No Chapter 4
Date Mekanika

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Kelas : B
Prodi : Pendidikan fisika

4.1. Penyelesaian :

(a) $\vec{F} = -\bar{\nabla} V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$

$\vec{F} = -C\,(\hat{i}yz + \hat{j}xz + \hat{k}xy)$

(b) $\vec{F} = -\bar{\nabla} V = -\hat{i}\,2\alpha x - \hat{j}\,2\beta y - \hat{k}\,2\gamma z$

(c) $\vec{F} = -\bar{\nabla} V = ce^{-(\alpha x + \beta y + \gamma z)}\left(\hat{i}\alpha + \hat{j}\beta + \hat{k}\gamma\right)$

(d) $\vec{F} = -\bar{\nabla} V = -\hat{e}_r\frac{\partial V}{\partial r} - \hat{e}_\theta\frac{1}{r}\frac{\partial V}{\partial \theta} - \hat{e}_\phi\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}$

$\vec{F} = -\hat{e}_r\, cnr^{n-1}$

4.2. Penyelesaian :

(a)
$$\bar{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix} = 0 \quad \text{(konservatif)}$$

(b)
$$\bar{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & -x & z^2 \end{vmatrix} = \hat{k}(-1-1) \neq 0$$

(c)
$$\bar{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & x & z^3 \end{vmatrix} = \hat{k}(1-0) = 0 \quad \text{(konservatif)}$$

(d)
$$\bar{\nabla}\times\vec{F} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{e}_r & \hat{e}_\theta r & \hat{e}_\phi r\sin\theta \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial \phi \\ -kr^{-n} & 0 & 0 \end{vmatrix} = 0 \quad \text{(konservatif)}$$

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4.3. Penyelesaian.

(a)
$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy & Cx^2 & z^3 \end{vmatrix} = \hat{k}\,(2Cx - x)$$

$$2Cx - x = 0$$
$$C = 1/2$$

(b)
$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z/y & Cxz/y^2 & x/y \end{vmatrix}$$
$$= \hat{i}\left(-\frac{x}{y^2} - \frac{Cx}{y^2}\right) + \hat{j}\left(\frac{1}{y} - \frac{1}{y}\right) + \hat{k}\left(\frac{Cz}{y^2} + \frac{z}{y^2}\right)$$

$$-\frac{x}{y^2} - \frac{Cx}{y^2} = 0 \Rightarrow C = 1$$

$$\frac{Cz}{y^2} + \frac{z}{y^2} = 0 \Rightarrow C = -1$$

4.4. Penyelesaian :

(a) $E = \text{constant} = V(x,y,z) + \frac{1}{2}mv^2$
$$E = 0 + \tfrac{1}{2}mv^2$$

pd 1.1.1 $\quad E = \alpha + \beta + \gamma + \frac{1}{2}mv^2 = \frac{1}{2}mv^2$

$$v^2 = v_0^2 - \frac{2}{m}(\alpha + \beta + \gamma)$$

$$v = \left[v_0^2 - \frac{2}{m}(\alpha + \beta + \gamma)\right]^{1/2}$$

(b) $v_0^2 - \frac{2}{m}(\alpha + \beta + \gamma) = 0$

$$v_0 = \left[\frac{2}{m}(\alpha + \beta + \gamma)\right]^{\frac{1}{2}}$$

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(c) $m\ddot{x} = F_x = -\frac{\partial V}{\partial x}$

$m\ddot{x} = -\alpha$

$m\ddot{y} = -\frac{\partial V}{\partial y} = -2\beta y$

$m\ddot{z} = -\frac{\partial V}{\partial z} = -3\gamma z^2$

4.5. Penyelesaian

(a) $\vec{F} = \hat{i}x + \hat{j}y$

•) $x = y \longrightarrow d\vec{r} = \hat{i}dx + \hat{j}dy$

$$\int_{(0,0)}^{(1,1)} \vec{F}\cdot d\vec{r} = \int_0^1 F_x\, dx + \int_0^1 F_y\, dy = \int_0^1 x\, dx + \int_0^1 y\, dy = 1$$

•) pd x - axis : $d\vec{r} = \hat{i}dx$

$x = 1 \quad d\vec{r} = \hat{j}dy$

$$\int_{(0,0)}^{(1,1)} \vec{F}\cdot d\vec{r} = \int_0^1 F_x\, dx + \int_0^1 F_y\, dy = 1$$

