

Nama: Chairani Kartini S. Harry

NPM : 2013022028

Kelas B

Mekanika

2.1 a) $F(x) = m\ddot{x} = F_0 + Ct$

$$m \frac{dv}{dt} = F_0 + Ct$$

$$m \cdot dv = (F_0 + Ct) dt$$

$$m \int_{v_0}^v dv = \int_0^t (F_0 + Ct) dt$$

$$m(v - v_0) = F_0(t - t_0) + \frac{C}{2}(t^2 - t_0^2)$$

Karena $v_0 = 0$ dan $t_0 = 0$, maka:

$$m v = F_0 t + \frac{Ct^2}{2}$$

$$v(t) = \frac{1}{m} \left(F_0 t + \frac{Ct^2}{2} \right) = \frac{dx}{dt}$$

$$\int_{x_0}^x dx = \int_{t_0}^t \frac{1}{m} \left(F_0 t + \frac{Ct^2}{2} \right) dt$$

$$x - x_0 = \frac{1}{m} \left(\frac{F_0}{2} (t^2 - t_0^2) + \frac{C}{6} (t^3 - t_0^3) \right)$$

$$x = \frac{1}{m} \left[\frac{F_0 t^2}{2} + \frac{Ct^3}{6} \right]$$

b) $F_x = m \frac{dv}{dt} = F_0 \sin Ct$

$$\int_0^v dv = \frac{F_0}{m} \int_0^t \sin ct dt$$

$$v(t) = \frac{-F_0}{mC} \cos Ct$$

$$x(t) = \int_0^x dt$$

$$x(t) = \frac{-F_0}{mC} \int_0^t \cos ct dt$$

$$x(t) = \frac{-F_0}{mC^2} \sin Ct$$

$$c). F_t = F_0 e^{ct}$$

$$V(t) = \frac{F_0}{m} \int_0^t e^{ct} dt$$

$$V(t) = \frac{F_0}{cm} e^{ct}$$

$$x(t) = \frac{F_0}{c^2 m} e^{ct}$$

$$2.2 \ a). F_x = F_0 + cx$$

$$F(x) = m v \frac{dv}{dx} = F_0 + cx$$

$$\int_{v_0}^v m v dv = \int_0^x (F_0 + cx) dx$$

$$\frac{mv^2}{2} = F_0 x + \frac{c}{2} x^2$$

$$v = \sqrt{\frac{2}{m} \left(F_0 x + \frac{c}{2} x^2 \right)}$$

$$b). F(x) = m v \frac{dv}{dx} = F_0 e^{-cx}$$

$$\frac{1}{2} m v^2 = -\frac{F_0}{c} e^{-cx}$$

$$v = \frac{2}{m} \left[\frac{-F_0}{c} e^{-cx} \right]^{1/2}$$

$$c). F(x) = m v \frac{dv}{dx} = F_0 e^{-cx}$$

$$\frac{1}{2} m v^2 = \int_0^x F_0 \cos cx$$

$$= \frac{F_0}{c} \sin cx$$

$$v = \frac{2}{m} \left[\frac{F_0}{c} \sin cx \right]^{1/2}$$

$$2.3 \ a). F_x = -\frac{dv}{dx} = F_0 + cx$$

$$u(x) = - \left[F_0 x + \frac{c}{2} x^2 \right] + C$$

$$b). F_x = -\frac{dv}{dx} = F_0 e^{-cx}$$

$$\int_{u(x_0)}^{u(x)} dv = -f_0 \int_{x_0}^x e^{-cx} dx$$

$$u(x) - u(x_0) = \frac{f_0}{c} [e^{-cx} - e^{-cx_0}]$$

$$c). F_x = \frac{-dv}{dx} = f_0 \cos cx$$

$$\int_{u(x_0)}^{u(x)} dv = -f_0 \int_{x_0}^x \cos(cx) dx$$

$$u(x) - u(x_0) = -\frac{f_0}{c} [\sin cx - \sin cx_0]$$

$$2.4 \quad V(x) = -\int F(x) dx$$

$$a). V(x) = -\int (-kx) dx$$

$$V(x) = \frac{kx^2}{2}$$

Jadi, energi Potensial $V(x) = \frac{kx^2}{2}$

b). dan c)

