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Kelas : B

Prodi : Pendidikan fisika

Tugas Mekanika

2.1 Find the velocity $\dot{x}$ and the position x as functions of the time t for a particle of mass m, which strarts from rest at x = 0 and t = 0, subject to the following force functions.

a. $f_x = f_0 + ct$

b. $f_x = f_0 \sin ct$

c. $f_x = f_0 e^{ct}$

Where f_0 and c are positive constants

Answer :

=> a). $\ddot{x} = \frac{1}{m}(f_0 + ct)$

$$\dot{x} = \int_0^x \frac{1}{m}(f + ct)dt = \frac{f_0}{m}t + \frac{c}{2m}t^2$$

$$x = \int_0^x \left(\frac{f_0}{m}t + \frac{c}{2m}t^2\right)dt = \frac{f}{m}t^2 + \frac{c}{6m}t^3$$

=> b). $\ddot{x} = \frac{f}{m}\sin ct$

$$\dot{x} = \int_0^x \frac{f_0}{m}\sin ct\ dt = \frac{f_0}{cm}\cos ct\Big|_0^t = \frac{f_0}{cm}(1-\cos ct)$$

$$x = \int_0^x \frac{f_0}{cm}(1-\cos ct)dt = \frac{f_0}{cm}\left(1-\frac{1}{c}\sin ct\right)$$

=> c). $\ddot{x} = \frac{f_0}{m}e^{ct}$

$$\dot{x} = \frac{f_0}{cm}e^{ct}\Big|_0^t = \frac{f_0}{cm}(e^{ct}-1)$$

$$x = \frac{f_0}{cm}\left(\frac{1}{c}e^{ct} - \frac{1}{c} - 1\right)$$

$$= \frac{f_0}{c^2m}(e^{ct} - 1 - ct)$$

2.2) a) $\ddot{x} = \frac{d\dot{x}}{dt} = \frac{d\dot{x}}{dx} \cdot \frac{dx}{dt} = \dot{x}\frac{d\dot{x}}{dx}$

$$\dot{x}\frac{d\dot{x}}{dx} = \frac{1}{m}(f_0 + cx)$$

$$\dot{x}\frac{d\dot{x}}{dx} = \frac{1}{m}(f + cx)dx$$

$$\frac{1}{2}\dot{x}^2 = \frac{1}{m}\left(f_0 x + \frac{cx^2}{2}\right)$$

$$\dot{x} = \left(\frac{x}{m}(2f_0 + cx)\right)^{\frac{1}{2}}$$

b) $\ddot{x} = \dot{x}\frac{d\dot{x}}{dx} = \frac{1}{m} f_0 e^{-cx}$

$$\dot{x}d\dot{x} = \frac{f_0}{cm}(e^{-cx} - 1) = \frac{f_0}{cm}(1 - e^{-cx})$$

$$\frac{1}{2}\dot{x}^2 = \frac{f_0}{cm}(e^{-cx} - 1) \cdot \frac{f_0}{cm}(1 - e^{-cx})$$

$$\dot{x} = \left[\frac{2f_0}{cm}(1 - e^{-cx})\right]^{\frac{1}{2}}$$

c) $\ddot{x} = \dot{x}\frac{d\dot{x}}{dx} = \frac{1}{m}(f \cos cx)$

$$\dot{x}d\dot{x} = \frac{f_0}{m}\cos cx\, dx$$

$$\frac{1}{2}\dot{x}^2 = \frac{f_0}{cm}\sin cx$$

$$\dot{x} = \left(\frac{2f_0}{cm}\sin cx\right)^{\frac{1}{2}}$$

2.3) a) $V(x) = -\int_{x_0}^{x} (f_0 + cx)\,dx$

$$= f_0 x - \frac{cx^2}{2} + C$$

b) $V(x) = -\int_{x_0}^{x} f_0 e^{-cx} dx$

$$= \frac{f_0}{c}\sin cx + C$$

c) $V(x) = -\int_{x}^{x} f_0 \cos cx\, dx$

$$= \frac{f_0}{c}\sin cx + C$$

2.4 a) $V(x) = -\int F(x)\,dx$

$V(x) = -\int(-kx)\,dx$

$V(x) = \frac{kx^2}{2}$

b) dan c

$E = T_0 = \frac{1}{2}kA^2$

$= T(x) + V(x)$

$T(x) = E - V(x)$

$= \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$= \frac{k}{2}(A^2 - x^2)$

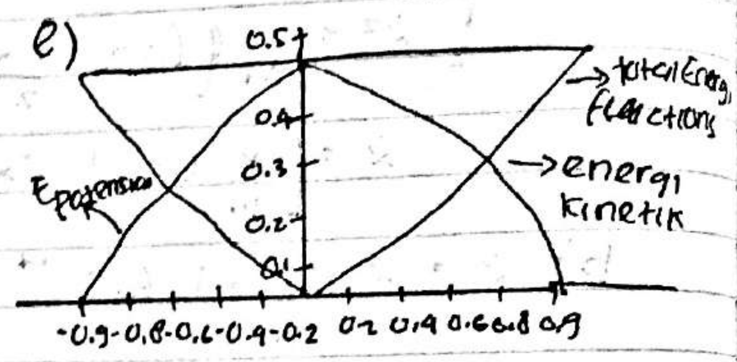
C). $E = T_0 = \frac{1}{2}kA^2$

d) $E = V(x)$

$\frac{1}{2}kA^2 = \frac{1}{2}kx^2$

$x = \pm A$

e)



