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4.1 Find the force for each of the following potential energy functions :

(a) $V = cxyz + C$

(b) $V = \alpha x^2 + \beta y^2 + \gamma z^2 + C$

(c) $V = Ce^{-(\alpha x + \beta y + \gamma z)}$

(d) $V = Cr^n$ in Spherical coordinates

4.2 By finding the curl, determine which of the following forces are conservative :

(a) $F = ix + jy + kz$

(b) $F = iy - jx + kz^2$

(c) $F = iy + jx + kz^3$

(d) $F = -kr^{-n}e$, in Spherical Coordinates

4.3 Find the value of the constant c such that each of the following forces is conservative :

(a) $F = ixy + jcx^2 + kz^3$

(b) $F = i(z/y) + cj(xz/y^2) + k(x/y)$

4.4 A particle of mass m moving in three dimensions under the potential energy function $V(x, y, z) = \alpha x + \beta y^2 + \gamma z^3$ has speed V_0 when it passes through the origin.

(a) What will its speed be if and when it passes through the point (1,1,1,) ?

(b) If the point (1,1,1) is a turning point in the motion (v=0), what is V_0?

(c) What are the component differential equations of motion of the particle ?

(Note : It is not necessary to solve the differential equations of motion in this problem)

4.5 Consider the two force functions

(a) $F = ix + jy$

(b) $F = iy - jx$

Verify that (a) is conservative and that (b) is nonconservative by showing that the integral $\int F \cdot dr$ is independent of the path of integration for (a), but not for (b), by taking two paths in which the starting point is the origin (0,0), and the endpoint is (1,1). For one path take the line x = y. For the other path take the x - axis out the point (1,0) and then the line x = 1 up to the point (1,1)

4.6 Show that the variation of gravity with height can be accounted for approximately by the following potential energy function :

$$V = mgz\left(1 - \frac{z}{r_e}\right)$$

In which r_e is the radius of the Earth. Find the force given by the above potential function. From this find the component differential equations of motion of a projectile under such a force. If the vertical component of the initial velocity is V_{0z}, how high does the projectile go? (compare with Example 2.3.2)

4.7 Particles of mud are thrown from the rim of a rolling wheel. If the forward speed of the wheel is V_0, and the radius of the wheel is b, show that the greatest heigh above the ground that the mud can go is

$$b + \frac{V_0^2}{2g} + \frac{gb^2}{2V_0^2}$$

At what point on the rolling wheel does this mud leave ?
(Note : It is necessary to assume that $V_0^2 \geq bg$)

4.8 A gun is located at the bottom of a hill of constant slope ϕ. show that the range of the gun measured up the slope of the hill is

$$\frac{2V_0^2 \cos a \sin(a - \phi)}{g \cos^2\phi}$$

where a is the angle of elevation of the gun, and that the maximum value of the slope range is

$$\frac{V_0^2}{g(1 + \sin\phi)}$$

4.9 A cannon that is capable of firing a shell at speed V_0 is mounted on a vertical tower of height h that overlooks a level plain below

(a) Show that the elevation angle a at which the cannon must be set to achive maximum range is given by the expression

$$\csc^2 a = 2\left(1 + \frac{gh}{V_0^2}\right)$$

(b) What is the maximum range R of the cannon ?

4.12 A baseball pitcher can throw a ball more easily horizontally than vertically. Assume that the pitcher's throwing speed varies with elevation angle approximately as $V_0 \cos \frac{1}{2}\theta_0$ m/s, where θ_0 is the initial elevation angle an V_0 is the initial velocity when the ball is thrown horizontally. Find the angle θ_0 at which the ball must be thrown to achive maximum (a) height and (b) range.

Find the values of the maximum (c) height and (d) range. Assume no air resistance and let $V_0 = 25$ m/s

1.13 A gun car fire an artillery shell with a speed V_0 in any direction. show that a sheel can strike any target within the surface given by

$$g^2 r^2 = V_0^4 - 2g V_0^2 z$$

Where z is the height of the target and r is its horizontal distance from the gun. Assume no air resistance.

