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Mekanika

1) a) $F_x = m a_x$

$F_0 + Ct = m \frac{dv_x}{dt}$

$dt(F_0 + Ct) = m\, dv_x$

$\int_0^t (F_0 + Ct)\,dt = m \int_0^{v_x} dv_x$

$\left| F_0 t + \frac{Ct^2}{2} \right|_0^t = m(v_x - 0)$

$F_0 t + \frac{Ct^2}{2} = m(v_x)$

$v_x = \frac{F_0 t}{m} + \frac{Ct^2}{2m}$

$\frac{v_x}{dx} = \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{F_0 t}{m} + \frac{Ct^2}{2m}$

$\int_0^x dx = \int_0^t \left(\frac{F_0 t}{m} + \frac{Ct^2}{2m} \right) dt$

$x - 0 = \frac{F_0 t^2}{2m} \Big|_0^t + \frac{Ct^3}{6m} \Big|_0^t$

$x = \frac{F_0 t}{2m} + \frac{Ct^2}{6t}$

b) $F_x = m a_x$

$F_0 \sin Ct = m \frac{dv_x}{dt}$

$dt\,(F_0 \sin Ct) = m\, dv_x$

$\int_0^t \left(\frac{F_0}{m} \sin(Ct) \right) dt = \int_0^{v_x} dv_x$

$\frac{F_0}{mC} \left(-\cos Ct \right) \Big|_0^t = v_x - 0$

$v_x = \frac{F_0}{mC} (1 - \cos(Ct))$

$v_x = \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{F_0}{mC} (1 - \cos Ct)$

$\int_0^x dx = \int_0^t \frac{F_0}{mC} (1 - \cos Ct)\, dt$

$x - 0 = \left[\frac{F_0 t}{mC} \right]_0^t - \left[\frac{F_0}{mC^2} \sin Ct \right]_0^t$

$x = \frac{F_0 t}{mC} - \frac{F_0}{mC^2} \sin Ct$

c) $F_x = m a_x$

$F_0 e^{CT} = m \frac{dv_x}{dt}$

$\int_0^t F_0 e^{Ct}\, dt = \int_0^{v_x} dv_x$

$\int_0^t \frac{F_0}{m} e^{Ct}\, dt = \int_0^{v_x} dv_x$

$\left[\frac{F_e}{mC} e^{Ct} \right]_0^t = [v_x - 0]$

$v_x = \frac{F_0}{mC} e^{Ct} - \frac{F_0}{mC} = \frac{F_0}{mC} (e^{CT} - 1)$

$\frac{dx}{dt} = \frac{F_0}{mC} (e^{Ct} - 1)$

$\int_0^x dx = \int_0^t \frac{F_0}{mC} (e^{Ct} - 1)\, dt$

$x - 0 = \left[\frac{F_0}{mC} e^{CT} \right]_0^t - \left[\frac{F_0 t}{mC} \right]_0^t$

$x = \frac{F_0}{mC^2} (e^{Ct} - 1) - \frac{F_0 t}{mC}$

2-2. a) $F_x = m a_x$

$$F_0 + Cx = m\frac{dv_x}{dt}$$

$$\frac{dv_x}{dt} = \frac{dv_x}{dt}\cdot\frac{dt}{dx} = \frac{dv_x}{dx}\cdot\frac{d}{dt}$$

$$\frac{dv_x}{dt} = \frac{dv_x}{dv_x}\cdot v_x$$

$$F_0 + Cx = m\frac{dv_x}{dx}\cdot v_x$$

$$\frac{1}{m}\int_0^x F_0 + (x)\,dx = \int_0^{v_x} v_x\,dv_x$$

$$\frac{1}{m}\left[f_0 x \Big|_0^x + \frac{cx^2}{2}\Big|_0^x \right] = \left[\frac{v_x^2}{2}\right]_0^{v_x}$$

