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Tugas Mekanika

2.1 a) $F_x = m\ddot{x} = F_0 + ct$

$$\frac{m\,dv}{dt} = F_0 + ct$$

$$m\,dv = (F_0 + ct)\,dt$$

$$m\int_{V_0}^{v} dv = \int_0^t (F_0 + ct)\,dt$$

$$m(v - V_0) = F_0(t - t_0) + \frac{c}{2}(t^2 - t_0^2)$$

karna $V_0 = 0$ dan $t_0 = 0$, maka :

$$mv = F_0 t + \frac{ct^2}{2}$$

$$V(t) = \frac{1}{m}\left(F_0 t + \frac{ct^2}{2}\right) = \frac{dx}{dt}$$

$$\int_{x_0}^{x} dx = \int_{t_0}^{t} \frac{1}{m}\left(F_0 t + \frac{ct^2}{2}\right) dt$$

$$x - x_0 = \frac{1}{m}\left(\frac{F_0}{2}(t^2 - t_0^2) + \frac{c}{6}(t^3 - t_0^3)\right)$$

$$x = \frac{1}{m}\left[\frac{F_0 t^2}{2} + \frac{ct^3}{6}\right]$$

b) $F_x = m\frac{dv}{dt} = F_0 \sin ct$

$$\int_0^v dv = \frac{F_0}{m}\int_0^t \sin ct\,dt$$

$$V(t) = -\frac{F_0}{mc}\cos ct$$

$$x(t) = \int_0^x dt$$

$$x(t) = -\frac{F_0}{mc}\int_0^t \cos ct\,dt$$

$$x(t) = -\frac{F_0}{mc^2} \sin ct$$

c) $F_x = F_0 e^{ct}$

$$v(t) = \frac{F_0}{m} \int_0^t e^{ct}\, dt$$

$$v(t) = \frac{F_0}{cm} e^{ct}$$

$$x(t) = \frac{F_0}{c^2 m} e^{ct}$$

2.2 a) $F_x = F_0 + cx$

$$F(x) = mv \frac{dv}{dx} = F_0 + cx$$

$$\int_{v_0}^{v} mv\, dv = \int_0^x (F_0 + cx))\, dx$$

$$\frac{mv^2}{2} = F_0 x + \frac{c}{2} x^2$$

$$v = \sqrt{\frac{2}{m} \left(F_0 x + \frac{c}{2} x^2\right)}$$

b) $F(x) = mv \frac{dv}{dx} = F_0 e^{-cx}$

$$\frac{1}{2} mv^2 = \frac{-F_0}{c} e^{-cx}$$

$$v = \frac{2}{m} \left[-\frac{F_0}{c} e^{-cx}\right]^{\frac{1}{2}}$$

c) $F(x) = mv \frac{dv}{dx} = F_0 \cos cx$

$$\frac{1}{2} mv^2 = \int_0^x F_0 \cos cx$$

$$= \frac{F_0}{2} \sin cx$$

$$v = \frac{2}{m} \left[\frac{F_0}{2} \sin cx\right]^{\frac{1}{2}}$$

(2.3) a) $F_x = -\frac{dv}{dx} = F_0 + Cx$

$U(x) = -\left[F_0 x + \frac{c}{2} x^2 \right] + C$

b) $F_x = -\frac{dv}{dx} = F_0 e^{-cx}$

$\int_{U(x_0)}^{U(x)} dv = -F_0 \int_{x_0}^{x} e^{-cx} dx$

c) $F_x = -\frac{dv}{dx} = F_0 \cos cx$

$\int_{U(x_0)}^{U(x)} dv = -F_0 \int_{x_0}^{x} \cos(x) dx$

$U(x) - U(x_0) = \frac{-F_0}{c} \left[\sin(x) - \sin(x_0) \right]$

(2.4) $V(x) = -\int f(x) dx$

a) $V(x) = -\int (-kx) dx$

$V(x) = \frac{kx^2}{2}$

Jadi, energi Potensial $V(x) = \frac{kx^2}{2}$

b) dan c)