2.8) $V = \frac{b}{x^3}$

$F = m\frac{du}{dt} = m\frac{dv}{dt}\cdot\frac{dx}{dt}$

$= mu\frac{dv}{dx}$

$\frac{dv}{dx} = -\frac{3c}{x^4}$

$F = m\left(\frac{b}{x^3}\right)\left(\frac{-3b}{x^4}\right)$

$= -3mb^2/x^7$

2.11) $a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = v\cdot\frac{dv}{dx} = -\frac{c}{m}v^{\frac{3}{2}}$

$= v^{-\frac{1}{2}}\,dv = -\frac{c}{m}\,dx$

$= \int_{v}^{v} v^{-\frac{1}{2}}\,dx = \int_{0}^{x} -\frac{c}{m}\,dx$

$= -2V_0^{\frac{1}{2}} = -\frac{c}{m}\,x\,max$

$x_{max} = \frac{2mv_0^{\frac{1}{2}}}{c}$

2.12) $M a_x = \Sigma f_x = \Sigma f_x$

$M a_x = -mg - C_2 v^2$

$a_x = \frac{dv}{dx} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dt} = v$

$m \frac{dv}{dx} = -mg = C_2 v^2$

$v \frac{dv}{dx} = -g - \frac{C_2}{m} v^2$

$K = \frac{C_2}{m}$

$\int_{v_0}^{v} \frac{v\,dv}{-g-Kv^2} = \int_0^x dx$

$y = -g - Kv^2$

$dy = -2Kv\,dv$

$v\,dv = -\frac{1}{2K} dy$

$\int_{v_0}^{v} -\frac{dy}{2Ky} = \int_0^x dx$

$-\frac{1}{2}K \ln(-g-Kv^2)\Big|_{v_0}^{v} = x$

$-\frac{1}{2K} \ln\left(\frac{-g-Kv^2}{-g-Kv_0^2}\right) = x$

$\frac{-g-Kv^2}{-g-Kv_0^2} = e^{-2Kx}$

$-g-Kv^2 = (-g-Kv_0^2)e^{-2Kx}$

$Kv^2 = (g+Kv_0^2)e^{-2Kx} - \frac{g}{K}$

$A = \frac{g}{K} + v_0^2$

$v^2 = Ae^{-2Kx} - \frac{g}{K}$

$Ma_x = \Sigma f_x$

$M a_x = -mg + C_2 v^2$

$a_x = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} \cdot v$

$mv \frac{dv}{dx} = -mg + C_2 v^2$

$\frac{dv}{dx} = -g + \frac{C_2}{m} \cdot v^2$

$K = \frac{C_2}{m}$

$v \frac{dv}{dx} = -g + Kv^2$

$\int_0^v \frac{v\,dv}{-g+Kv^2} = \int_0^x dx$

$y = -g + Kv^2$

$dy = 2Kv\,dv$

$v\,dv = \frac{1}{2}K\,dy$

$\int_0^v \frac{1}{2}K \frac{dy}{y} = \int_{x_0}^{x} dx$

$\frac{1}{2K} \ln(y)\Big|_0^v = x\Big|_{x_0}^{x}$

$\frac{1}{2K} \ln(-g+Kv^2)\Big|_0^v = x - x_0$

$\frac{1}{2K} \ln\left(\frac{-g+Kv^2}{-g}\right) = x - x_0$

$\frac{-g+Kv^2}{g} = e^{2K(x+x_0)}$

$-g + Kv^2 = -ge^{2K(x-x_0)}$

$Kv^2 = -ge^{2K(x-x_0)}$

$v^2 = -\frac{g}{K} e^{2Kx_0} e^{2Kx} + \frac{g}{K}$

$B = \frac{g}{K} e^{2Kx_0}$

$v^2 = \frac{g}{K} - Be^{2Kx}$

2.13) $V^2 = \left(\frac{g}{K} + V_0^2\right) e^{-2Kx} - \frac{g}{K}$ ke atas

$U^2 = \frac{-g}{K} \cdot e^{-2Kx_0} e^{2Kx} + \frac{g}{K}$ kebawah

$V = 0$ gerak ke atas

$0 = \left(\frac{g}{K} + V_0^2\right) e^{-2Kx} - \frac{g}{K}$

$e^{-2Kx} = \frac{\frac{g}{K}}{\frac{g}{K} + V_0^2}$

h posisi awal untuk gerakan ke bawah (+) $a = 0$

$V^2 = \frac{-g}{K} \cdot e^{-2Kh} e^{2K0} + \frac{g}{K}$ subsitusi $x = 0$

$V^2 = \frac{-g}{K} \cdot \frac{\frac{g}{K}}{\frac{g}{K} + V_0^2} \cdot 1 + \frac{g}{K}$

