxyz + C$
b. $V = \alpha x^2 + \beta y^2 + \gamma z^2 + c.$
c. $V = ce^{-(\alpha x + \beta y + \gamma z)}.$
d. $V = cr^n$ in spherical coordinates.

4.2 By finding the curl, determine which of the following Forces are conservative :

a. $F = ix + jy + kz$
b. $F = iy - jx + kz^2$
c. $F = iy + jx + kz^3$
d. $F = -kr^{-n} e_r$ in spherical coordinates.

4.3 Find the value of the constant C such that each of the following forces is conservative :

a. $F = ixy + jcx^2 + kz^2$
b. $F = i(z/y) + cj(xz/y^2) + k(x/y).$

4.4 A Particle of massa m moving in the three dimensions under the Potential energy function $V(x,y,z) = \alpha x + \beta y^2 + \gamma z^3$ has speed if and when it passes through the origin.

a. What will its speed V_0 when it passes though the point $(1,1,1)$?
b. If the point $(1,1,1)$ is a turning point in the motion $(v = 0)$. What is V_0?

c. What are the component differential equations of motion of the particle?
<note: it is not necessary to solve the differential equation in this problem.

45. Consider the two functions.
a. $F = ix + jy$
b. $F = iy - jx$.
verify that <a> is conservative and that <b> is nonconservative by showing that the integral $\int F \cdot dr$ is independent of the path of integration for <a>, but not for <b>, by taking two paths in wich the starting points is the origin <0,0> and the endpoint is <1,1>. For one path take the line $x = y$. For the other path take the x-axis out to the points <1,0> and the lime $x=1$ up to the point <1,1>.

46. Show that the variation of gravity with beight can be accuanted for approximately by the following potential energy function:

$$V = mgz\left[1 - \frac{z}{r_e}\right]$$

In which r_e is the radius of the earth. Find the force given by the above potential function. From this find the component differential equations of motion of a projectile under such a force. if the vertical component of the initial velocity is V_{0z}, how high does the projectile go <compare with example 2.3.2>.

47. Particles of mud are thrown from the rim of a rolling wheel. if the greatest height above the ground that the mud can go is

$$b+\frac{v_0^2}{2g}+\frac{gb^2}{2v_0^2}$$

at what Point on the rolling wheel does this mud Leave?
<note: it is necessary to assume that $v_0^2 \geq bg$>.

4.8 A gun is located at the bottom of a hill of constant slope ϕ. Show that the range of the gun measured up the slope of the hill is.

$$\frac{2v_0^2 \cos\alpha \sin\langle\alpha-\phi\rangle}{g\cos^2\phi}$$

where α is the angle elevation of the gun, and that the maximum Value of ~~t~~ the slope range is

$$\frac{v_0^2}{g\langle 1+\sin\phi\rangle}$$

4.9. A cannon that is capable of firing a shell at speed v_0 is mounted on a vertical tower of height h that overlooks a level plain below.

a. Show that the elevation angle α at which the cannon must be set to achieve maximum range is given by the expression.

$$\csc^2\alpha = 2\left[1+\frac{gh}{v_0^2}\right]$$

b. What is the maximum range R of the cannon?

4.12. A Baseball Pitcher can throw a ball more easily horizontally than vertically, assume that the Pitcher's throwing speed varies with elevation angle approximately as $v_0 \cos\frac{1}{2}\theta_0$ m/s. where θ_0 is the initial elevation angle and v_0 is the initial when the ball is thrown horizontally. Find the angle θ_0 at which the ball

must be thrown to. achieve maximum <a> height and <b> range.
find the values of the maximum <c> height and <d> range. assume no air resistance and Let $v_0 = 25$ m/s.

4.13. A gun can fire an artillery shell with a speed v_0 in any direction. Show that a shell can strike any target within the surface given by.

$$g^2 r^2 = v_0^4 - 2 g v_0^2 z.$$

where z is the height of the target and r is its horizontal distance from the gun. assume no air resistance.