$$\frac{1}{m}\left(F_0 x + \frac{cx^2}{2}\right) = \frac{v_x^2}{2}$$

$$v_x^2 = \frac{2}{m}\left(F_0 x + \frac{Cx^2}{2}\right)$$

$$v_x = \sqrt{\frac{2}{m}\left(F_0 x + \frac{Cx^2}{2}\right)}$$

b). $F_x = F_0 e^{-Cx}$

$$F_x = m a_x$$

$$F_0 e^{-Cx} = m\frac{dv_x}{dt}$$

$$\frac{dv_x}{dt} = \frac{dv_x}{dt}\cdot\frac{dx}{dx} = \frac{dv_x}{dx}\cdot\frac{dx}{dt} = \frac{dv_x}{dx}\cdot v_x$$

sehingga $F_0 e^{-Cx} = m\frac{dv_x}{dx}\cdot v_x$

$$\frac{1}{m}F_0 e^{-cx} = \frac{dv_x}{dx}\cdot v_x$$

$$\frac{1}{m}\int_0^x f_0 e^{-cx}\,dx = \int_0^{v_x} v_x\,dv_x$$

$$\frac{1}{m}\left(-\frac{F_0}{c}e^{-cx}\Big|_0^x\right) = \frac{v_x^2}{2}\Big|_0^{v_x}$$

$$\frac{v_x^2}{2} = \frac{F_0}{cm}(1-e^{-cx})$$

$$v_x^2 = \frac{2F_0}{cm}(1-e^{-cx}$$

$$v_x = \sqrt{\frac{2f_0}{cm}(1-e^{-cx})}$$

c) $F_x = m a_x$

$$F_0\cos Cx = m\frac{dv_x}{dt}$$

$$\frac{dv_x}{dt} = \frac{dv_x}{dt}\cdot\frac{dx}{dx} = \frac{dv_x}{dx}\cdot\frac{dx}{dt} = \frac{dv_x}{dx}\cdot v_x$$

$$F_0\cos Cx = m\cdot\frac{dv_x}{dx}\cdot v_x$$

$$F_0\cos Cx = m\cdot\frac{dv_x}{dx}\cdot v_x$$

$$\frac{1}{m}\int_0^x F_0\cos Cx\,dx = \int_0^{v_x} v_x\,dv_x$$

$$\frac{1}{m}\left(-\frac{F_0}{c}\sin cx\Big|_0^x\right) = \frac{v_x^2}{2}\Big|_0^{v_x}$$

$$\frac{v_x^2}{2} = \frac{F_0\sin cx}{cm}$$

$$v_x^2 = 2F_0\sin cx\,/cm$$

$$v_x = \sqrt{\frac{2F_0\sin cx}{cm}}$$

2.3 a) $F_x = F_0 + Cx$

$$V(x) = -\int(F_0 + Cx)\,dx$$

$$V(x) = -\left\{\left(F_0 x + \frac{Cx^2}{2}\right) + c\right\}$$

$$V(x) = -F_0 x - \frac{Cx^2}{2} + c$$

b) $F_x = F_0 e^{-Cx}$

$$V(x) = -\int F(x)$$

$$= -\int (F_0 e^{-Cx})\,dx$$

$$= -\int \frac{F_0 e^{-Cx}}{-c}$$

$$= -\frac{F_0 e^{-Cx}}{-c} + c$$

$$= \frac{F_0 e^{-Cx}}{c} + c$$

c) $F_x = -\int F(x)\,dx$

$$V(x) = -\int(F_0\cos(x))\,dx$$

$$= -\frac{F_0}{c}\sin(x) + c$$

$\boxed{2.4}$ a) Fungsi energi potensial v(x) untuk gaya

$$V(x) = -\int F(x)\,dx$$
$$= -\int (-kx)\,dx$$
$$= \frac{kx^2}{2}$$
$$V(x) = \frac{1}{2}kx^2$$