$\vec{F}$ adalah konservatif

(b) $\vec{F} = \hat{i}y - \hat{j}x$

•) $x = y$

$$\int_{(0,0)}^{(1,1)} \vec{F}\cdot d\vec{r} = \int_0^1 F_x\, dx + \int_0^1 F_y\, dy = \int_0^1 y\, dx - \int_0^1 x\, dy$$

•) $x = y \quad \int_{(0,0)}^{(1,1)} \vec{F}\cdot d\vec{r} = \int_0^1 x\, dx - \int_0^1 y\, dy = 0$

•) x - axis $\int_{(0,0)}^{(1,0)} \vec{F}\cdot d\vec{r} = \int_0^1 F_x\, dx = \int_0^1 y\, dx$

dg $y = 0$, x - axis $\int_{(0,0)}^{(1,0)} \vec{F}\cdot d\vec{r} = 0$

•) $x = 1 \quad \int_{(1,0)}^{(1,1)} \vec{F}\cdot d\vec{r} = \int_0^1 F_y\, dy = \int_0^1 x\, dy$

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by $x = 1$ $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_0^1 dy = 1$

on this path $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = 0 + 1 = 1$

Jadi, $\vec{F}$ bukan konservatif

(2-3-2)

4.6. •) $V(z) = -mg \dfrac{r_e^2}{(r_e + z)}$

$= -mgr_e \left(1 + \dfrac{z}{r_e}\right)^{-1}$

•) from Appendix D, $(1+x)^{-1} = 1 - x + x^2 + \ldots$

$V(z) = -mgr_e \left(1 - \dfrac{z}{r_e} + \dfrac{z^2}{r_e^2} + \ldots\right.$

$= -mgr_e + mgz - \dfrac{mgz^2}{r_e} + \ldots$

by $-mgr_e$ additive constant,

$V(z) \approx mgz \left(1 - \dfrac{z}{r_e}\right)$

$\vec{F} = -\vec{\nabla} V = -\hat{k} \dfrac{\partial}{\partial z} V(z)$

$= -\hat{k} mg \left[1 - \dfrac{z}{r_e} + z\left(-\dfrac{1}{r_e}\right)\right]$

$\vec{F} = -\hat{k} mg \left(1 - \dfrac{2z}{r_e}\right)$

$\Rightarrow m\ddot{x} = F_x = 0$ $\Rightarrow m\ddot{y} = F_y = 0$ $\Rightarrow m\ddot{z} = -mg\left(1 - \dfrac{2z}{r_e}\right)$

$m\dot{z} \dfrac{d\dot{z}}{dz} = -mg\left(1 - \dfrac{2z}{r_e}\right)$

$\int_{V_z}^{0} \dot{z}\, dz = -g \int_0^h \left(1 - \dfrac{2z}{r_e}\right) dz$

$-\dfrac{1}{2} V_{0z}^2 = -g\left(h - \dfrac{h^2}{r_e}\right)$

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$$h^2 - r_e h + \frac{r_e V_{0z}^2}{2g} = 0$$

$$h = \frac{r_e}{2} - \frac{1}{2}\sqrt{r_e^2 - \frac{2 r_e V_{0z}^2}{g}} \quad (h, z \ll r_e)$$

$$h = \frac{r_e}{2} - \frac{r_e}{2}\sqrt{1 - \frac{2V_{0z}^2}{g r_e}}$$

From Appendix D, $(1+x)^{\frac{1}{2}} = 1 + \frac{x}{2} - \frac{x^2}{8} + \dots$

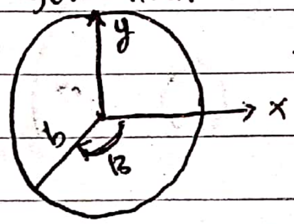
$$h \approx \frac{r_e}{2} - \frac{r_e}{2} + \frac{V_{0z}^2}{2g} + \frac{V_{0z}^4}{4 g r_e^2} + \dots$$

$$h \approx \frac{V_{0z}^2}{2g}\left(1 + \frac{V_{0z}^2}{2 g r_e}\right)$$

From (2.3.2.) $\Rightarrow h = \frac{V_0^2}{2g}\left(1 - \frac{V_0^2}{2 g r_e}\right)^{-1}$

dan $(1-x)^{-1} \simeq 1 + x$, $h \simeq \frac{V_0^2}{2g}\left(1 + \frac{V_0^2}{2 g r_e}\right)$