$U^2 = \frac{g}{x}$

$= -Vt^2 \cdot \frac{Vt^2}{Vt^2 + V_0^2} + Vt^2$

$= Vt^2 \left(1 - \frac{Vt^2}{Vt^2 + V_0^2}\right)$

$U^2 = \frac{U_t^2 \; V_0^2}{Vt + V_0^2}$

$V = \frac{Vt \; V_0}{\sqrt{Vt^2 + V_0}}$

2.14) $max = \Sigma f x$

$= -Kx^{-2}$

$a_x = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dx} \cdot v$

$mv \frac{dv}{dx} = \frac{-K}{m} \int_0^K x^{-2} \, dx$

$\frac{V^2}{2} = \frac{K}{m} \left(x^{-1} - b^{-1}\right)$

$V^2 = \frac{2K}{m} \left(\frac{b - x}{bx}\right)$

$V = \frac{dx}{dt}$

$$\sqrt{\frac{2K}{m}\left(\frac{b-x}{m}\right)} = \frac{dx}{dt}$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2K}{m}\cdot\left(\frac{b-x}{bx}\right)}} = \int_0^t dt$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2K}{m}\left(\frac{b-x}{bx}\right)}} = \sqrt{\frac{Mb}{2K}} \int_b^0 \sqrt{\frac{x}{b-x}}\, dx$$

$$u = \frac{\sqrt{b-x}}{\sqrt{x}}$$

$$dv = \frac{\frac{1}{2\sqrt{b-x}}(-1)\cdot\sqrt{x}\,dx \cdot \frac{1}{2\sqrt{x}}}{x}$$

$$dx = \frac{dv}{\frac{1}{-2\sqrt{2}\cdot v\sqrt{x}} - \frac{v\sqrt{x}}{2x^{\frac{3}{2}}}}$$

$$= \frac{dv}{-\frac{1}{2\sqrt{x}} - \frac{v}{2x}}$$

$$= \frac{dv}{\frac{1+v2}{2xv}}$$

$$= \sqrt{b-x} = v\sqrt{x}$$

$$b - x = v^2 x$$

$$b = x^2(v^2+1)$$

$$x = \frac{b}{4^2} + 1$$

$$dx = -\frac{2bv\,dv}{(v^2+1)^2}$$

$$\sqrt{\frac{mb}{2K}} \int_b^0 \sqrt{\frac{x}{b-x}}\,dx = \sqrt{\frac{mb}{2K}} \int_b f(x) \frac{2bv\,dv}{(v^2+1)^2}$$

$$= \sqrt{\frac{mb}{2K}} \int_b^0 \frac{2bv\,dv}{(v^2+1)^2}$$

$$\int \frac{dv}{(av+b)^n} = \frac{2n-3}{2b(n-1)} \int \frac{dv}{(av^2+b)^{n+1}} + \frac{v}{2b(n-1)(av^2+b)^{n-1}}$$

$$= \frac{u}{2(v^2+1)} + \frac{1}{2}\int \frac{dv}{v^2+1}$$

$$\int_b^0 \frac{dv}{(v^2+1)^2} = \frac{v}{2(v^2+1)}\Big|_b^0 + \frac{1}{2}\arctan v\Big|_b^0$$

$$-\sqrt{\frac{mb}{2K}} \int_b^0 \frac{2bdv}{(v^2+1)^2} = -2b\sqrt{\frac{mb}{2K}}\left(\frac{v}{2(v^2+1)}\Big|_b^0 + \frac{1}{2}\arctan u\Big|_b^0\right)$$

$$= -2b\sqrt{\frac{mb}{2K}}\left(\frac{v}{2(v^2+1)}\Big|_0^{-\infty} + \frac{1}{2}\arctan v\Big|_0^{-\infty}\right)$$

$$= -2b\sqrt{\frac{mb}{2K}}\left(0 - 0 + \frac{1}{2}\cdot\left(-\frac{\pi}{2} - 0\right)\right) = \pi\sqrt{\frac{mb^2}{8K}}$$

$$\pi\sqrt{\frac{mb^3}{8K}} = \int_0^t dt \qquad \boxed{t = \pi\sqrt{\frac{mb^3}{8K}}}$$

2.15) $ma = \Sigma F$

$$ma = mg - C_1 v - C_2 v^2$$

$$mg - C_1 v - C_2 v^2 = 0$$

$$C_2 v^2 - C_1 v - mg = 0$$

$$v_{1,2} = \frac{-C \pm \sqrt{C_1^2 + 4C_2 mg}}{2C_2}$$

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$$v_1 = \frac{-C_1 + \sqrt{C_1^2 + 4C_2 mg}}{2C^2}$$

$$v_1 = v_t = \frac{-C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C_2}}$$

$$v_t = \frac{-C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C_2}}$$