4.17. A small Lead ball of massa m is suspended by means of six light springs as shown in figure 4.4.1 The stiffness Constants are in the ratio 1:4:9, so that the potensial energy function can be expressed as.

$$V = \frac{k}{2} \langle x^2 + 4y^2 + 9z^2 \rangle.$$

at time $t = 0$ the ball receives a push in the $\langle 1,1,1 \rangle$ Direction that imparts to it a speed v_0 at the origin

2.20. An electron moves in a force field due to a uniform electric field E and a uniform magnetic field B that is at right angles to E. Let $E = jE$ and $B = kB$ take initial position of the electron at the origin with initial velocity $v_0 = i v_0$ in the x direction. Find the resulting motion of the particle. Show that the path of motion is a cycloid:

$$x = a \sin \omega t + bt$$
$$y = a \langle 1 - \cos \omega t \rangle$$

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$z = 0.$

Cycloidal motion of electrons is used in an electronic tube called a magnetron to produce the microwaves in a microwave oven.

answer

4.1. (a) $\vec{F} = -\bar{\nabla}V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}.$

$\vec{F} = -c\langle \hat{i}yz + \hat{j}xz + \hat{k}xy\rangle$

(b) $\vec{F} = -\bar{\nabla}V = -\hat{i}\,2\alpha x - \hat{j}\,2\beta y - \hat{k}\,2\gamma z.$

(c) $\vec{F} = -\bar{\nabla}V = ce^{-\langle \alpha x+\beta y+\gamma z\rangle}\langle \hat{i}\alpha + \hat{j}\beta + \hat{k}\gamma$

(d) $\vec{F} = -\bar{\nabla}V = -\hat{e}_r\frac{\partial V}{\partial r} - \hat{e}_\theta\frac{1}{r}\frac{\partial V}{\partial \theta} - \hat{e}_\phi\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}.$

$\vec{F} = -\hat{e}_r\, cnr^{n-1}$

4.2. (a)

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0.$$

(b)

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & z^2 \end{vmatrix} = \hat{k}\langle -1-1\rangle \neq 0$$

(c)

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} i & j & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x & z^2 \end{vmatrix} = \hat{k}\,\langle 1-1\rangle = 0.$$

(d)

$$\vec{\nabla}\times\vec{F} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{e}_r & \hat{e}_\theta r & \hat{e}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \theta} \\ -kr^{-n} & 0 & 0 \end{vmatrix} = 0$$

4.3. (a)

$$\bar{\nabla}\times\bar{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & Cx^2 & z^3 \end{vmatrix} = k\langle 2Cx - x\rangle$$

$2Cx - x = 0$

$C = \frac{1}{2}.$

(b).

$$\bar{\nabla}\times\bar{F} \begin{vmatrix} i & j & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{z}{y} & \frac{Cxz}{y^2} & \frac{x}{y} \end{vmatrix}$$

$$= i\left[-\frac{x}{y^2} - \frac{Cx}{y^2}\right] + j\left[\frac{1}{y} - \frac{1}{y}\right] + \hat{k}\left[\frac{Cz}{y^2} + \frac{z}{y^2}\right]$$

$-\frac{x}{y^2} - \frac{Cx}{y^2} = 0.$ $C = -1$

also $\frac{Cz}{y^2} + \frac{z}{y^2} = 0$ implies that $C = -1$ as it must.

4.4. (a) . $E = \text{constant} = V\langle x,y,z\rangle + \frac{1}{2}mv^2$

at the origin $E = 0 + \frac{1}{2}mv^2$

at $\langle 1.1.1\rangle$ $E = \alpha + \beta + \gamma + \frac{1}{2}mv^2 = \frac{1}{2}mv^2.$

$v^2 = v_0^2 - \frac{2}{m}\langle\alpha + \beta + \gamma\rangle.$

$v = \left\{v^2 - \frac{2}{m}\langle\alpha + \beta + \gamma\rangle\right]^{\frac{1}{2}}$