4.7. Penyelesaian :



$\rightarrow V_0 \quad \vec{r} = \hat{i}\, b\cos\theta - \hat{j}\, b\sin\theta$

$\theta = \omega t = \frac{V_0 t}{b}$ so $\vec{r} = \hat{i}\, V_0 \sin\theta - \hat{j}\, V_0\cos\theta$

Relative to the ground, $v = \hat{i}\, V_0(1 - \sin\theta) - \hat{j}\, V_0\cos\theta$

Untuk partikel

$y_0 = -b\sin\theta$ dan $V_{0y} = -V_0\cos\theta$

Jadi, $V_y = V_{0y} - gt = -V_0\cos\theta - gt$

$y = -b\sin\theta - V_0 t\cos\theta - \frac{1}{2} g t^2$

~ Pada ketinggian maksimum, $V_y = 0$:

$$t = -\frac{V_0\cos\theta}{g}$$

$$h = -b\sin\theta - V_0\left(-\frac{V_0\cos\theta}{g}\right)\cos\theta - \frac{1}{2}g\left(-\frac{V_0\cos\theta}{g}\right)^2$$

$$h = -b\sin\theta + \frac{V_0^2\cos^2\theta}{2g}$$

~ h maksimum

$$\frac{dh}{d\theta} = 0 = -b\cos\theta - \frac{2V_0^2\cos\theta\sin\theta}{2g}$$

$$\sin\theta = -\frac{gb}{V_0^2}$$

$$\cos^2\theta = 1 - \sin^2\theta = \frac{V_0^4 - g^2b^2}{V_0^4}$$

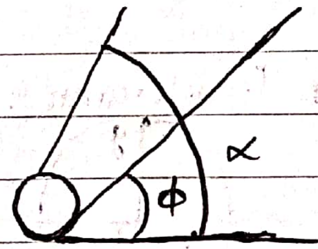
$$h_{max} = \frac{gb^2}{V_0^2} + \frac{V_0^4 - g^2b^2}{2gV_0^2} = \frac{gb^2}{2V_0^2} + \frac{V_0^2}{2g}$$

Measured from the ground,

$$h'_{max} = b + \frac{gb^2}{2V_0^2} + \frac{V_0^2}{2g}$$

The mud leaves the wheel at $\theta = \sin^{-1}\left(-\frac{gb}{V_0^2}\right)$

4.8 Penyelesaian :



•) $x = R\cos\phi$ dan $x = V_{0x}t = (V_0\cos\alpha)t$

Jadi, $t = \frac{R\cos\phi}{V_0\cos\alpha}$

•) $y = R\sin\phi$ dan $y = V_{0y}t - \frac{1}{2}gt^2$

$$= (V_0\sin\alpha)t - \frac{1}{2}gt^2$$

$$R\sin\phi = (V_0\sin\alpha)\frac{R\cos\phi}{V_0\cos\alpha} - \frac{1}{2}g\left(\frac{R\cos\phi}{V_0\cos\alpha}\right)^2$$

$$\sin\phi = \tan\alpha\cos\phi - \frac{gR\cos^2\phi}{2V_0^2\cos^2\alpha}$$

$$R = \frac{2V_0^2\cos^2\alpha}{g\cos^2\phi}(\tan\alpha\cos\phi - \sin\phi)$$

$$= \frac{2V_0^2\cos\alpha}{g\cos\phi}(\sin\alpha\cos\phi - \cos\alpha\sin\phi)$$

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from appendix B, $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$$R = \frac{2V_0^2 \cos\alpha}{g\cos^2\phi}\sin(\alpha-\phi)$$

R maksimum, untuk $\frac{dR}{d\alpha} = 0 = \frac{2V_0^2}{g\cos^2\phi}\left[-\sin\alpha\sin(\alpha-\phi) + \cos\alpha\cos(\alpha-\phi)\right]$