$$E = T_0 = \frac{1}{2} kA^2$$

$$E = T(x) + V(x)$$

$$T(x) = E - V(x)$$

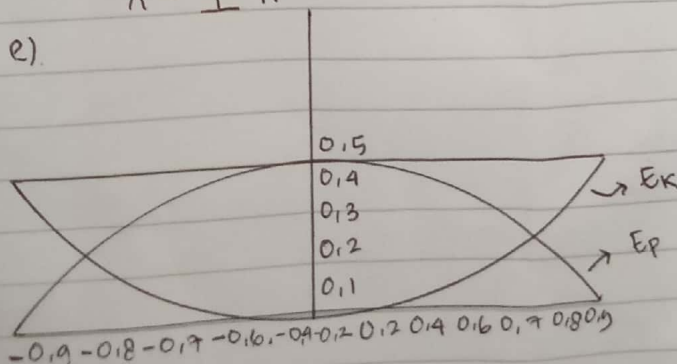
$$T(x) = \frac{1}{2} kA^2 - \frac{1}{2} kx^2$$

$$d). E = Vc x$$

$$\frac{1}{2} kA^2 = \frac{1}{2} kx^2$$

$$x = \pm A$$

e).



Hasilnya: a) $V(x) = \frac{kx^2}{2}$

$$b) T(x) = \frac{k}{2} (A^2 - x^2)$$

$$c) E = T_0 = \frac{1}{2} kA^2$$

PAPERLINE

$$2.8 \quad V = \frac{b}{x^3}$$

$$F = m \frac{dv}{dt} = m \frac{dv}{dx} \cdot \frac{dx}{dt} \\ = m v \frac{dv}{dx}$$

$$\frac{dv}{dx} = \frac{-3b}{x^4}$$

$$F = m \left(\frac{b}{x^3} \right) \left(\frac{-3b}{x^4} \right) = -3 \frac{mb^2}{x^7}$$

$$2.11 \quad F(v) = -Cv^{3/2}$$

$$= m \frac{dv}{dt} = m \frac{dv}{dx} \cdot \frac{dx}{dt} \\ = m \cdot \frac{dv}{dx}$$

$$-Cv^{3/2} = m \frac{dv}{dx}$$

$$\int_0^x dx = \int_{v_0}^v \frac{-m v}{C v^{3/2}} dv$$

$$x = -\frac{m}{C} \int_{v_0}^v \frac{dv}{\sqrt{v}} = -\frac{m}{C} \int_{v_0}^v v^{-1/2} dv$$