(b) . $v^2 - \frac{2}{m}\langle\alpha + \beta + \gamma\rangle = 0.$

$v = \left[\frac{2}{m}\langle\alpha + \beta + \gamma\rangle\right]^{\frac{1}{2}}$

(c) . $m\ddot{x} = F_x = -\frac{\partial V}{\partial x}$

$m\ddot{x} = -\alpha$

$m\ddot{y} = -\frac{\partial V}{\partial y} = -2\beta y$

A Champion is someone who gets up even when they can't

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$$m\ddot{z} = -\frac{\partial V}{\partial z} = -3yz^2.$$

4.5. (a) $\bar{F} = \hat{i}x + \hat{j}y$

on the path $x = y$ $\quad d\bar{r} = \hat{i}dx + \hat{j}dy.$

$$\int_{(0,0)}^{(1,1)} \bar{F}\cdot d\bar{r} = \int_0^1 F_x\, dx + \int_0^1 F_y\, dy = \int_0^1 x\,dx + \int_0^1 y\,dy = 1$$

on the Path along the x-axis: $\quad d\bar{r} = \hat{i}\,dx$

and on the line $x = 1$: $\quad d\bar{r} = \hat{j}\,dy.$

$$\int_{(0,0)}^{(1,1)} \bar{F}\cdot d\bar{r} = \int_1^1 F_x\, dx + \int_0^1 F_y\, dy = 1$$

$\bar{F}$ is conservative

(b) $\bar{F} = \hat{i}y - \hat{j}x$

on the path $x = y$:

$$\int_{(0,0)}^{(1,1)} \bar{F}\cdot d\bar{r} = \int_0^1 F_x\, dx + \int_0^1 F_y\, dy = \int_0^1 y\,dx - \int_0^1 x\,dy.$$

and with $x = y$. $\int_{0,0}^{1,1} \bar{F}\cdot d\bar{r} = \int_0^1 x\,dx - \int_0^1 y\,dy = 0.$

one the x-axis $\int_{0,0}^{1,0} F\cdot dr = \int_0^1 F_x\, dx = \int_0^1 y\,dx$

and, with $y = 0$ on the x-axis $\int_{0,0}^{1,0} \bar{F}\cdot d\bar{r} = 0.$

one the line $x = 1$ $\int_{1,0}^{1,1} \bar{F}\cdot d\bar{r} = \int_0^1 F_y\, dy = \int_0^1 x\,dy.$

And with $x = 1$ $\int_{1,0}^{1,1} F\,dr = \int dy = 1$

on this Path: $\int_{0,0}^{1,1} F\cdot dr = 0 + 1 = 1$

F is not conservative.

9.5. From example 2.32. $V(z) = -mg \dfrac{r_e^2}{(r_e+z)}$

$$V(z) = -mgr_e \left(1 + \frac{z}{r_e}\right)^{-1}$$

from Appendix D $(1+x)^{-1} = 1 - x + x^2 + \ldots$

$$V(z) = -mgr_e \left[1 - \frac{z}{r_e} + \frac{z^2}{r_e^2} + \ldots\right]$$

$$V(z) = -mgr_e + mgz - \frac{mgz^2}{r_e} + \ldots$$

with $-mgr_e$ an additive constant.

$$V(z) = mgz\left[1 - \frac{z}{r_e}\right]$$

$$\bar{F} = -\bar{\nabla}V = -\hat{k}\frac{\partial}{\partial z} V(z).$$

$$= -\hat{k}mg\left[1 - \frac{z}{r_e} + z\left(-\frac{1}{r_e}\right)\right]$$

$$\bar{F} = -\hat{k}mg\left(1 - \frac{2z}{r_e}\right)$$

$m\ddot{x} = F_x = 0$ $\quad m\ddot{y} = F_y = 0$ $\quad m\ddot{z} = -mg\left(1 - \frac{2z}{r_e}\right)$