1.17 A small lead ball of mass m is suspended by means of six light springs as shown in figure 4.4.1. The stiffness constants are in the ratio 1:4:9, so that the potential energy function can be expressed as

$$V = \frac{k}{2}(x^2 + 4y^2 + 9z^2)$$

At time $t = 0$ the ball receives a push in the (1,1,1) direction that imparts to it a speed V_0 at the origin. If $k = \pi^2 m$, numerically find x, y, and z as functions of the time t. Does the ball ever retrace its path ? If so, for what value of t does it first return to the origin with the same velocity that it had at $t = 0$?

1.20 An electron moves in a force field due to a uniform electric field E and a uniform magnetic field B that is at right angles to E. let $E = jE$ and $B = kB$. Take the initial position of the electron at the origin with initial velocity $V_0 = iV_0$ in the x direction. Find the resulting motion of the particle. Show that the path of motion is a cycloid:

$$x = a \sin \omega t + bt$$
$$y = a(1 - \cos \omega t)$$
$$z = 0$$

Cycloidal motion of electrons is used in an electronic tube called a magnetron to produce the microwaves in a microwave oven.

Penyelesaian

4.1 (a) $\vec{F} = -\vec{\nabla}V = -\hat{i}\frac{\partial V}{\partial x} - \hat{j}\frac{\partial V}{\partial y} - \hat{k}\frac{\partial V}{\partial z}$

$\vec{F} = -c(\hat{i}yz + \hat{j}xz + \hat{k}xy)$

(b) $\vec{F} = -\vec{\nabla}V = -\hat{i}2\alpha x - \hat{j}2\beta y - \hat{k}2\gamma z$

(c) $\vec{F} = -\vec{\nabla}V = Ce^{-(\alpha x + \beta y + \gamma z)}(\hat{i}\alpha + \hat{j}\beta + \hat{k}\gamma)$

(d) $\vec{F} = -\vec{\nabla}V = -\hat{e}_r\frac{\partial V}{\partial r} - \hat{e}_\theta\frac{1}{r}\frac{\partial V}{\partial \theta} - \hat{e}_\phi\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}$

$\vec{F} = -\hat{e}_r Cnr^{\alpha-1}$

4.2 (a)

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0$$

Konservatif

(b)

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & z^2 \end{vmatrix} = \hat{k}(-1-1) \neq 0$$

~~Konservatif~~

(c)

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x & z^3 \end{vmatrix} = \hat{k}(-1-1) = 0$$

Konservatif

(d)

$$\vec{\nabla}\times\vec{F} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{e}_r & \hat{e}_\theta r & \hat{e}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ -kr^{-n} & 0 & 0 \end{vmatrix} = 0$$

Konservatif

4.3 (a)

$$\vec{\nabla} \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & cx^2 & z^3 \end{vmatrix} = k\,(2cx - x)$$

$$2cx - x = 0$$

$$c = \frac{1}{2}$$

(b)

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{z}{y} & \frac{cxz}{y^2} & \frac{x}{y} \end{vmatrix}$$

$$= \hat{i}\left(-\frac{x}{y^2} - \frac{cx}{y^2}\right) + \hat{j}\left(\frac{1}{y} - \frac{1}{y}\right) + \hat{k}\left(\frac{cz}{y^2} + \frac{z}{y^2}\right)$$

$$-\frac{x}{y^2} - \frac{cx}{y^2} = 0 \qquad c = -1$$

Juga $\frac{cz}{y^2} + \frac{z}{y^2} = 0$ menyiratkan bahwa $c = -1$ seperti yang harus

4.4 (a) E = Konstan $= V(x, y, z) + \frac{1}{2} mv^2$

di asalnya $E = 0 + \frac{1}{2} mv^2$

pada $(1,1,1)$ $E = \alpha + \beta + \gamma + \frac{1}{2} mv^2 = \frac{1}{2} mv^2$

