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Mekanika

2.1 a) $F_x = m\ddot{x} = F_0 + Ct$

$$m\frac{dv}{dt} = F_0 + Ct$$

$$m.dv = (F_0 + Ct)\,dt$$

$$m\int_{v_0}^{v} dv = \int_0^t (F_0 + Ct)\,dt$$

$$m(v - v_0) = F_0(t - t_0) + \frac{c}{2}(t^2 - t_0^2)$$

karena $v_0 = 0$; $t_0 = 0$, maka

$$mv = F_0 t + \frac{ct^2}{2}$$

$$v_{(t)} = \frac{1}{m}\left(F_0 t + \frac{ct^2}{2}\right) = \frac{dx}{dt}$$

$$\int_{x_0}^{x} dx = \int_{t_0}^{t} \frac{1}{m}\left(F_0 t + \frac{ct^2}{2}\right) dt$$

$$x - x_0 = \frac{1}{m}\left(\frac{F_0}{2}(t^2 - t_0^2) + \frac{c}{6}(t^3 - t_0^3)\right)$$

$$x = \frac{1}{m}\left[\frac{F_0 t^2}{2} + \frac{Ct^3}{6}\right]$$

b) $F_x = m\frac{dv}{dt} = F_0 \sin Ct$

$$\int_0^v dv = \frac{F_0}{m}\int_0^t \sin ct\,dt$$

$$v(t) = -\frac{F_0}{mc}\cos ct$$

$$x(t) = \int_0^x dt$$

$$x(t) = -\frac{F_0}{mc}\int_0^t \cos ct\,dt$$

$$x(t) = -\frac{F_0}{mc^2}\sin ct$$

2.2 a) $F_x = F_0 + Cx$

$$F_{(x)} = mv\frac{dv}{dx} = F_0 + Cx$$

$$\int_{v_0}^{v} mv\,dv = \int_0^x (F_0 + Cx)\,dx$$

$$\frac{mv^2}{2} = F_0 x + \frac{C}{2}x^2$$

$$v = \sqrt{\frac{2}{m}\left(F_0 x + \frac{C}{2}x^2\right)}$$

2.3 a) $F_x = -\frac{dv}{dx} = F_0 + Cx$

$$u(x) = -\left[F_0 x + \frac{C}{2}x^2\right] + C$$

b) $F_x = -\frac{dv}{dx} = F_0 e^{-cx}$

$$\int_{u(x_0)}^{u(x)} du = -F_0 \int_{x_0}^{x} e^{-cx}\,dx$$

$$u(x) - u(x_0) = \frac{F_0}{c}\left[e^{-cx} - e^{-cx_0}\right]$$

2.4 $V_{(x)} = -\int f(x)\,dx$

a) $V_{(x)} = -\int (-kx)\,dx$

$$= \frac{kx^2}{2}$$

b) dan c)

$$E = T_0 = \frac{1}{2}kA^2$$

$$E = T_{(x)} + V_{(x)}$$

$$T_{(x)} = E - V_{(x)}$$

$$= \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$= \frac{k}{2}\left(A^2 - x^2\right)$$