$$m\dot{z}\frac{d\dot{z}}{dz} = -mg\left[1 - \frac{2z}{r_e}\right]$$

$$\int_v^0 \dot{z}\,d\dot{z} = -g\int_0^h \left[1 - \frac{2z}{r_e}\right]dz.$$

$$-\frac{1}{2}V_z^2 = -g\left[h - \frac{h^2}{r_e}\right].$$

$$h^2 - r_e h + \frac{r_e v_z^2}{2g} = 0.$$

$$h = \frac{r_e}{2} - \frac{1}{2}\sqrt{r_e^2 - \frac{2r_e v_z^2}{g}} \quad (h \cdot z \ll r_e)$$

$$h = \frac{r_e}{2} - \frac{r_e}{2}\sqrt{1 - \frac{2v^2}{gr_e}}.$$

From appendix D. $(1+x)^{\frac{1}{2}} = 1 + \frac{x}{2} - \frac{x^2}{8} + \ldots$

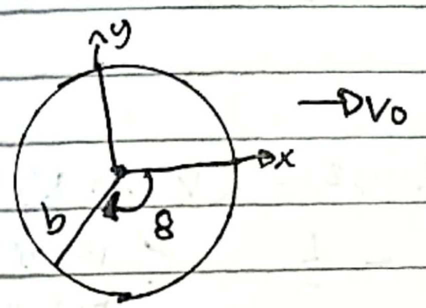
$$h = \frac{r_e}{2} - \frac{r_e}{2} + \frac{V_{02}^2}{2g} + \frac{V_{02}^4}{4gr_e} + \dots$$

$$h = \frac{V_{02}^2}{2g}\left(1 + \frac{V_{02}^2}{2gr_e}\right)$$

From example 2.3.2 $h = \frac{V_0^2}{2g}\left(1 - \frac{V^2}{2gr}\right)^{-1}$

and with $(1-x)^{-1} \simeq 1+x$, $h = \frac{V^2}{2g}\left(1 + \frac{V^2}{2gr}\right)$

2.7.



For a point rim measured from the center of the wheel:

$\bar{r} = ib\cos\theta - jb\sin\theta$

$\theta = \omega t = \frac{V_0 t}{b}$ so $\bar{r} = -iV\sin\theta - jV\cos\theta$

relative to the ground,

$\vec{V} = iV(1-\sin\theta) - jV\cos\theta$

For a particle of mud leaving the rim:

$y = -b\sin\theta$ and, $V_y = -V_1\cos\theta$

so $V_y = V_{0y} - gt = -V_1\cos\theta - gt.$

and $y = -b\sin\theta - V_0 t\cos\theta - \frac{1}{2}gt^2$

at maximum height, $V_y = 0.$

$$t = -\frac{V_0\cos\theta}{g}.$$

$$h = -b\sin\theta - V_0\left(-\frac{V_0\cos\theta}{g}\right)\cos\theta\,\frac{1}{2}g\left(-\frac{V_0\cos\theta}{g}\right)$$

$$h = -b\sin\theta + \frac{V_1^2\cos^2\theta}{2g}$$

$$\sin\theta = -\frac{gb}{V^2}$$

$$\cos^2\theta = 1 - \sin^2\theta = \frac{V^2 - g^2b^2}{V^4}$$

$$h_{max} = \frac{gb^2}{v^2} + \frac{v^4 - g^2b^2}{2gv^2} = \frac{gb^2}{2v^2} + \frac{v^2}{2g}$$

measured from the ground.

$$h_{max} = b + \frac{gb^2}{2v^2} + \frac{v^2}{2g}.$$

the mud Leaves the wheel at $\theta = \sin^{-1}\left(-\frac{gb}{v^2}\right)$

4?. $x = R\cos\phi$ an $x = v_x \cdot T = (v_0 \cos\alpha)T$,

$$\text{so } T = \frac{R\cos}{v\cos\alpha}$$

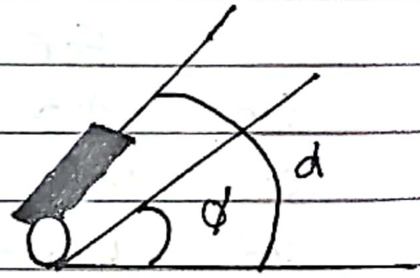
$y = R\sin\phi$ and $y = v_1 t - \frac{1}{2}gt^2 = (v\sin\alpha)t - \frac{1}{2}gt^2$,