$$v^2 = v^2 - \frac{2}{m}(\alpha + \beta + \gamma)$$

$$v = \left(v^2 - \frac{2}{m}(\alpha + \beta + \gamma)\right)^{\frac{1}{2}}$$

(b) $v^2 - \frac{2}{m}(\alpha + \beta + \gamma) = 0$

$$v_0 = \left(\frac{2}{m}(\alpha + \beta + \gamma)\right)^{\frac{1}{2}}$$

(c) $m\ddot{x} = F_x = -\frac{\partial V}{\partial x}$

$$m\ddot{x} = -\alpha$$

$$m\ddot{y} = -\frac{\partial V}{\partial y} = -2\beta y$$

$$m\ddot{z} = -\frac{\partial V}{\partial z} = -3\gamma z^2$$

4.5 (a) $\vec{F} = \hat{i}x + \hat{j}y$

di jalan $x = y$: $\qquad d\vec{r} = \hat{i}dx + \hat{j}dy$

$$\int_{(0,0)}^{(1,1)} \vec{F}\cdot d\vec{r} = \int_0^1 F_x\,dx + \int_0^1 F_y\,dy = \int_0^1 x\,dx + \int_0^1 y\,dy = 1$$

Pada lintasan sepanjang sumbu x : $\qquad d\vec{r} = \hat{i}dx$

dan pada garis $x = 1$: $\qquad d\vec{r} = \hat{j}dy$

$$\int_{(0,0)}^{(1,1)} \vec{F}\cdot d\vec{r} = \int_0^1 F_x\,dx + \int_0^1 F_y\,dy = 1$$

$\vec{F}$ adalah konservatif

(b) $\vec{F} = \hat{i}y - \hat{j}x$

di jalan $x = y$:

$$\int_{(0,0)}^{(1,1)} \vec{F}\cdot d\vec{r} = \int_0^1 F_x\,dx + \int_0^1 F_y\,dy = \int_0^1 y\,dx - \int_0^1 x\,dy$$

dan, dengan $x = y$ $\qquad \int_{(0,0)}^{(1,1)} \vec{F}\cdot d\vec{r} = \int_0^1 x\,dx - \int_0^1 y\,dy = 0$

pada sumbu x $\qquad \int_{(0,0)}^{(1,0)} \vec{F}\cdot d\vec{r} = \int_0^1 F_x\,dx = \int_0^1 y\,dy = 0$

dan, dengan $y = 0$ pada sumbu x $\int_{(0,0)}^{(1,0)} \vec{F}\cdot d\vec{r} = 0$

pada garis $x = 1$: $\qquad \int_{(1,0)}^{(1,1)} \vec{F}\cdot d\vec{r} = \int_0^1 F_y\,dy = \int_0^1 x\,dy$

dan, dengan $x = 1$ $\qquad \int_{(1,0)}^{(1,1)} \vec{F}\cdot d\vec{r} = \int_0^1 dy = 1$

dijalan ini $\int_{(0,0)}^{(1,1)} \vec{F}\cdot d\vec{r} = 0 + 1 = 1$

$\vec{F}$ tidak konservatif

4.6 dari contoh 2.3.2, $V(z) = -mg\dfrac{r_e^2}{(r_e + z)}$

$$V(z) = -mgr_e\left(1 + \frac{z}{r_e}\right)^{-1}$$

Dari lampiran D $\qquad (1+x)^{-1} = 1 - x + x^2 + \dots$

$$V(z) = -mgr_e\left(1 - \frac{z}{r_e} + \frac{z^2}{r_e^2} + \dots\right)$$

$$V(z) = -mgr_e + mgz - \frac{mgz^2}{r_e} + \dots$$

Dengan $-mgr_e$ dengan konstanta tambahan

$$V(z) \approx mgz\left(1-\frac{z}{r_e}\right)$$

$$\vec{F} = -\vec{\nabla}V = -\hat{k}\frac{\partial}{\partial z}V(z)$$

$$= -\hat{k}mg\left[1-\frac{z}{r_e}+z\left(-\frac{1}{r_e}\right)\right]$$

$$\vec{F} = \hat{k}mg\left(1-\frac{2z}{r_e}\right)$$

$$m\ddot{x} = F_x = 0 \qquad m\ddot{y} = F_y = 0 \qquad m\ddot{z} = -mg\left(1-\frac{2z}{r_e}\right)$$

$$m\dot{z}\frac{d\dot{z}}{dz} = -mg\left(1-\frac{2z}{r_e}\right)$$

$$\int_{r_0}^{0}\dot{z}\,dz = -g\int_0^{h}\left(1-\frac{2z}{r_e}\right)dz$$

$$-\frac{1}{2}V_{oz}^2 = -g\left(h-\frac{h^2}{r_e}\right)$$

$$h^2 - r_e h + \frac{r_e V_{oz}^2}{2g} = 0$$

$$h = \frac{r_e}{2} - \frac{1}{2}\sqrt{r_e^2 - \frac{2r_e V_{oz}^2}{g}} \qquad (h \cdot z \ll r_e)$$

$$h = \frac{r_e}{2} - \frac{r_e}{2}\sqrt{1-\frac{2V_{oz}^2}{gr_e}}$$

Dari lampiran D. $(1+x)^{\frac{1}{2}} = 1 + \frac{x}{2} - \frac{x^2}{8} + \ldots$