Mensyaratkan bahwa, $\cos\alpha\cos(\alpha-\phi) - \sin\alpha\sin(\alpha-\phi) = 0$

from appendix B, $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

Jadi $\cos(2\alpha-\phi) = 0$

$$2\alpha - \phi = \frac{\pi}{2} \Leftrightarrow \alpha = \frac{\pi}{4} + \frac{\phi}{2}$$

$$R_{max} = \frac{2V_0^2}{g\cos^2\phi}\cos\left[\frac{\pi}{4}+\frac{\phi}{2}\right]\sin\left(\frac{\pi}{4}-\frac{\phi}{2}\right)$$

Sekarang, $\sin\left(\frac{\pi}{4}-\frac{\phi}{2}\right] = \cos\left[\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{\phi}{2}\right)\right] = \cos\left[\frac{\pi}{4}+\frac{\phi}{2}\right]$

$$R_{max} = \frac{2V_0^2}{g\cos^2\phi}\cos^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)$$

Gunakan appendix B, $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$$R_{max} = \frac{2V_0^2}{g\cos^2\phi}\left[\frac{1}{2}\cos\left[\frac{\pi}{2}+\phi\right]+\frac{1}{2}\right] = \frac{V_0^2}{g\cos^2\phi}\left[\cos\left(\frac{\pi}{2}+\phi\right)+1\right]$$

Gunakan : $\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$$R_{max} = \frac{V_0^2}{g(1-\sin^2\phi)}(1-\sin\phi)$$

$$R_{max} = \frac{V_0^2}{g(1+\sin\phi)}$$

4.9. Penyelesaian :

a) $R = \frac{2V_0^2}{g}\frac{\cos\alpha\sin(\alpha+\phi)}{\cos^2\phi}$

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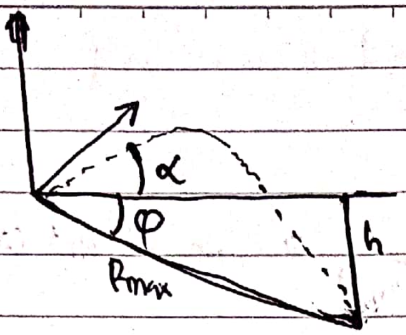


•) $R_{max} = \frac{V_0^2}{g} \frac{(1+\sin\varphi)}{\cos^2\varphi}$ dan $2\alpha = \frac{\pi}{2} - \varphi$

→) $R_{max} = \frac{h}{\sin\varphi} = \frac{V_0^2}{g}\frac{(1+\sin\varphi)}{\cos^2\varphi} = \frac{V_0^2}{g(1-\sin\varphi)}$

$\sin\varphi = \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2}\right)$

$\sin\varphi = \sin\left(\frac{\pi}{2} - 2\alpha\right) = \cos 2\alpha = 1 - 2\sin^2\alpha$

Thus, $1 - 2\sin^2\alpha = \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2}\right)$

$2\sin^2\alpha = \frac{2}{\csc^2\alpha} = 1 - \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2}\right) = \frac{1}{1 + gh/V_0^2}$

Jadi, $\csc^2\alpha = 2\left(1 + \frac{gh}{V_0^2}\right)$

b) Penyelesaian w/ R_{max}

$R_{max} = \frac{h}{\sin\varphi} = \frac{h}{1 - 2\sin^2\alpha} = \frac{h}{1 - 2/\csc^2\alpha}$

substitusi $\csc^2\alpha$ ⇒ $R_{max} = \frac{V_0^2}{g}\left(1 + \frac{gh}{V_0^2}\right)$

4.12. Penyelesaian :

$x = V_0 t \cos\frac{1}{2}\theta_0$ dan $z = V_0 t \cos\frac{1}{2}\theta_0 \sin\theta_0 - \frac{1}{2}gt^2$

→ $V_x = V_0 \cos\frac{1}{2}\theta$

The horizontal range is $R = \frac{V_0^2}{g}\cos^2\frac{1}{2}\theta_0 \sin 2\theta_0$

The maximum range occurs @ $dR/d\theta_0 = 0$

$$\frac{dR}{d\theta_0} = \frac{V_0^2}{g}\left(2\cos^2\frac{1}{2}\theta_0 \cos 2\theta_0 - \cos\frac{1}{2}\theta_0 \sin\frac{1}{2}\theta_0 \cdot \sin 2\theta_0\right) = 0$$

Thus, $2\cos^2\frac{1}{2}\theta_0 \cos 2\theta_0 = \cos\frac{1}{2}\theta_0 \sin\frac{1}{2}\theta_0 \sin 2\theta_0$