$$R\sin\phi = (v_1 \sin\alpha)\frac{R\cos\phi}{v\cos\alpha} - \frac{1}{2}g\left(\frac{R\cos\phi}{v\cos\alpha}\right)$$

$$\sin\phi = \tan\alpha\cos\phi - \frac{gR\cos^2\phi}{2v^2\cdot\cos^2\alpha}$$

$$R = \frac{2v^2\cdot\cos\alpha}{g\cos^2\phi}(\tan\alpha\cos\phi - \sin\phi)$$

$$= 2v^2\cdot\cos\alpha\cos\phi - \cos\alpha\sin\phi)$$



From appendix B, $\cos(\theta.\phi) = 0$.

implies that $\cos\alpha\cos(\alpha-\phi) - \sin\alpha\sin(\alpha-\phi\cdots) = 0$.

From appendix B, $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$.

So $\cos(2\alpha - \phi) = 0$.

$$2\alpha - \phi = \frac{\pi}{2}. \qquad \alpha = \frac{\pi}{4} + \frac{\phi}{2}.$$

$$R_{max} = \frac{2v^2}{g\cos^2\phi}\cos\left[\frac{\pi}{4} + \frac{\phi}{2}\right]\sin\left[\frac{\pi}{4} - \frac{\phi}{2}\right]$$

again using Appendix B, $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$$R_{max} = \frac{2v^2}{g\cos^2\phi}\left[\frac{1}{2}\cos\left(\frac{\pi}{2} + \phi\right)\right] + \frac{1}{2}\right] = \frac{v_0^2}{g\cos^2\phi}$$

$$\left[\cos\left(\frac{\pi}{2} + \phi\right) + 1\right]$$

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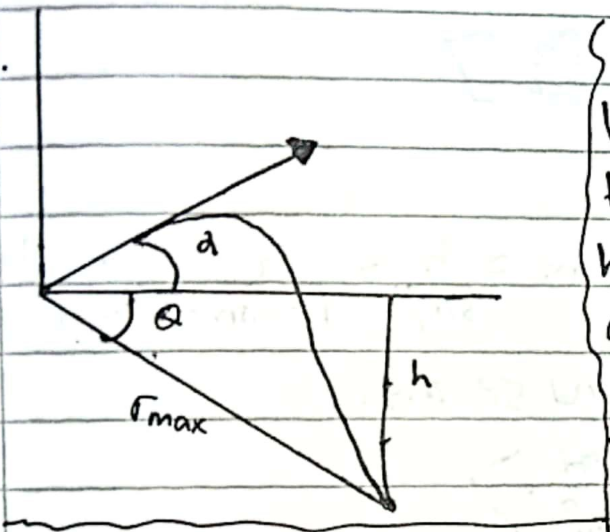
using $\cos\langle \frac{\pi}{2} + \theta \rangle = -\sin\theta$

$$R_{max} = \frac{v^2}{g\langle 1-\sin^2\theta \rangle} \langle 1-\sin\phi \rangle$$

$$R_{max} = \frac{v^2}{g\langle 1+\sin\phi \rangle}$$

49.



a). here we not that the projectile is launched "downhill" towards the target. which is located a distance h below the cannon along a line at an angle ϕ below the horizon. α is the angle of projection that yields maximum range. Rmax we can use the result from problem 4.8 for this problem. we simply have to replace the angle ϕ in the above result with the angle $-\phi$ to acount for the downhill slope. thus we get for the downhill range...