$E = T_0 + \frac{1}{2} kA^2$

$E = T(x) + V(x)$

$T(x) = E - V(x)$

$T(x) = \frac{1}{2} kA^2 - \frac{1}{2} kx^2$

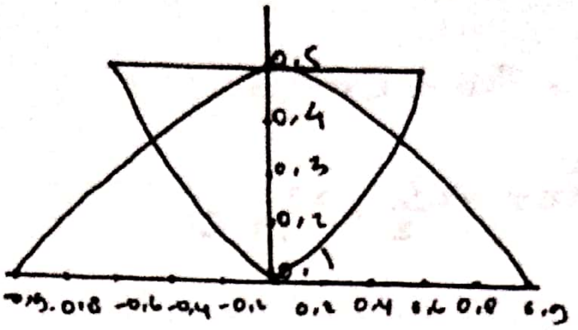
$T(x) = \frac{k}{2} (A^2 - x^2)$

d) $E = V_{cx}$

$\frac{1}{2} kA^2 = \frac{1}{2} kx^2$

$x = \pm A$

e)



Hasilnya : a) $V(x) = \frac{kx^2}{2}$

b) $T(x) = \frac{k^2}{2}(A - x^2)$

c) $E = T_0 = \frac{1}{2} KA^2$

2.8 $V = \frac{b}{V^3}$

$$F = \frac{m\,dv}{dt} = m \frac{dv}{dx} \frac{dx}{dt}$$

$$= mv \frac{dv}{dx}$$

$$\frac{dv}{dx} = \frac{-3b}{x^4}$$

$$f = m\left(\frac{b}{x^3}\right)\left(\frac{-3b}{x^4}\right) = -\frac{3mb^2}{x^7}$$

2.11 $F(v) = -cv^{\frac{1}{2}}$

$$= m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt}$$

$$= m\frac{dv}{dt}$$

$$-cv^{\frac{1}{2}} = m\frac{dv}{dx}$$

$$\int_0^x dx = \int_{v_0}^{v} -\frac{mv}{cv^{\frac{1}{2}}}\,dv$$

$$x = -\frac{m}{c}\int_{v_0}^{v}\frac{dv}{\sqrt{v}} = -\frac{m}{c}\int_{v_0}^{v} v^{-\frac{1}{2}}\,dv$$

$$= -\frac{m}{c}\left.\frac{v^{\frac{1}{2}}}{\frac{1}{2}}\right|_{v_0}^{v}$$

$$= \frac{2m}{c}\left(\sqrt{v_0} - \sqrt{v}\right)$$

Pers berhenti, maka $v = 0$, jadi $x_{max} = \frac{2m\sqrt{v_0}}{c}$

2.12 $m.a_x = \Sigma F_x$

$m.a_x = -mg - C_2 v^2$

$$a_x = \frac{dv}{dt} = \frac{dv\,dx}{dx\,dt} = \frac{dv}{dx} . v$$

$$m \frac{dv}{dx} = -mg - C_2 v^2$$

$$v \frac{dv}{dx} = -g - \frac{C^2}{m} v^2$$

$$k = \frac{C^2}{m}$$

$$\int_{v_0}^{v} \frac{v\,dv}{-g . kv^2} = \int_0^x du$$

$y = -y - kv^2$

$du = -2kv\,dv$

$v\,dv = -\frac{1}{2k}\,dy$

$$\int_{v_0}^{v} \frac{-1\,dy}{2ky} = \int_0^x dx$$

$$\frac{-1}{2k} \ln(y) \Big|_{v_0}^{v} = x \Big|_0^x$$

$$-\frac{1}{2k} \ln(-g - kv^2) \Big|_{v_0}^{v} = x$$

$$-\frac{1}{2k} \ln\left(\frac{-g . kv^2}{-g - kv_0^2}\right) = x$$

$$\frac{-g . kv^2}{-g . kv_0^2} = e^{-2kx}$$

$-g - kv^2 = (-g - kv_0^2)\, e^{-2kx}$

$kv^2 = (g + kv_0^2)\, e^{-2kx} - \frac{g}{k}$

$$A = \frac{g}{k} + v_0^2$$

$$\boxed{v^2 = A e^{-2kx} - \frac{g}{k}}$$

$m.a_x = \Sigma F_x$

$m.a_x = -mg + C_2 v^2$

$$a_x = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx} v$$

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$$mv\frac{dv}{dx} = -mg + C_2 V^2$$