d) $E = V_c\ x$

$$\frac{1}{2}kA^2 = \frac{1}{2}kx^2$$

$$x = \pm A$$

2.8 $v = \dfrac{b}{x^3}$

$$F = m\frac{dv}{dt} = m\frac{dv}{dx}\,\frac{dx}{dt}$$

$$= m v \frac{dv}{dx}$$

$$\frac{dv}{dx} = \frac{-3b}{x^4}$$

$$F = m\left(\frac{b}{x^3}\right)\left(\frac{-3b}{x^4}\right)$$

$$= \frac{-3mb^2}{x^7}$$

2.11 $F(v) = -cv^{3/2}$

$$= m\frac{dv}{dt} = m\frac{dv}{dx}\cdot\frac{dx}{dt}$$

$$= m\frac{dv}{dx}$$

$$-cv^{3/2} = m\frac{dv}{dx}$$

$$\int_0^x dx = \int_{v_0}^{v} -\frac{mv}{cv^{3/2}}\,dv$$

$$x = -\frac{m}{c}\int_{v_0}^{v}\frac{dv}{\sqrt{v}}$$

$$= -\frac{m}{c}\int_{v_0}^{v} v^{-1/2}\,dv$$

$$= -\frac{m}{c}\,\frac{v^{1/2}}{1/2}\Bigg|_{v_0}^{v}$$

$$= \frac{2m}{c}\left(\sqrt{v_0} - \sqrt{v}\right)$$

partikel berhenti, maka $v = 0$

$$\therefore\ x_{max} = \frac{2m\sqrt{v_0}}{c}$$

2.12 $m.\,a_x = \Sigma F_x$

$$m.\,a_x = -mg - c_2 v^2$$

$$a_x = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{dv}{dx}\cdot v$$

$$m.\frac{dv}{dx} = -mg - c_2 v^2$$

$$v\frac{dv}{dx} = -g - \frac{c_2}{m}v^2$$

$$k = \frac{c^2}{m}$$

$$dy = 2kv\,dv$$

$$v\,dv = \frac{1}{2k}\,dy$$

$$\int_0^v \frac{1}{2k}\frac{dy}{y} = \int_{x_0}^{x} dx$$

$$\frac{1}{2k}\ln(y)\Big|_0^v = x\Big|_{x_0}^{x}$$

$$\frac{1}{2k}\ln\left(-g + kv^2\right)\Big|_0^v = x - x_0$$

$$\frac{1}{2k}\ln\left(\frac{-g + kv^2}{-g}\right) = x - x_0$$

$$\frac{-g + kv^2}{g} = e^{2k(x-x_0)}$$

$$-g + kv^2 = -g e^{2k(x-x_0)}$$

$$kv^2 = -g e^{2k(x-x_0)}$$

$$v^2 = -\frac{g}{k} e^{2kx_0}\cdot e^{2kx} + \frac{g}{k}$$

$$B = \frac{g}{k} e^{2kx_0}$$

$$\boxed{v^2 = \frac{g}{k} - Be^{2kx}}$$

2.13 $v^2 = \left(\dfrac{g}{k} + v_0^2\right)e^{-2kx} - \dfrac{g}{k}$ → ke atas

$v^2 = -\dfrac{g}{k}e^{-2kx_0}e^{2kx} + \dfrac{g}{k}$ → kebawah

$v = 0$ pada gerak keatas

$$0 = \left(\frac{g}{k} + v_0^2\right)e^{-2kx} - \frac{g}{k}$$

$$e^{-2kh} = \frac{g/k}{\frac{g}{k} + v_0^2}$$

h adalah posisi awal untuk gerakan kebawah dan (t) a = 0,

$$v^2 = -\frac{g}{k} e^{-2kh} e^{2kx} + \frac{g}{k} \quad \rightarrow \text{subs } x = 0$$

$$v^2 = -\frac{g}{k} \frac{g/k}{\frac{g}{k} + v_0^2} \cdot 1 + \frac{g}{k}$$

$$v_t^2 = \frac{g}{k}$$

$$v^2 = -v_t^2 \frac{v_t^2}{v_t^2 + v_0^2} + v_t^2$$

$$v^2 = v_t^2 \left(1 - \frac{v_t^2}{v_t^2 + v_0^2} \right)$$

$$v^2 = v_t^2 \left(\frac{v_t^2 + v_0^2 - v_t^2}{v_t^2 + v_0^2} \right)$$

$$v^2 = \frac{v_t^2 v_0^2}{v_t + v_0^2}$$

$$v = \frac{v_t v_0}{\sqrt{v_t^2 + v_0}}$$

(2.19) $m a_x = \sum F_x$

$$m a_x = -k x^{-2}$$

$$a_x = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} \cdot v$$

$$mv \frac{dv}{dx} = -k x^{-2}$$

$$v \frac{dv}{dx} = \frac{-k}{m} x^{-2}$$

$$\int_0^v v \, dv = \frac{-k}{m} x^{-2} dx$$

$$\frac{v}{2} \Big|_0^v = -\frac{k}{m} (-1) x^{-1} \Big|_0^x$$

$$\frac{v^2}{v} = \frac{k}{m} (x^{-1} - b^{-1})$$

$$v^2 = \frac{2k}{m} \left(\frac{b - x}{bx} \right)$$

$$v = \frac{dx}{dt}$$

$$\sqrt{\frac{2k}{m} \left(\frac{b - x}{bx} \right)} = \frac{dx}{dt}$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2k}{m} \left(\frac{b-x}{bx} \right)}} = \int_0^t dt$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2k}{m} \frac{(b-x)}{bx}}} = \sqrt{\frac{mb}{2k}} \int_b^0 \sqrt{\frac{x}{b-x}} \, dx$$