$$R = \frac{2 v_0^2}{g} \frac{\cos\alpha \sin\langle \alpha+\phi \rangle}{\cos^2\phi}.$$

the maximum range and the is α are obaitned from the problem above again by replacing ϕ with the angle, $-\phi$

$$R_{max} = \frac{V_0^2 \langle 1+\sin\phi \rangle}{g \cos^2\phi} \text{ and}$$

$$2\alpha = \frac{\pi}{2} - \phi.$$

we can now calculate α.. $R_{max} = \frac{h}{\sin\phi} = \frac{V_0^2}{g} \cdot \frac{\langle 1+\sin\phi \rangle}{\cos^2\phi}$

$$= \frac{V_0^2}{g\langle 1 \sin\phi \rangle}$$

Solving for $\sin\phi$.. $\sin\phi = \frac{gh}{V_0^2} / \left[1 + \frac{gh}{V_0^2} \right]$

But from the above $\sin\phi = \sin\left[\frac{\pi}{2} - 2\alpha\right] = \cos 2\alpha$

$= 1 - 2\sin^2\alpha$

thus... $1 - 2\sin^2\alpha = \frac{gh}{v_0^2} / \langle 1 + \frac{gh}{v_0^2} \rangle$

$$2\sin^2\alpha = \frac{2}{\csc^2\alpha} = 1 - \frac{gh}{v_0^2} / \left[1 + \frac{gh}{v_0^2}\right] = \frac{1}{1+\frac{gh}{v_0^2}}.$$

Finally... $\csc^2\alpha = 2\left[1 + \frac{gh}{v_0^2}\right]$

b. Solving for R_{max} ... $R_{max} = \frac{h}{\sin\phi} = \frac{h}{1-2\sin^2\alpha} = \frac{h}{1-2/\csc^2\alpha}$

Subtituting for $\csc^2\alpha$ and solving.

$$R_{max} = \frac{v_0^2}{g} \langle 1 + \frac{gh}{v_0^2} \rangle$$

4.12 The x and z Positions of the ball vs time are

$x = v_1 t \cos\frac{1}{2}\theta$ $\quad z = v_1 t \cos\frac{1}{2}\theta \sin\theta - \frac{1}{2}gt^2.$

since $v_2 = v_0 \cos\frac{1}{2}\theta$

The horizontal range is $R = \frac{v^2}{g} \cos^2\frac{1}{2}\theta, \sin 2\theta$

The maximum range occurs @ $\frac{dR}{d\theta} = 0.$

$$\frac{dR}{d\theta} = \frac{v^2}{g} \langle 2\cos^2\frac{1}{2}\theta, \cos 2\theta, -\cos\frac{1}{2}\theta, \sin\frac{1}{2}\theta, \sin 2\theta \rangle$$

$= 0.$

thus: $2\cos^2\frac{1}{2}\theta \cos 2\theta = \cos\frac{1}{2}\theta \sin\frac{1}{2}\theta, \sin 2\theta$

using the identities $2\cos^2\theta = 1 + \cos 2\theta$, and $\sin 2\theta = 2\sin\theta, \cos\theta$, we get.

$$\langle 1 + \cos\theta \rangle \langle 2\cos^2\theta - 1 \rangle = \sin\theta, \sin\theta, \cos\theta$$

$= \langle 1 - \cos^2\theta \rangle . \cos\theta$ or.

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$\langle 1+\cos\theta\rangle\langle 3\cos^2\theta-\cos\theta-1\rangle=0.$

This $\cos\theta_1=-1$ $\cos\theta=\frac{1}{6}\langle 1\pm\sqrt{13}\rangle.$

Only the positive root applies for the θ_1 range $= 0\le\theta_1\frac{\pi}{2}.$

$\cos\theta_1=\frac{1}{6}\langle 1+\sqrt{13}\rangle=0.7676.$ $\underline{\theta_0=39°51'.}$

Thus $\langle 6\rangle$ for $V_0=25ms^{-2}$ $\underline{R_{max}=55.9m \quad \theta_0=39°51'}$

(a) The maximum height occurs at $\frac{dz}{dt}=0.$

$V_0\cos\frac{1}{2}\theta=gT$ or at $T=\frac{V\cos\frac{1}{2}\theta\cdot\sin\theta}{g}.$

or $H=\frac{V_0^2}{2g}\cos^2\frac{1}{2}\theta\cdot\sin^2\theta.$ Maximum at fixed θ.