Gunakan : $2\cos^2\theta_0 = 1+\cos 2\theta_0$ dan $\sin 2\theta_0 = (1-\cos^2\theta_0)\cos\theta_0$

Akan kita peroleh :

$$(1+\cos\theta_0)(2\cos^2\theta_0 - 1) = \sin\theta_0 \sin\theta_0 \cos\theta_0 - (1-\cos^2\theta_0)\cos\theta_0$$

or $(1+\cos\theta_0)(3\cos^2\theta_0 - \cos\theta_0 - 1) = 0$

Thus, $\cos\theta_0 = -1$ dan $\cos\theta_0 = \frac{1}{6}(1 \pm \sqrt{13})$

Hanya range : $0 \leq \theta_0 < \frac{\pi}{2}$

$\cos\theta_0 = \frac{1}{6}(1+\sqrt{13}) = 0,7676$ $\theta_0 = 39°51'$

Thus (b) for V_0 = 25 m/s, R_{max} = 55,4 m @ $\theta_0 = 39°51'$

(a) the ketinggian maksimum pada $\frac{dz}{dt} = 0$

$V_0 \cos\frac{1}{2}\theta_0 \sin\theta_0 = gT$ or at $T = \frac{V_0\cos\frac{1}{2}\theta_0\sin\theta_0}{g}$

or $H = \frac{V_0^2}{2g}\cos^2\frac{1}{2}\theta_0 \sin^2\theta_0$

Gunakan identitas trigonometri, kita peroleh :

$$(1+\cos\theta_0)\sin\theta_0\cos\theta_0 = \frac{1}{2}\sin\theta_0\sin^2\theta_0 = \frac{1}{2}\sin\theta_0(1-\cos^2\theta_0)$$

or

$$\sin\theta_0(1+\cos\theta_0)(3\cos\theta_0 - 1) = 0$$

Jadi, H_{max} = 18,9 m

@ $\theta_0 = \cos^{-1}\frac{1}{3} = 70°32'$

4-13. Penyelesaian :

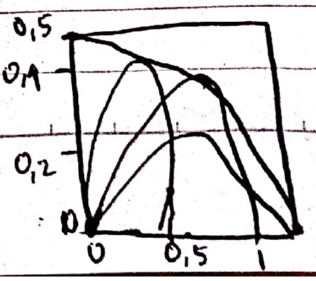
$z = \frac{\dot{z}_0}{\dot{r}_0} r - \frac{g}{2\dot{r}_0^2}$, dimana $\dot{r}_0 = V_0\cos\theta_0$
$\dot{z}_0 = V_0\sin\theta_0$

Thus, $z = r\tan\theta_0 - \frac{gr^2}{2V_0^2}\sec^2\theta_0$

$\sec^2\theta_0 = 1+\tan^2\theta_0$

Kita punya : $\frac{gr^2}{2V_0^2}\tan^2\theta_0 - r\tan\theta_0 + z + \frac{gr^2}{2V_0^2} = 0$

$\rightsquigarrow \tan\theta_0 = \frac{1}{gr}\left[V_0^2 \pm (V_0^4 - 2gzV_0^2 - g^2r^2)^{1/2}\right]$



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The roots are only $\rightarrow$ $V_0^4 - 2gzV_0^2 - g^2r^2 \geq 0$

Sehingga, The critical surface is $\Rightarrow$ $V_0^4 - 2gzV_0^2 - g^2r^2 = 0$

4.17 Penyelesaian :

$\rightsquigarrow m\ddot{x} = F_x = -\dfrac{\partial V}{\partial x} = -kx = -\pi^2 mx$

$\dot{x} = A\cos\left(\sqrt{\dfrac{k}{m}}t + \alpha\right) = A\cos(\pi t + \alpha)$

$\rightsquigarrow m\ddot{y} = -\dfrac{\partial V}{\partial y} = -4\pi^2 mx$

$y = B\cos(2\pi t + \beta)$

$\rightsquigarrow m\ddot{z} = -\dfrac{\partial V}{\partial z} = -9\pi^2 mz$

$z = C\cos(3\pi t + \gamma)$

$\Rightarrow$ $x = y = z = 0$ pada $t = 0$, $\alpha = \beta = \gamma = -\pi/2$

$x = A\cos\left(\pi t - \dfrac{\pi}{2}\right) = A\sin \pi t$

$\dot{x} = A\pi\cos\pi t$

$\Rightarrow$ $V_0^2 = \dot{x}_0^2 + \dot{y}_0^2 + \dot{z}_0^2$ dan $\dot{x}_0 = \dot{y}_0 = \dot{z}_0$.