$$u = \frac{\sqrt{b-x}}{\sqrt{x}}$$

$$dv = \frac{\frac{1}{2\sqrt{b-x}} (-1) \cdot \sqrt{x} \, dx - \frac{1}{2\sqrt{x}}}{x}$$

$$dx = \frac{dv}{-\frac{1}{2\sqrt{x} \cdot v\sqrt{x}} - \frac{v\sqrt{x}}{2x^{1/2}}}$$

$$= \frac{dv}{-\frac{1}{2ux} - \frac{u}{2x}}$$

$$= -\frac{dv}{\frac{1 + v^2}{2xv}}$$

$$= -\frac{2xu \, du}{u^2 + 1}$$

$$\sqrt{b - x} = v \sqrt{x}$$

$$b - x = u^2 x$$

$$b = x^2 (v^2 + 1)$$

$$x = \frac{b}{u^2 + 1}$$

$$dx = -\frac{2bv \, du}{(v^2 + 1)^2}$$

$$\sqrt{\frac{mb}{2k}} \int_b^0 \sqrt{\frac{x}{b-x}} \, dx = \sqrt{\frac{mb}{2k}} \int_b^0 \frac{1}{v} \frac{(-1)^2 bv du}{(v^2+1)^2}$$

$$= \sqrt{\frac{mb}{2k}} \int_0^t \frac{2 \, bv du}{(v^2 + 1)^2}$$

$$\int \frac{du}{(av + b)^n} = \frac{2n - 3}{2b(n-1)} \int \frac{du}{(av^2 + b)^{n-1}} + \frac{v}{2b(n-1)(av^2 + b)^{n-1}}$$

$$= \frac{u}{2(v^2+1)} + \frac{1}{2}\int \frac{dv}{v^2+1}$$

$$\int_b^0 \frac{dv}{(v^2+1)^2} = \frac{v}{2(v^2+1)}\Bigg|_b^0 + \frac{1}{2}\int_b^0 \frac{dv}{v^2+1}$$

$$\int_b^0 \frac{dv}{(v^2+1)^2} = \frac{u}{2(v^2+1)}\Bigg|_b^0 + \frac{1}{2} \arctan v \Bigg|_b^0$$

$$-\sqrt{\frac{mb}{2k}}\int_b^0 \frac{2b\,du}{(v^2+1)^2} = -2b\sqrt{\frac{mb}{2k}}\left(\frac{v}{2(v^2+1)}\Bigg|_b^0 + \frac{1}{2}\arctan u\Bigg|_b^0\right)$$

$$= -2b\sqrt{\frac{mb}{2k}}\cdot\left(\frac{v}{2(v^2+1)}\Bigg|_0^{-\infty} + \frac{1}{2}\arctan u\Bigg|_0^{-\infty}\right)$$

$$= -2b\sqrt{\frac{mb}{2k}}\left(0-0+\frac{1}{2}\cdot\left(\frac{-\pi}{6}-0\right)\right)$$

$$= \pi\sqrt{\frac{mb^3}{8k}}$$

$$\pi\sqrt{\frac{mb^3}{8k}} = \int_0^t dt$$

$$= t-0$$

$$t = \pi\sqrt{\frac{mb^3}{8k}}$$

(2.15) $m.a = \Sigma F$

$ma = mg - C_2 v - C_2 v^2$

$ma = 0$

$mg - C_1 v - C_2 V^2 = 0$

$C_2 v^2 - C_1 v - mg = 0$

$$V_{1.2} = \frac{-C_1 \pm \sqrt{C_1^2 + 4C_2 mg}}{2C_2}$$

pilih (+)

$$V_1 = \frac{-C_1 + \sqrt{C_1^2 + 4C_2 mg}}{2C_2}$$

$$V_1 = V_t = \frac{-C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right) + \frac{mg}{C_2}}$$

$$V_t = \frac{-C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C_2}}$$