The maximum possible height occurs $\frac{dH}{da}=0.$

$$\frac{dH}{d\alpha}=\frac{V^2}{2g}\langle 2\cos^2\frac{1}{2}\theta,\sin\theta,\cos\theta,-\cos\frac{1}{2}\theta,\sin\frac{1}{2}\theta,\sin^2\theta^2\rangle=0.\quad \text{or}\quad \sin\theta\langle 1+\cos\theta\rangle\langle 3\cos\theta-1\rangle=0.$$

There are 3-roots: $\sin\theta=0,$ $\cos=-1$ $\cos\theta=\frac{1}{3}.$

The first two roots give minimum height: the last gives the maximum thus. $H_{max}=18.9m$ $\theta=\cos^{-1}\frac{1}{3}=70°32'$

4.13 The trajectory of the shell is given by Eq. 4.3.11 with r replacing x

$$z=\frac{\dot z}{\dot r}r-\frac{g}{2\dot r^2}r^2 \quad \text{whe}\quad \dot r_0=V_0\cos\theta_0 \quad \dot z=V_0\sin\theta.$$

Thus, $z=r\tan\theta-\frac{gr^2}{2v^2}\sec^2\theta.$

Since $\sec^2\theta=1+\tan^2\theta.$

we have:

$$\frac{gr^2}{2v^2}\tan^2\theta-r\tan\theta+z+\frac{gr^2}{2v^2}=0.$$

(r, z) are target coordinates.
The above equation yields
two possible roots:

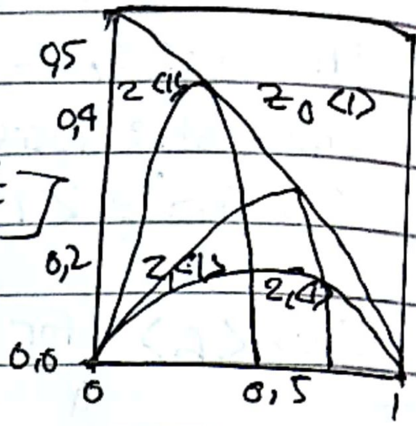
$$\tan\theta = \frac{1}{gr}\left[v^2 \pm (v^4 - 2gzv^2 - g^2r^2)^{\frac{1}{2}}\right]$$

$\therefore$ the roots are only real if

$v^4 - 2gzv^2 - g^2r^2 \geq 0$

The critical surface is therefore:

$v^4 - 2gzv^2 - g^2r^2 = 0$.



4.17 $m\ddot{x} = F, = -\frac{\partial V}{\partial x} = -kx = -\pi^2 mx$.

$x = A\cos\left[\sqrt{\frac{k}{m}}\,t + \alpha\right] = A\cos(\pi t + \alpha)$.

$m\ddot{y} = -\frac{\partial V}{\partial y} = -4\pi^2 my$.

$y = \beta\cos(2\pi t + \beta)$.

$m\ddot{z} = -\frac{\partial V}{\partial z} = -9\pi^2 mz$.

$z = C\cos(3\pi t + \gamma)$.

since $x = y = z = 0$ at $t = 0$. $\alpha = \beta = \gamma = -\frac{\pi}{2}$.

$x = A\cos\left[\pi t - \frac{\pi}{2}\right] = A\sin\pi t$

$\dot{x} = A\pi\cos\pi t$.

since $V_0^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$ an $\dot{x} = \dot{y} = \dot{z}$.