b) Energi kinetik

$E_k : T_0 = \frac{1}{2}kA^2$

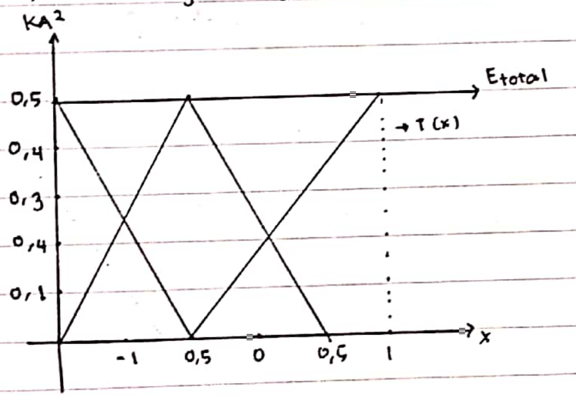
c) Energi total partikel sebagai fungsi posisi

$$E_k = T(x) + V(x)$$
$$T(x) = E_k - V(x)$$
$$= \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$
$$T(x) = \frac{k}{2}(A^2 - x^2)$$

d) titik balik gerakan

$$E = V(x)$$
$$\frac{1}{2}kA^2 = \frac{1}{2}kx^2$$
$$A^2 = x^2$$
$$x = \pm A$$

e) sistem fungsi energi potensial dan total



$\boxed{2.8}$ Ditanya: Gaya yang bekerja pada partikel sebagai fungsi dari x.

Jawaban:

$F = m.a$ $\quad\quad \dot{x} = bx^{-3}$

$= m.\ddot{x}$ $\quad\quad \frac{d\dot{x}}{dx} = -bx^{-4}$

$= m.\dot{x}\frac{d\dot{x}}{dx}$

$$F = m\dot{x}\frac{d\dot{x}}{dx}$$
$$= m(bx^{-3}) . -3bx^{-4}$$
$$= -3\,mb\,bx^{-3} . x^{-4}$$
$$= -3\,mb^2x^{-7}$$

$\boxed{2.11}$ Ditanya: Tunjukkan bahwa balok tidak dapat bergerak lebih jauh dari $2\,mv_0^{3/2}/c$

Jawaban:

$$F = ma$$
$$-cv^{3/2} = ma$$
$$-cv^{3/2} = m\frac{dv}{dt}$$
$$\frac{dv}{dt}.\frac{dx}{dx} = \frac{dv}{dx}.\frac{dx}{dt} = \frac{dv}{dt}.v$$
$$m.\frac{dv}{dx}.v = -cv^{3/2}$$
$$v^{-1/2}\,dv = -\frac{c}{m}\,dx$$
$$\int_{v_0}^{v} v^{-\frac{1}{2}}\,dx = -\frac{c}{m}\int_0^x dx$$
$$2v^{1/2}\Big|_{v_0}^{v} = -\frac{c}{m}x\Big|_0^x$$
$$2(\sqrt{v} - \sqrt{v_0}) = -\frac{cx}{m}$$
$$\sqrt{v_2} = V_0 - \frac{Cx}{2m}$$
$$V = \left(\sqrt{0} - \frac{Cx}{2m}\right)^2$$

Maksimum v dari jarak x, jadi jika temukan derivation dari x:

$$\frac{dv}{dx} = 0$$
$$2\left(\sqrt{v_0} - \frac{cx}{2m}\right)\left(\frac{-c}{2m}\right) = 0$$
$$\sqrt{v_0} - \frac{cx}{2m} = 0$$
$$-\frac{cx}{2m} = -\sqrt{v_0}$$
$$x = \frac{2m}{c}\sqrt{v_0}$$

2.12 $\Sigma F_x = m a_x$

$-mg - C_2 v^2 = m a_x$

$a_x = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} \cdot v$

$mv \frac{dv}{dx} = -mg - C_2 v^2$

$v \cdot \frac{dv}{dx} = -mg - C_2 v^2$

$v \frac{dv}{dx} = -g - \frac{C_2 v^2}{m}$

$k = \frac{C_2}{m}$

$v \frac{dv}{dx} = -g - kv^2$

$\int_{v_0}^{v} \frac{v\,dv}{-g-kv} = \int_0^x dx$

- $y = -g - kv^2$ (substitusi)
- $dy = -2kv\,dv$ (differensial)