$$c.\ \frac{dv}{dx} = -g + \frac{C_2}{m} V^2$$

$$k = \frac{C_2}{m}$$

$$v\frac{dv}{dx} = -g + kv^2$$

$$\int_0^v \frac{v\,dv}{-g+kv^2} = \int_0^x dx$$

$$y = -g + kv^2$$

$$dy = 2kv\ dv$$

$$v\ dv = \frac{1}{2k}\ dy$$

$$\int_0^v \frac{1}{2k}\frac{dy}{y} = \int_{x_0}^x dx$$

$$\frac{1}{2k}\ln(y)\Big|_0^v = x\Big|_{x_0}^x$$

$$\frac{1}{2k}\ln(-g+kv^2)\Big|_0^v = x - x_2$$

$$\frac{1}{2k}\ln\left(\frac{-g+kv^2}{-g}\right) = x - x_2$$

$$\frac{-g+kv^2}{g} = e^{2k(x-x_0)}$$

$$-g+kv^2 = -g\,e^{2k(x-x_0)}$$

$$kv^2 = -g\,e^{2k(x-x_0)}$$

$$v^2 = \frac{-g}{k} e^{2kx_0} e^{2kx} + \frac{g}{k}$$

$$B = \frac{g}{k} e^{2kx_0}$$

$$\boxed{V^2 = \frac{g}{k} - Be^{2kx}}$$

(2.19) $V^2 = \left(\frac{g}{k} + V_0^2\right) e^{-2kx} - \frac{g}{k}$ ke atas

$V^2 = -\frac{g}{k} e^{-2kx_0} e^{2kx} + \frac{g}{k}$ kebawah

$V = 0$ pada gerak ke atas

$$0 = \left(\frac{g}{k} + V_0^2\right) e^{-2kx} - \frac{g}{k}$$

$$e^{-2kh} = \frac{\frac{g}{k}}{\frac{g}{k} + V_0^2}$$

h adalah posisi awal untuk gerakan ke bawah dan (+) $q = 0$.

$V^2 = -\frac{g}{k} e^{-2kh} e^{2k0} + \frac{g}{k}$ sub $x = 0$

$$V^2 = -\frac{g}{k} \cdot \frac{\frac{g}{k}}{\frac{g}{k} + V_0^2} \cdot 1 + \frac{g}{k}$$