•) $\dot{x}_0 = \dfrac{V_0}{\sqrt{3}} = A\pi$

$A = \dfrac{V_0}{\pi\sqrt{3}}$

$x = \dfrac{V_0 \sin \pi t}{\pi\sqrt{3}}$

•) $y = B\sin 2\pi t$, $\dot{y} = 2B\pi\cos 2\pi t$

$\dot{y}_0 = \dfrac{V_0}{\sqrt{3}} = 2\pi B$

$B = \dfrac{V_0}{2\pi\sqrt{3}}$, $y = \dfrac{V_0}{2\pi\sqrt{3}}\sin 2\pi t$

•) $z = C\sin 3\pi t$, $\dot{z} = 3C\pi\cos 3\pi t$

$\dot{z}_0 = \dfrac{V_0}{\sqrt{3}} = 3C\pi$

$C = \dfrac{V_0}{3\pi\sqrt{3}}$, $z = \dfrac{V_0}{3\pi\sqrt{3}}\sin 3\pi t$

$\Rightarrow$ $\omega_x = \pi - \omega_y = 2\pi$ dan $\omega_z = 3\pi$

$t_{min} = \dfrac{2\pi n_1}{\omega_x} = \dfrac{2\pi n_2}{\omega_y} = \dfrac{2\pi n_3}{\omega_z}$

The minimum time occurs at $n_1 = 1, n_2 = 2, n_3 = 3$

$t_{min} = \dfrac{2\pi}{\pi} = 2$

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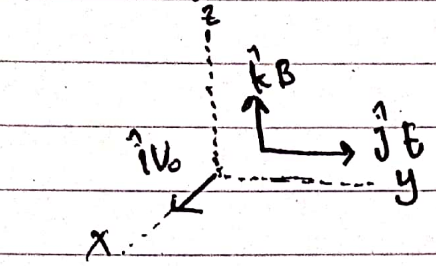
4.20 Penyelesaian :



$$\vec{F} = q(\bar{E} + \vec{v} \times \vec{B})$$

$$\vec{v} \times \vec{B} = (\hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z}) \times \hat{k}B = \hat{i}\dot{y}B - \hat{j}\dot{x}B$$

$$\vec{F} = \hat{i}q\dot{y}B + \hat{j}q(E - \dot{x}B)$$

$$m\ddot{x} = F_x = q\dot{y}B$$

$$\dot{x} - \dot{x}_0 = \frac{qB}{m} y$$

$$\Rightarrow m\ddot{y} = F_y = qE - q\dot{x}B = qE - qB\left(\dot{x}_0 + \frac{qB}{m} y\right)$$

$$\ddot{y} = \frac{qE}{m} - \frac{qB\dot{x}_0}{m} - \left(\frac{qB}{m}\right)^2 y = -\frac{eE}{m} + \frac{eB\dot{x}_0}{m} - \left(\frac{eB}{m}\right)^2 y$$

$$\ddot{y} + \omega^2 y = -\frac{eE}{m} + \omega\dot{x}_0, \qquad \omega = \frac{eB}{m}$$

$$y = \frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}_0\right) + A\cos(\omega t + \theta_0)$$

$$\dot{y} = -A\omega\sin(\omega t + \theta_0)$$

$$\dot{y}_0 = 0, \text{ so } \theta_0 = 0$$

$$y_0 = 0, \text{ so } A = -\frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}_0\right)$$

$$y = a(1 - \cos\omega t), \qquad a = \frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}_0\right)$$

$$\dot{x} = \dot{x}_0 + \frac{qB}{m} y = \dot{x}_0 - \omega y = \dot{x}_0 - \omega a(1 - \cos\omega t)$$

$$\dot{x} = (\dot{x}_0 - \omega a) + \omega a\cos\omega t$$

$$x = (\dot{x}_0 - \omega a)t + a\sin\omega t$$

$$x = a\sin\omega t + bt, \qquad b = \dot{x}_0 - \omega a$$

$$m\ddot{z} = F_z = 0$$

$$z = \dot{z}_0 t + z_0 = 0$$