$v\,dv = \frac{-1}{2k} dy$

$\int_{v_0}^{v} \frac{-1}{2k} \frac{dy}{y} = \int_0^x dx$ (substitusikan ke y)

$-\frac{1}{2k} \ln(y) \Big|_{v_0}^{v} = x \Big|_0^x$

$-\frac{1}{2k} \ln\left(\frac{-g-kv^2}{-g-kv^2}\right) = x$

$\frac{-g-kv^2}{-g-kv_0^2} = e^{-2kx}$

$-g-kv^2 = (-g-kv_0^2)e^{-2kx}$

$kv^2 = (g+kv_0^2)e^{-2kx} - g$

$v^2 = \left(\frac{g}{k} + v_0^2\right) e^{-2kx} - \frac{g}{k}$

$A = \frac{g}{k} + v_0^2$

$v^2 = Ae^{-2kx} - \frac{g}{k}$

Kemudian kita mencari persamaan gerak kebawah dengan cara yang sama seperti di atas.

$m a_x = -mg + C_2 v^2$

$mv \frac{dv}{dx} = -mg + C_2 v^2$

$v \frac{dv}{dx} = -g + \frac{C_2 v^2}{m}$; $k = \frac{C^2}{m}$

$v \cdot \frac{dv}{dx} = -g + kv^2$

$\int_0^v \frac{v\,dv}{-g+kv^2} = \int_0^x dx$ (diintegralkan)

$y = -g + kv^2$

$dy = 2kv\,dv$

$v\,dv = \frac{1}{2} dy$

$\int_0^v \frac{1}{2}k \frac{dy}{y} = \int_{x_0}^x dx$

$\frac{1}{2} \ln(y) \Big|_0^v = x \Big|_{x_0}^x$

$\frac{1}{2k} \ln(y)(-g+kv^2) \Big|_0^v = x - x_0$

$\frac{1}{2k} \ln\left(\frac{-g+kv^2}{-g}\right) = x - x_0$

$\frac{-g+kv^2}{-g} = e^{2k(x-x_0)}$

$-g + kv^2 = -ge^{2k(x-x_0)}$

$kv^2 = -ge^{2k(x-x_0}$

$v^2 = \frac{-g}{k} e^{-2kx_0} e^{2kx} + \frac{g}{k}$

Misalkan : $B = \frac{g}{k} e^{-2kx_0}$

Maka $v^2 = \frac{g}{k} \cdot -Be^{2kx}$

$\boxed{2.13}$ Solution!

* $v^2 = \frac{g}{k} e^{-2kx_0} \cdot e^{2kx} + \frac{g}{k}$ (Gerak kebawah) ...(1)

* $v^2 = \left(\frac{g}{k} + V_0^2\right) \cdot -e^{-2kx} - \frac{g}{k}$ (Gerak ke-atas) ...(2)