$$\dot{x} = \frac{V_0}{\sqrt{3}} = A\pi$$

$$A = \frac{V_0}{\pi\sqrt{3}}$$

$$x = \frac{V_0}{\pi\sqrt{3}}\sin\pi t$$

$y = \beta\sin 2\pi t \qquad \dot{y} = 2\beta\pi\cos 2\pi t$

$$\dot{y}_0 = \frac{V_0}{2\pi\sqrt{3}}$$

$$B = \frac{V_0}{2\pi\sqrt{3}} \sin 2\pi t$$

$$z = C \sin 3\pi t \qquad \dot z = 3C\pi \cos 3\pi t.$$

$$\dot z = \frac{V_0}{\sqrt{3}} = 3C\pi$$

$$C = \frac{V_0}{3\pi\sqrt{3}}$$

$$z = \frac{V_0}{3\pi\sqrt{3}} \sin 3\pi t.$$

since $\omega_x = \pi$, $\omega_y = 2\pi$. and $\omega_z = 3\pi$ the ball does retrace its path.

$$t_{min} = \frac{2\pi n_1}{\omega_x} = \frac{2\pi n_2}{\omega_y} = \frac{2\pi n_3}{\omega_z}$$

the minimum time occurs at $n_1 = 1$ $n_2 = 2$ $n_3 = 3$

$$t_{min} = \frac{2\pi}{\pi} = 2.$$

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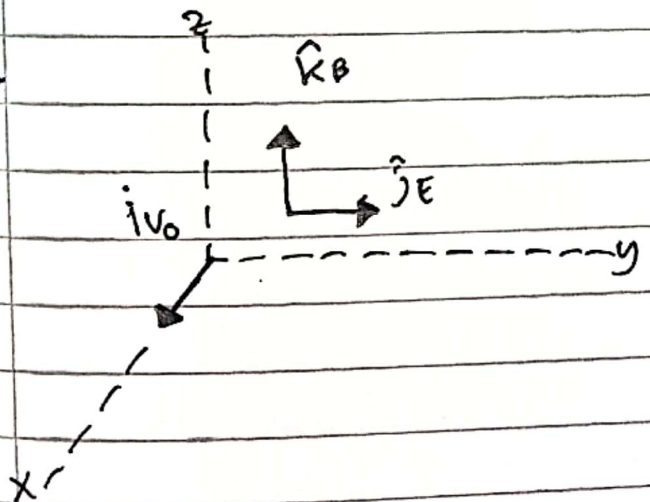

$$\vec F = q(\vec E + \vec v \times \vec B)$$

$$\vec v \times \vec B = (\hat i \dot x + \hat j \dot y + \hat k \dot z) \times \hat k B$$

$$= \hat i \dot y B - \hat j \dot x B$$

$$\vec F = \hat i q \dot y B + \hat j q(E - \dot x B).$$

$$m\ddot x = F_x = q\dot y B$$

$$\dot x - \dot x = \frac{qB}{m} y$$

$$m\ddot y = F_y = qE - q\dot x B = qE - qB(\dot x + \frac{qB}{m} y).$$

$$\ddot y = \frac{qE}{m} - \frac{qB\dot x}{m} - \left[\frac{qB}{m}\right]^2 y = -\frac{eE}{m} + \frac{eB\dot x}{m} - \left[\frac{eB}{m}\right]^2 y$$

$$\ddot y + \omega^2 y = -\frac{eE}{m} + \omega \dot x \qquad \omega = \frac{eB}{m}.$$

$$y = \frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega \dot x\right) + A\cos(\omega t + \theta).$$

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$y = a\langle 1-\cos\omega t\rangle.\quad a = \frac{1}{\omega^2}\langle \frac{1}{}\frac{eE}{m} + \omega\dot{x}\rangle.$

$y = a\langle 1-\cos\omega t\rangle,\quad a = \frac{1}{\omega^2}\langle -\frac{eE}{m} + \omega\dot{x}\rangle.$

$\dot{x} = \dot{x} + \frac{qB}{m}y = \dot{x} - \omega y = \dot{x} - \omega a\langle 1-\cos\omega t\rangle.$

$\dot{x} = \langle \dot{x} - \omega a\rangle + \omega a\cos\omega t.$

$\dot{x} = \langle \dot{x} - \omega a\rangle t + a\sin\omega t$

$\dot{x} = a\sin\omega t + bt.\quad b = \dot{x} - \omega a.$

$mz = F_2 = 0$

$z = \dot{z}t + z_0 = 0.$