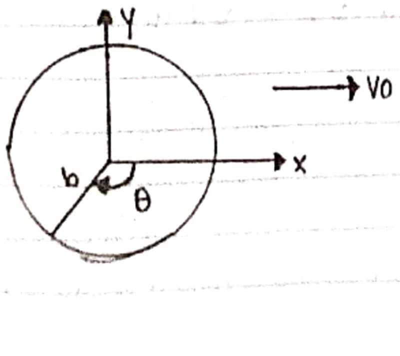
$$h = \frac{r_e}{2} - \frac{r_e}{2} + \frac{V_{oz}^2}{2g} + \frac{V_{oz}^4}{4gr_e} + \ldots$$

$$h = \frac{V_{oz}^2}{2g}\left(1+\frac{V_{oz}^2}{2gr_e}\right)$$

Dari Contoh 2.3.2. $h = \frac{V_0^2}{2g}\left(1-\frac{V_0^2}{2gr_e}\right)^{-1}$

dan, dengan $(1-x)^{-1} \approx 1+x$, $\quad h \approx \frac{V_0^2}{2g}\left(1+\frac{V_0^2}{2gr_e}\right)$

4.7



Untuk suatu titik pada pelek yang diukur dari pusat roda:

$$\vec{r} = \hat{i}b\cos\theta - \hat{j}b\sin\theta$$

$$\theta = \omega t = \frac{V_0 t}{b} \qquad \text{Jadi } \dot{\vec{r}} = -\hat{i}v_0\sin\theta - \hat{j}v_0\cos\theta$$

Relatif terhadap tanah, $\vec{v} = \hat{i}v_0(1-\sin\theta) - \hat{j}v_0\cos\theta$

Untuk partikel lumpur yang meninggalkan rim

$$Y_0 = -b \sin\theta \text{ and } V_{oy} = -V_0 \cos\theta$$

jadi $V_y = V_{oy} - gt = -V_0 \cos\theta - gt$

dan $Y = -b \sin\theta - V_0 t \cos\theta \frac{+}{2} gt^2$

Pada ketinggian height. $V_y = 0$:

$$t = \frac{V_0 \cos\theta}{g}$$

$$h = -b \sin\theta - V_0 \left(-\frac{V_0 \cos\theta}{g}\right) \cos\theta - \frac{1}{2} g \left(-\frac{V_0 \cos\theta}{g}\right)^2$$

$$h = -b \sin\theta + \frac{V_0^2 \cos^2\theta}{2g}$$

h maksimum terjadi untuk $\frac{dh}{d\theta} = 0 = -b \cos\theta - \frac{2V_0^2 \cos\theta \sin\theta}{2g}$

$$\sin\theta = -\frac{gb}{V_0^2}$$

$$\cos^2\theta = 1 - \sin^2\theta = \frac{V_0^4 - g^2 b^2}{V_0^4}$$

$$h_{max} = \frac{gb^2}{V_0^2} + \frac{V_0^4 - g^2 b^2}{2gV^2} = \frac{gb^2}{2V^2} + \frac{V_0^2}{2g}$$

Diukur dari tanah

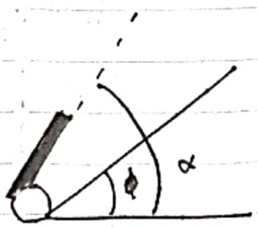
$$h'_{max} = b + \frac{gb^2}{2V_0^2} + \frac{V_0^2}{2g}$$