Substitusikan $v=0$ ke persamaan gerak ke atas untuk menemukan nilai tertinggi

$$v^2 = \left(\frac{g}{k} + V_0^2\right) e^{-2kx} - \frac{g}{k}$$

$$0^2 = \left(\frac{g}{k} + V_0^2\right) e^{-2kx} - \frac{g}{k}$$

$$\frac{g}{k} = \left(\frac{g}{k} + V_0^2\right) e^{-2kx}$$

$$e^{-2kx} = \frac{\frac{g}{k}}{\frac{g}{k} + V_0^2} \quad \ldots (3)$$

Jika $x_0 = h$, $x = 0$, maka substitusi ke persamaan gerak ke bawah

$$v^2 = -\frac{g}{k} e^{-2kx_0} e^{2kx} + \frac{g}{k}$$

$$= -\frac{g}{k} e^{-2kh} e^{2k0} + \frac{g}{k}$$

$$= -\frac{g}{k} e^{-2kh} e^{0} + \frac{g}{k}$$

$$v^2 = -\frac{g}{k} e^{-2kh} + \frac{g}{k} \quad \ldots (4)$$

Substitusikan persamaan 3 ke pers. 4

$$v^2 = -\frac{g}{k} e^{-2kh} + \frac{g}{k}$$

$$= -\frac{g}{k} \cdot \frac{\frac{g}{k}}{\frac{g}{k} \cdot V_0^2} + \frac{g}{k} \quad \left(\text{karena } V_t^2 = \frac{g}{k} \text{ Maka}\right)$$

$$v^2 = -V_t^2 \cdot \frac{V_t^2}{V_t^2 \cdot V_0^2} + V_t^2$$

$$= V_t^2 \cdot \left(\frac{1 - V_t^2}{V_t^2 + V_0^2}\right)$$

$$= V_t^2 \left(\frac{V_t^2 + V_0^2 - V_t^2}{V_t^2 + V_0^2}\right)$$

$$v^2 = \frac{V_t^2 \cdot V_0^2}{V_t^2 + V_0^2}$$

$$v = \sqrt{\frac{V_t^2 \cdot V_0^2}{V_t^2 + V_0^2}} \Rightarrow \text{TERBUKTI}$$

$\boxed{2.14}$ Solution

* $F = m \cdot a$

$$m \cdot a_x = \Sigma F(x)$$

$$m \cdot a_x = -kx^{-2} \Rightarrow a_x = \frac{dv}{dt} = \frac{dv}{dt} \cdot \frac{dx}{dt} = \frac{dv}{dt} \cdot v$$

$$m \cdot \frac{dv}{dt} v = -kx^{-2}$$

$$\frac{dv}{dt} v = -kx^{-2}$$

$$\int_0^v v \, dv = -\frac{k}{m} \int_b^x x^{-2} \, dx$$

$$\left.\frac{v^2}{2}\right|_0^v = -\frac{k}{m} \left(\left. -1 x^{-1} \right|_b^x \right)$$

$$\frac{v^2}{2} = \frac{k}{m} \left(x^{-1} - b^{-1}\right)$$

$$v^2 = \frac{2k}{m} \left(\frac{b-x}{bx}\right)$$

$$v = \sqrt{\frac{2k}{m} \cdot \left(\frac{b-x}{bx}\right)}$$

* $v = \frac{dx}{dt}$

$$\frac{dx}{dt} = v$$

$$\frac{dx}{dt} = \sqrt{\frac{2k}{m} \frac{(b-x)}{bx}}$$

$$\int_0^t dt = \int_b^0 \frac{dx}{\left(\frac{2k}{m}\right)\left(\frac{b-x}{bx}\right)}$$

$$\int_b^0 \frac{dx}{\left(\frac{2k}{m}\right)\left(\frac{b-x}{bx}\right)} = \sqrt{\frac{m \cdot b}{2k}} \int_b^0 \sqrt{\frac{x}{b-x}} \, dx$$

Misal : $u = \frac{\sqrt{b-x}}{\sqrt{x}}$

$$du = \frac{\frac{1}{2\sqrt{b-x}} (-1) - \sqrt{x} \, dx - \frac{1}{2\sqrt{x}} \cdot \sqrt{b \cdot x} \, dx}{x}$$

$$du = \frac{-dx}{2\sqrt{x} \cdot \sqrt{b-x}} \cdot \frac{\sqrt{b-x}}{2x^{3/2}} dx$$

$$dx = \frac{du}{\frac{1}{-2\sqrt{x} \cdot \sqrt{b-x}} - \frac{\sqrt{b-x}}{2x^{3/2}}} \Rightarrow \sqrt{b-x} = u\sqrt{x}$$