$$= -\frac{m}{C} \left. \frac{v^{1/2}}{1/2} \right|_{v_0}^v$$

$$= \frac{2m}{C} (\sqrt{v_0} - \sqrt{v})$$

Partikel berhenti, maka $v=0$, jadi

$$x_{\max} = \frac{2m\sqrt{v_0}}{C}$$

$$2.12 \quad m - a_x = \sum F_x$$

$$m \cdot a_x = -mg - C_2 v^2$$

$$a_x = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dx} \cdot v$$

$$m \cdot \frac{dv}{dx} = -mg - C_2 v^2$$

$$v \cdot \frac{dv}{dx} = -g - \frac{C_2}{m} v^2$$

$$\int_{v_0}^v \frac{V dv}{-g - kV^2} = \int_0^x dx$$

$$y = -g - kV^2$$

$$dy = -2kV dv$$

$$V dv = -\frac{1}{2k} dy$$

$$\int_{v_0}^v \frac{V dv}{-2ky} = \int_0^x dx$$

$$-\frac{1}{2k} \ln(y) \Big|_{v_0}^v = x \Big|_0^x$$

$$-\frac{1}{2k} \ln(-g - kV^2) \Big|_{v_0}^v = x$$

$$-\frac{1}{2k} \ln \left(\frac{-g - kV^2}{-g - kV_0^2} \right) = x$$

$$\frac{-g - kV^2}{-g - kV_0^2} = e^{-2kx}$$

$$-g - kV^2 = (-g - kV_0^2) e^{-2kx}$$

$$kV^2 = (g + kV_0^2) e^{-2kx} - \frac{g}{k}$$

$$A = \frac{g}{k} + V_0^2$$

$$V^2 = A e^{-2kx} - \frac{g}{k}$$

$$m a_x = \Sigma F_x$$

$$m \cdot a_x = -mg + C_2 V^2$$

$$a_x = \frac{dv}{dt} = \frac{dv}{dt} \frac{dx}{dt} = \frac{dv}{dx} \cdot v$$

$$m v \frac{dv}{dx} = -g + \frac{C_2}{m} v^2$$

$$k = \frac{C_2}{m}$$

$$V \frac{dv}{dx} = -g + kV^2$$

$$\int_0^v \frac{V dv}{-g + kV^2} = \int_0^x dx$$

$$y = -g + kV^2$$

$$dy = 2kV dv$$

$$v dv = \frac{1}{2k} dy$$

$$\int_0^v \frac{1}{2k} \frac{dy}{y} = \int_{x_0}^x dx$$

$$\frac{1}{2k} \ln(y) \Big|_0^v = x \Big|_{x_0}^x$$

$$\frac{1}{2k} \ln(-g + kv^2) \Big|_0^v = x - x_0$$

$$\frac{1}{2k} \ln \left(\frac{-g + kv^2}{-g} \right) = x - x_0$$

$$\frac{-g + kv^2}{-g} = e^{2k(x-x_0)}$$

$$-g + kv^2 = -ge^{2k(x-x_0)}$$

$$kv^2 = -ge^{2k(x-x_0)} + g$$

$$v^2 = \frac{-g}{k} e^{2kx_0} e^{2kx} + \frac{g}{k}$$

$$B = \frac{g}{k} e^{2kx_0}$$

$$v^2 = \frac{g}{k} - Be^{2kx}$$

$$2.13 \quad v^2 = \left(\frac{g}{k} + v_0^2 \right) e^{-2kx} - \frac{g}{k} \quad (\text{keatas})$$

$$v^2 = \frac{-g}{k} e^{-2kx_0} e^{2kx} + \frac{g}{k} \quad (\text{kebawah})$$

$v=0$ pada gerak keatas

$$0 = \left(\frac{g}{k} + v_0^2 \right) e^{-2kx_0} - \frac{g}{k}$$

$$e^{-2kx_0} = \frac{g/k}{g/k + v_0^2}$$

h adalah posisi awal untuk gerakan kebawah dan

$(+) a=0$,

$$v^2 = \frac{-g}{k} e^{-2kh} e^{2kx} + \frac{g}{k} \quad \text{Sub } x=0$$

$$v^2 = \frac{-g}{k} \cdot \frac{g/k}{g/k + v_0^2} \cdot 1 + \frac{g}{k}$$

$$v_t^2 = g/k$$

$$V^2 = \frac{V_t^2 \cdot V_0^2}{V_t^2 + V_0^2} + V_t^2$$

$$V^2 = V_t^2 \left(\frac{V_t^2 + V_0^2}{V_t^2 + V_0^2} \right)$$

$$V^2 = V_t^2 \left(\frac{V_t^2 + V_0^2 - V_t^2}{V_t^2 + V_0^2} \right)$$

$$V^2 = \frac{V_t^2 V_0^2}{V_t^2 + V_0^2}$$

$$V = \frac{V_t V_0}{\sqrt{V_t^2 + V_0^2}}$$

$$2.14 \quad \text{Max} = \sum F_x$$

$$\text{Max} = -Kx^{-2}$$

$$a_x = \frac{dv}{dt} = \frac{dv}{dt} \frac{dx}{dt} = \frac{dv}{dx} \cdot v$$

$$mv \frac{dv}{dx} = -Kx^{-2}$$

$$v \frac{dv}{dx} = \frac{-K}{m} x^{-2}$$

$$\int_0^v v dv = \frac{-K}{m} \int_b^x x^{-2} dx$$

$$\frac{v^2}{2} \Big|_0^v = \frac{-K}{m} (-1) x^{-1} \Big|_b^x$$

$$\frac{v^2}{2} = \frac{K}{m} (x^{-1} - b^{-1})$$

$$v^2 = \frac{2K}{m} \left(\frac{b-x}{bx} \right)$$

$$v = \frac{dx}{dt}$$

$$\sqrt{\frac{2K}{m} \left(\frac{b-x}{m} \right)} = \frac{dx}{dt}$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2K}{m} \left(\frac{b-x}{bx} \right)}} = \frac{dx}{dt}$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2K}{m} \left(\frac{b-x}{bx} \right)}} = \sqrt{\frac{mb}{2K}} \int_b^0 \sqrt{\frac{x}{b-x}} dx$$