Lumpur meninggalkan roda di $\theta = \sin^{-1}\left(-\frac{gb}{V_0^2}\right)$

4.8 $X = R \cos\phi$ dan $x = V_0 . t = (V_0 \cos\alpha) t$

Jadi $t = \frac{R \cos\phi}{V \cos\alpha}$

$Y = R \sin\phi$ dan $y = V_0 t - \frac{1}{2} gt^2 = (V_0 \sin\alpha) t - \frac{1}{2} gt^2$



$$R \sin\phi = (V_0 \sin\alpha) \frac{R \cos\phi}{V_0 \cos\alpha} - \frac{1}{2} g \left(\frac{R \cos\phi}{V_0 \cos\alpha}\right)^2$$

$$\sin\phi = \tan\alpha \cos\phi - \frac{gR \cos^2\phi}{2V_0^2 \cos^2\alpha}$$

$$R = \frac{2V_0^2 \cos^2\alpha}{g \cos^2\phi} (\tan\alpha \cos\phi - \sin\phi) = \frac{2V_0^2 \cos\alpha}{g \cos^2\phi} (\sin\alpha \cos\phi - \cos\alpha \sin\phi)$$

Dari lampiran B, $\sin(\theta + \phi) = \sin\theta \cos\phi + \cos\theta \sin\phi$

$$R = \frac{2V_0^2 \cos\alpha}{g \cos^2\phi} \sin(\alpha - \phi)$$

R adalah maksimum untuk $\frac{dR}{d\alpha} = 0 = \frac{2V_0^2}{g\cos^2\phi}\left[-\sin\alpha\sin(\alpha-\phi) + \cos\alpha\cos(\alpha-\phi)\right]$

Menyiratkan bahwa $\cos\alpha\cos(\alpha-\phi) - \sin\alpha\sin(\alpha-\phi) = 0$

Dari lampiran B, $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

Jadi $\cos(2\alpha-\phi) = 0$

$$2\alpha - \phi = \frac{\pi}{2} \qquad \alpha = \frac{\pi}{4} + \frac{\phi}{2}$$

$$R_{max} = \frac{2V_0^2}{g\cos^2\phi}\cos\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\sin\left(\frac{\pi}{4}-\frac{\phi}{2}\right)$$

Sekarang $\sin\left(\frac{\pi}{4}-\frac{\phi}{2}\right) = \cos\left[\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{\phi}{2}\right)\right] = \cos\left(\frac{\pi}{4}+\frac{\phi}{2}\right)$

$$R_{max} = \frac{2V_0^2}{g\cos^2\phi}\cos^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)$$

Lagi menggunakan lampiran B, $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

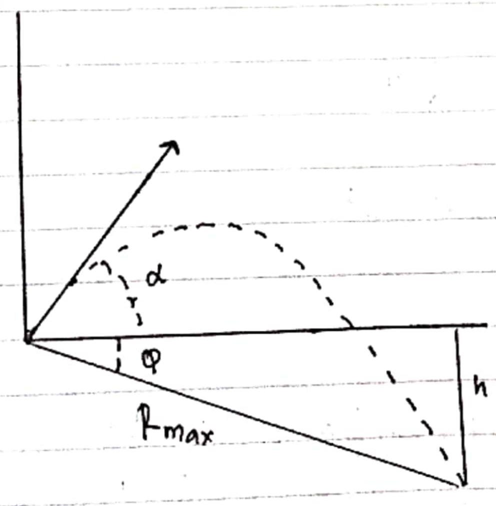
$$R_{max} = \frac{2V_0^2}{g\cos^2\phi}\left[\frac{1}{2}\cos\left(\frac{\pi}{2}+\phi\right)+\frac{1}{2}\right] = \frac{V_0^2}{g\cos^2\phi}\left[\cos\left(\frac{\pi}{2}+\phi\right)+1\right]$$

Menggunakan $\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$$R_{max} = \frac{V_0^2}{g(1-\sin^2\phi)}(1-\sin\phi)$$

$$R_{max} = \frac{V_0^2}{g(1+\sin\phi)}$$

4.9



(a) Disini kami mencatat bahwa proyektil diluncurkan "Menurun" menuju target. Yang terletak pada jarak h dibawah garis meriam di bawah garis membentuk sudut φ dibawah cakrawala. α adalah sudut proyeksi yang menghasilkan jangkauan maksimum R_{max}. Kita dapat menggunakan hasil dari masalah 4.8 dari masalah ini kita hanya perlu mengganti sudutnya φ, untuk memperhitungkan kemiringan lereng Jadi, kita dapatkan untuk rentang menurun.

$$R = \frac{2V_0^2}{g}\frac{\cos\alpha\sin(\alpha+\varphi)}{\cos^2\varphi}$$

Jangkauan maksimum dan sudut adalah α diperoleh dari soal di atas lagi dengan $2\alpha = \frac{\pi}{2} - \varphi$.