$$V_t^2 = \frac{g}{k}$$

$$V^2 = -V_t^2 \cdot \frac{V_t^2}{V_t^2 + V_0^2} + V_t^2$$

$$V^2 = V_t^2 \left(\frac{1 - V_t^2}{V_t^2 + V_0^2}\right)$$

$$V^2 = V_t^2 \left(\frac{V_t^2 + V_0^2 - V_t^2}{V_t^2 + V_0^2}\right)$$

$$V^2 = \frac{V_t^2 + V_0^2}{V_t + V_0^2}$$

$$V = \frac{V_t + V_0}{\sqrt{V_t^2 + V_0}}$$

Dipindai dengan CamScanner

2.14 $m a_x = \Sigma F_x$

$m \cdot a_x = -k x^{-2}$

$$a_x = \frac{dv}{dt} = -k x^{-2}$$

$$v \frac{dv}{dx} = -\frac{k}{m} x^{-2}$$

$$\int_0^v v\,dv = -\frac{k}{m}\int_b^x x^{-2}\,dx$$

$$\frac{2}{v}\Big|_0^v = -\frac{k}{m}(-1)x^{-1}\Big|_0^x$$

$$\frac{v^2}{2} = \frac{k}{m}(x^{-1} - b^{-1})$$

$$v^2 = \frac{2k}{m}\left(\frac{b-x}{bx}\right)$$

$$v = \frac{dx}{dt}$$

$$\sqrt{\frac{2k}{m}\left(\frac{b-x}{m}\right)} = \frac{dx}{dt}$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2k}{m}\left(\frac{b-x}{bx}\right)}} = \int_0^t dt$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2k}{m}\left(\frac{b-x}{bx}\right)}} = \sqrt{\frac{mb}{2k}}\int_b^0 \sqrt{\frac{x}{b-x}}\,dx$$

$$u = \frac{\sqrt{b-x}}{\sqrt{x}}$$

$$du = \frac{\frac{1}{2\sqrt{b-x}}(-1)\cdot\sqrt{x}\,dx\,\frac{1}{2\sqrt{x}}}{x}$$

$$dx = \frac{dv}{\frac{1}{-2\sqrt{x}\cdot v\sqrt{x}} - \frac{v\sqrt{x}}{2x^{3/2}}}$$

$$= \frac{dv}{\frac{1}{2vx} - \frac{v}{2x}}$$

$$= \frac{-dv}{\frac{1+v^2}{2xv}}$$

$$= -\frac{2xv\,dv}{v^2+1}$$

$$dx = -\frac{2bv\,dv}{(v^2+1)^2}$$

$$\sqrt{\frac{mb}{2k}}\int_b^0 \sqrt{\frac{x}{b-x}}\,dx = \sqrt{\frac{mb}{2k}}\int_b^0 (-1)^2\frac{bv\,dv}{(v^2+1)^2}$$

$$= \sqrt{\frac{mb}{2k}}\int_b^0 \frac{2bv\,dv}{(v^2+1)^2}$$

$$\int \frac{dv}{(av+b)^2} = \frac{v}{2(v^2+1)}\Big|_b^0 + \frac{1}{2}\int_b^0 \frac{dv}{v^2+1}$$

$$\int_b^0 \frac{dv}{(v^2+1)^2} = \frac{v}{2(v^2+1)}\Big|_b^0 + \frac{1}{2}\arctan v\Big|_b^0$$

$$-\sqrt{\frac{mb}{2k}}\int_b^0 \frac{2b\,dv}{(v^2+1)^2} = -2b\sqrt{\frac{mb}{2k}}\left(\frac{v}{2(v^2+1)}\Big|_b^0 + \frac{1}{2}\arctan v\Big|_b^0\right)$$

$$= -2b\sqrt{\frac{mb}{2k}}\cdot\left(\frac{v}{2(v^2+1)}\Big|_0^{-\infty} + \frac{1}{2}\arctan v\Big|_0^{-\infty}\right)$$

$$= -2b\sqrt{\frac{mb}{2k}}\left(0-0+\frac{1}{2}\cdot\left(\frac{-\pi}{0}-0\right)\right)$$

$$\pi\sqrt{\frac{mb^3}{8k}} = \int_0^t dt \qquad = -\pi\sqrt{\frac{mb^3}{8k}}$$

$$t = \pi\sqrt{\frac{mb^3}{8k}}$$

(2.15) $ma = \sum F$

$ma = mg - C_1 V - C_2 V^2$

$ma = 0$

$mg - C_1 V - C_2 V^2 = 0$

$C_2 V^2 - C_2 V - mg = 0$

$$V_{1,2} = \frac{-C_1 \pm \sqrt{C_1^2 + 4C_2 mg}}{2C_2}$$

Pilih (+)

$$V_1 = \frac{-C_1 + \sqrt{C_1^2 + 4C_1 mg}}{2C_2}$$

$$V_1 = V_t = -\frac{C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C^2}}$$

$$V_t = \frac{-C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C^2}}$$