$$u = \frac{\sqrt{b-x}}{\sqrt{x}}$$

$$dv = \frac{1}{2\sqrt{b-x}} (-1) \cdot \sqrt{x} dx - \frac{1}{2\sqrt{x}}$$

$$dx = \frac{du}{\dots}$$

$$= \frac{1}{-2\sqrt{x} - \sqrt{x}} - \frac{v\sqrt{x}}{2x^{3/2}}$$

$$= \frac{1/2\sqrt{x} - v/2x}{du}$$

$$= -\frac{1+v^2}{2xv}$$

$$= -\frac{2xv dv}{v^2+1}$$

$$\sqrt{b-x} = v\sqrt{x}$$

$$b-x = v^2 x$$

$$b = x^2(v^2+1)$$

$$x = \frac{b}{v^2+1}$$

$$dx = -\frac{2bv dv}{(v^2+1)^2}$$

$$\sqrt{\frac{mb}{2k}} \int_b^0 \sqrt{\frac{x}{b-x}} dx = \sqrt{\frac{mb}{2k}} \int_b^0 \frac{1}{v} \frac{(-1) 2bv dv}{(v^2+1)^2}$$

$$\int \frac{dv}{(av^2+b)^n} = \frac{2n-3}{2b(n-1)} \int \frac{dv}{(av^2+b)^{n-1}} + \frac{v}{2b(n-1)(av^2+b)^{n-1}}$$

$$= \frac{u}{2(v^2+1)} + \frac{1}{2} \int \frac{dv}{v^2+1}$$

$$\int_b^0 \frac{dv}{(v^2+1)^2} = \frac{v}{2(v^2+1)} \Big|_b^0 + \frac{1}{2} \int_b^0 \frac{dv}{v^2+1}$$

$$\int_b^0 \frac{dv}{(v^2+1)^2} = \frac{u}{2(v^2+1)} \Big|_b^0 + \frac{1}{2} \arctan v \Big|_b^0$$

$$-\sqrt{\frac{mb}{2k}} \int_b^0 \frac{2bv dv}{(v^2+1)^2} = -2b \sqrt{\frac{mb}{2k}} \left(\frac{v}{2(v^2+1)} \Big|_b^0 + \frac{1}{2} \arctan u \Big|_b^0 \right)$$

$$= -2b \sqrt{\frac{mb}{2k}} \left(\frac{v}{2(v^2+1)} \right) \Big|_0^{-\infty} + \frac{1}{2} \arctan v \Big|_0^{-\infty}$$

$$= -2b \sqrt{\frac{mb}{2k}} \left(0 - 0 + \frac{1}{2} \cdot \left(\frac{\pi}{2} - 0 \right) \right)$$

$$= \pi \sqrt{\frac{mb^3}{8k}}$$

$$\pi \sqrt{\frac{mb^3}{8k}} = \int_0^t dt$$

$$t = \pi \sqrt{\frac{mb^3}{8k}}$$

2.15 $Ma = \leq F$

$$Ma = mg - C_1 V - C_2 V^2$$

$$Ma = 0$$

$$mg - C_1 V - C_2 V^2 = 0$$

$$C_2 V^2 - C_1 V - mg = 0$$

$$V_{1,2} = \frac{-C_1 \pm \sqrt{C_1^2 + 4C_2 mg}}{2C_2}$$

Pilih (+)

$$V_1 = \frac{-C_1 + \sqrt{C_1^2 + 4C_2 mg}}{2C_2}$$

$$V_1 = V_t = \frac{-C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2} \right)^2 + \frac{mg}{C_2}}$$

$$V_t = \frac{-C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2} \right)^2 + \frac{mg}{C_2}}$$