Menggantikan φ dengan sudut $-\varphi$ ... $R_{max} = \frac{V_0^2}{g} \frac{(1+\sin\varphi)}{\cos^2\varphi}$ dan ...

Kita tidak dapat menghitung α ... $R_{max} = \frac{h}{\sin\varphi} = \frac{V_0^2}{g} \frac{(1+\sin\varphi)}{\cos^2\varphi} = \frac{V_0^2}{g(1-\sin\varphi)}$

Pemecahan untuk $\sin\varphi$ ... $\sin\varphi = \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2}\right)$

tapi, dari atas ... $\sin\varphi = \sin\left(\frac{\pi}{2} - 2\alpha\right) = \cos 2\alpha = 1 - 2\sin^2\alpha$

Dengan demikian ... $1 - 2\sin^2\alpha = \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2}\right)$

$$2\sin^2\alpha = \frac{2}{\csc^2\alpha} = 1 - \frac{gh}{V_0^2} \Big/ \left(1 + \frac{gh}{V_0^2}\right) = \frac{1}{1 + \frac{gh}{V_0^2}}$$

Akhirnya ... $\csc^2\alpha = 2\left(1 + \frac{gh}{V_0^2}\right)$

(b)

Pemecahan untuk R_{max} ... $R_{max} = \frac{h}{\sin\varphi} = \frac{h}{1 - 2\sin^2\alpha} = \frac{h}{1 - 2/\csc^2\alpha}$

menggantikan untuk $\csc^2\alpha$ dan penyelesaian ...

$$R_{max} = \frac{V_0^2}{g}\left(1 + \frac{gh}{V_0^2}\right)$$

4. 12 Posisi x dan z bola vs waktu adalah

$$x = V_0 t \cos\tfrac{1}{2}\theta \qquad z = V_0 t \cos\tfrac{1}{2}\theta \cdot \sin\theta - \tfrac{1}{2} g t^2$$

Sejak $V_0 = V_0 \cos\frac{1}{2}\theta$

Jangkauan mendatar adalah $R = \frac{V_0^2}{g} \cos^2\frac{1}{2}\theta \cdot \sin 2\theta$

Jangkauan maksimum terjadi @ $\frac{dR}{d\theta} = 0$

$$\frac{dR}{d\theta} = \frac{V_0^2}{g}\left(2\cos^2\tfrac{1}{2}\theta \cdot \cos 2\theta - \cos\tfrac{1}{2}\theta \cdot \sin\tfrac{1}{2}\theta \cdot \sin 2\theta\right) = 0$$

Dengan demikian, $2\cos^2\frac{1}{2} \cdot \cos 2\theta = \cos\frac{1}{2}\theta \cdot \sin\frac{1}{2}\theta \cdot \sin 2\theta$

Menggunakan identitas : $2\cos^2\theta = 1 + \cos 2\theta$ dan $\sin 2\theta = 2\sin\theta \cdot \cos\theta$
kita mendapatkan :

$$(1+\cos\theta)(2\cos^2\theta - 1) = \sin\theta \cdot \sin\theta \cdot \cos\theta = (1 - \cos^2\theta)\cos\theta$$

atau $(1+\cos\theta_0)(3\cos^2\theta_0 - \cos\theta_0 - 1 = 0$

Dengan demikian $\cos\theta = -1$. $\cos\theta = \frac{1}{6}(1 \pm \sqrt{13})$

Hanya akar positif yang berlaku untuk θ_0 jangkauan : $0 \le \theta_0 \le \frac{\pi}{2}$

$$\cos\theta_0 = \frac{1}{6}(1+\sqrt{13}) = 0{,}7676$$

$\theta_0 = 39'51'$

Dengan demikian (b) untuk $V_0 = 25\ ms^{-1}$ $R_{max} = 55{.}4\ m$ @ $\theta_0 = 39'51'$