$$dx = \frac{du}{-\frac{1}{2ux} - \frac{u\sqrt{x}}{2x^{3/2}}}$$

$$dx = \frac{du}{-\frac{1}{2ux} - \frac{u}{2x}}$$

$$dx = \frac{\frac{du}{1+u^2}}{2xu}$$

$$dx = \frac{-2xu\ du}{u^2+1}$$

$* \sqrt{b-x} = u\sqrt{x}$

$$b-x = u^2x$$

$$b = \frac{b}{u^2+1}$$

$$x = \frac{b}{u^2+1}$$

$$dx = \frac{-2x\ udu}{u^2+1}$$

$$dx = \frac{-2bu\ du}{(u^2+1)^2}$$

$$\sqrt{\frac{Mb}{2k}} \int_b^0 \sqrt{\frac{x}{b-x}}\ dx = \sqrt{\frac{mb}{2k}} \int_b^0 \frac{1}{u} (-1)\ \frac{2\ budu}{(u^2+1)^2}$$

$$= \sqrt{\frac{mb}{2k}} \int_b^0 -\frac{2b\,du}{(u+1)^2}$$

Mereduksi $\int_b^0 \frac{du}{(u^2+1)^2}$

$$\int \frac{du}{(au^2+b)^n} = \frac{2n-3}{2b(n-1)} \int \frac{du}{(au^2+b)^{n-1}} + \frac{u}{2b(n-1)(au^2+b)^{n-1}}$$

$$= \frac{u}{2(u^2+1)} + \frac{1}{2} \int \frac{du}{u^2+1}$$

substitusikan $a=1$, $b=1$, $n=2$

$$\int_b^0 \frac{du}{(u^2+1)^2} = \frac{u}{2(u^2+1)}\Big|_b^0 + \frac{1}{2}\int_b^0 \frac{du}{u^2+1}$$

$$= \frac{u}{2(u^2+1)}\Big|_b^0 + \frac{1}{2} \arctan u$$

$$-\sqrt{\frac{mb}{2k}} \int_b^0 \frac{2b\,du}{(u^2+1)^2} = 2b\sqrt{\frac{mb}{2k}} \left(\frac{u}{2(u^2+1)}\right)_b^0 + \frac{1}{2} \arctan u \Big|_b^0$$

$$= -2b\sqrt{\frac{mb}{2k}} \left(0 - 0 + \frac{1}{2}\left(-\frac{\pi}{2} - 0\right)\right)$$

$$= \pi \sqrt{\frac{mb^2}{8k}}$$

$$\pi\sqrt{\frac{mb^2}{8k}} = \int_0^t dt$$

$$\pi\sqrt{\frac{mb^3}{8k}} = t - 0$$

$$= \pi\sqrt{\frac{mb^3}{8k}}$$

$$t = \pi\left(\frac{mb^3}{8k}\right)^{\frac{1}{2}} \Rightarrow \text{(TERBUKTI)}$$

2.15 Solution !

$ma = \Sigma F$

$ma = mg - C_1 V - C_2 V^2$ (gaya drag)

$0 = mg - C_1 V - C_2 V^2$ ($V_0 = 0$)

$C_2 V^2 + C_1 V_1 - mg = 0$ ($a = C_2$, $b = C_1$, $c = m.g$)

$$V_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{ac}$$

$$= \frac{C_1 \pm \sqrt{C_1^2 + 4C_2 mg}}{2C_2}$$

pilih yang tanda positif cause negatif bukan solusi fisik

$$V_1 = \frac{-C_1 + \sqrt{C_1^2 + 4C_1 mg}}{2C_2}$$

$$V_1 = \frac{-C}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right) + \frac{mg}{C_2}} \Rightarrow V_1 = V_t$$

$$V_t = \frac{-C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C_2}}$$

$$V_t = \left[\left(\frac{mg}{2}\right) + \left(\frac{C_1}{2C_2}\right)^2\right]^{\frac{1}{2}} - \frac{C_1}{2C_2}$$

$\Downarrow$

TERBUKTI