(a) Ketinggian maksimum terjadi pada $\frac{dz}{dt} = 0$

$V_0 \cos\frac{1}{2}\theta_0 \sin\theta_0 = gT$ atau di $T = \frac{V_0\cos\frac{1}{2}\theta\,.\sin\theta}{g}$

Atau $H = \frac{V_0^2}{2g}\cos^2\frac{1}{2}\theta\,.\sin^2\theta$ maksimum di tetap

Ketinggian maksimum yang mungkin terjadi @ $\frac{dH}{d\alpha} = 0$

$$\frac{dH}{d\alpha} = \frac{V_0^2}{2g}\left(2\cos^2\frac{1}{2}\theta, \sin\theta, \cos\theta - \cos\frac{1}{2}\theta\,.\sin\frac{1}{2}\theta, \sin^2\theta_0\right) = 0$$

Menggunakan Identitas trigonometri diatas, kita dapat

$$(1+\cos\theta_0)\sin\theta\,.\cos\theta_0 = \frac{1}{2}\sin\theta\,,\sin^2\theta = \frac{1}{2}\sin\theta\,.(1-\cos^2\theta_0)$$

atau $\sin\theta_0(1+\cos\theta)(3\cos\theta - 1) = 0$

Ada 3 akar : $\sin\theta_0 = 0$, $\cos\theta_0 = -1$, $\cos\theta_0 = \frac{1}{3}$

Dua akar pertama memberikan ketinggian minimum : yang terakhir memberikan maksimum

Dengan demikian, $H_{max} = 18{,}9\ m$ @ $\theta = \cos^{-1}\frac{1}{3} = 70'32'$

4.13 Lintasan kulit diberikan oleh persamaan 4.3.11 dengan r menggantikan x

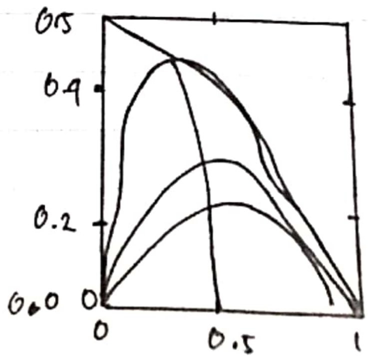
$$z = \frac{\dot{z}_0}{\dot{r}_c} r - \frac{g}{2\dot{r}_c^2} r^2$$ dimana $\dot{r}_c = V_0\cos\theta$, $\dot{z}_0 = V_0 \sin\theta$.

Dengan demikian $z = r\tan\theta - \frac{gr^2}{2V_0^2}\sec^2\theta_0$

Sejak $\sec^2\theta = 1 + \tan^2\theta$

kita dapat :

$$\frac{gr^2}{2V_0^2}\tan^2\theta - r\tan\theta + z + \frac{gr^2}{2V^2} = 0$$



(r,z) adalah koordinat target

Persamaan diatas menghasilkan dua akar yang mungkin :

$$\tan\theta_0 = \frac{1}{gr}\left[V_0^2 \pm (V_0^2 - 2gzV_0^2 - g^2r^2)^{\frac{1}{2}}\right]$$

Akar-akarnya hanya nyata jika

$$V_0^4 - 2gzV_0^2 - g^2r^2 \geq 0$$

Permukaan kritis ada untuk :

$$\underline{V_0^4 - 2gzV_0^2 - g^2r^2 = 0}$$

4.19 $m\ddot{x} = F_x = -\frac{\partial V}{\partial x} = -kx = -\pi^2 mx$

$$x = A \cos\left[\sqrt{\frac{k}{m}}\, t + \alpha\right] = A \cos(\pi t + \alpha)$$

$$m\ddot{y} = -\frac{\partial V}{\partial y} = -4\pi^2 mx$$

$$y = B\cos(2\pi t + \beta)$$

$$m\ddot{z} = -\frac{\partial V}{\partial z} = -9\pi^2 mz$$

$$z = C\cos(3\pi t + \gamma)$$

Sejak $x = y = z = 0$ at $t = 0$, $\alpha = \beta = \gamma = -\frac{\pi}{2}$

$$x = A\cos\left(\pi t - \frac{\pi}{2}\right) = A\sin \pi t$$

$$\dot{x} = A\pi \cos \pi t$$

Sejak $V_0^2 = \dot{x}_0^2 + \dot{y}_0^2 + \dot{z}_0^2$ dan $\dot{x}_1 = \dot{y}_1 = \dot{z}_1$

$$\dot{x}_0 = \frac{V_0}{\sqrt{3}} = A\pi$$

$$A = \frac{V_0}{\pi\sqrt{3}}$$

$$x = \frac{V_0}{\pi\sqrt{3}} \sin \pi t$$

$$y = B\sin 2\pi t; \qquad \dot{y} = 2B\pi \cos 2\pi t$$

$$\dot{y}_1 = \frac{V_0}{\sqrt{3}} = 2\pi B$$

$$B = \frac{V_0}{2\pi\sqrt{3}}$$

$$y = \frac{V_0}{2\pi\sqrt{3}} \sin 2\pi t$$

$$z = C\sin 3\pi t. \qquad \dot{z} = 3C\pi \cos 3\pi t$$

$$\dot{z}_1 = \frac{V_0}{\sqrt{3}} = 3C\pi$$

$$C = \frac{V_0}{3\pi\sqrt{3}}$$

$$z = \frac{V_0}{3\pi\sqrt{3}} \sin 3\pi t$$

No.
Date

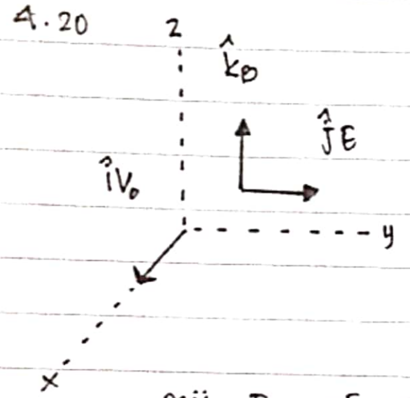
Sejak $\omega_x = \pi$, $\omega_y = 2\pi$, dan $\omega_z = 3\pi$ bola mengikuti jalannya.

$$t_{min} = \frac{2\pi n_1}{\omega_x} = \frac{2\pi n_2}{\omega_y} = \frac{2\pi n_3}{\omega_z}$$

Waktu minimum terjadi pada $n_1 = 1$, $n_2 = 2$, $n_3 = 3$.

$$t_{min} = \frac{2\pi}{\pi} = 2$$

4.20



$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{v} \times \vec{B} = (\hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z}) \times \hat{k}B = \hat{i}\dot{y}B - \hat{j}\dot{x}B$$

$$\vec{F} = \hat{i}q\dot{y}B + \hat{j}q(E - \dot{x}B)$$

$$m\ddot{x} = F_x = q\dot{y}B$$

$$\dot{x} - \dot{x}_0 = \frac{qB}{m} y$$

$$m\ddot{y} = F_y = qE - q\dot{x}B = qE - qB\left(\dot{x}_0 + \frac{qB}{m} y\right)$$

$$\ddot{y} = \frac{qE}{m} - \frac{qB\dot{x}}{m} - \left(\frac{qB}{m}\right)^2 y = -\frac{eE}{m} + \frac{eB\dot{x}}{m} - \left(\frac{eB}{m}\right)^2 y$$

$$\ddot{y} + \omega^2 y = -\frac{eE}{m} + \omega\dot{x}_0, \qquad \omega = \frac{eB}{m}$$

$$y = \frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}\right) + A\cos(\omega t + \theta)$$

$$\dot{y} = A\omega \sin(\omega t + \theta)$$

$\dot{y} = 0$, jadi $\theta_0 = 0$

$y_0 = 0$, jadi $A = -\frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega\dot{x}_0\right)$

$$\dot{x} = \dot{x}_0 + \frac{qB}{m} y = \dot{x}_0 - \omega y = \dot{x}_0 - \omega\alpha(1 - \cos\omega t)$$

$$\dot{x} = (\dot{x}_0 - \omega\alpha) + \omega\alpha\cos\omega t$$

$$x = (\dot{x}_0 - \omega\alpha)t + \alpha\sin\omega t$$

$x = \alpha\sin\omega t + bt$. $\quad b = \dot{x}_0 - \omega\alpha$

$$m\ddot{z} = F_z = 0$$

$$z = \dot{z}_0 t + z_0 = 0$$